SPACELIKE FACTORABLE SURFACES IN FOUR-DIMENSIONAL MINKOWSKI SPACE

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Abstract. In the current work, we study factorable surfaces in Minkowski four space. We describe such surfaces in terms of their Gaussian and mean curvature functions. We classify flat and minimal spacelike factorable surfaces in $\mathbb{E}^4_1$.

1. Introduction

In $\mathbb{E}^4_1$, the Lorentzian inner product is defined by
$$\langle u, v \rangle = -u_0v_0 + u_1v_1 + u_2v_2 + u_3v_3$$
for all $u, v \in \mathbb{E}^4_1$. A surface $M : F = F(s, t) : (s, t) \in D (D \subset \mathbb{E}^2)$ in $\mathbb{E}^4_1$ is said to be spacelike if $\langle \cdot, \cdot \rangle$ induces a Riemannian metric on $M$. Therefore, we know the following decomposition at each point $p$ of a spacelike surface $M$:
$$\mathbb{E}^4_1 = T_pM \oplus T^\perp_p M.$$ 

The Levi-Civita connections on $M$ and $\mathbb{E}^4_1$ are represented by $\nabla$ and $\nabla$, respectively. Let $X_1$ and $X_2$ be tangent vector fields and $\eta$ be a normal vector field of $M$. $\nabla X_1 \eta$ and $\nabla X_1 X_2$ are separated into tangential and normal components by the Weingarten and Gauss formulas;
$$\nabla X_1 \eta = -A_\eta X_1 + D X_1 \eta,$$
$$\nabla X_1 X_2 = \nabla X_1 X_2 + h(X_1, X_2).$$

Thus, these formulas introduce the second fundamental tensor $h$ and the shape operator $A_\eta$ corresponding to $\eta$ [3].

Denote $H$ the mean curvature vector field of $M$, then $H = \frac{1}{2}trh$. Consequently, we have $H = \frac{1}{2}((h(X_1, X_1) + h(X_2, X_2))$ with respect to a local orthonormal frame $\{X_1, X_2\}$.

Let $M : F = F(s, t) : (s, t) \in D (D \subset \mathbb{E}^2)$ be a local parametrization on a spacelike surface in Minkowski 4-space. In accordance with $\langle F_s, F_s \rangle > 0, \langle F_t, F_t \rangle > 0$, we have $\langle F_s, F_t \rangle > 0$.

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Moreover, the second fundamental tensor can be written as linear combinations of the vector fields $F_s$, $F_t$, $\eta_1$, $\eta_2$:

\begin{align*}
\nabla_{F_s} F_s &= F_{ss} = \Gamma^1_{11} F_s + \Gamma^2_{11} F_t - c^1_{11} \eta_1 + c^2_{11} \eta_2, \\
\nabla_{F_s} F_t &= F_{st} = \Gamma^1_{12} F_s + \Gamma^2_{12} F_t - c^1_{12} \eta_1 + c^2_{12} \eta_2, \\
\nabla_{F_t} F_t &= F_{tt} = \Gamma^1_{22} F_s + \Gamma^2_{22} F_t - c^1_{22} \eta_1 + c^2_{22} \eta_2,
\end{align*}

where \( \{F_s, F_t, \eta_1, \eta_2\} \) is positively oriented in \( \mathbb{E}^4 \), (see, [6]). \( c^k_{ij}, i, j, k = 1, 2 \) are given by

\begin{align*}
\frac{c^1_{11}}{11} &= \langle F_{ss}, \eta_1 \rangle, & \frac{c^1_{12}}{12} &= \langle F_{st}, \eta_1 \rangle, & \frac{c^1_{22}}{22} &= \langle F_{tt}, \eta_1 \rangle, \\
\frac{c^2_{11}}{11} &= \langle F_{ss}, \eta_2 \rangle, & \frac{c^2_{12}}{12} &= \langle F_{st}, \eta_2 \rangle, & \frac{c^2_{22}}{22} &= \langle F_{tt}, \eta_2 \rangle,
\end{align*}

(see, [6]).

The second fundamental tensor \( h \) of \( M \) defined as (see [6])

\begin{align*}
h(F_s, F_s) &= -c^1_{11} \eta_1 + c^2_{11} \eta_2, \\
h(F_s, F_t) &= -c^1_{12} \eta_1 + c^2_{12} \eta_2, \\
h(F_t, F_t) &= -c^1_{22} \eta_1 + c^2_{22} \eta_2.
\end{align*}

Moreover, the second fundamental tensor can be written as

\[ h(X_1, X_2) = -\langle A_{\eta_1} (X_1), X_2 \rangle \eta_1 + \langle A_{\eta_2} (X_1), X_2 \rangle \eta_2. \]

The \( k \)-th component of \( H \) denoted by \( H_k \) is obtained by \( H_k = \langle H, \eta_k \rangle = \frac{\Gamma^k (\eta_k)}{2} \). Hence we get

\[ H_k = \frac{c^k_{11} g - 2 c^k_{12} f + c^k_{22} e}{2(eg - f^2)}. \]

According to the normal basis, the mean curvature vector field \( H \) becomes

\[ H = -H_1 \eta_1 + H_2 \eta_2. \]

Mean curvature function of \( M \) is the norm of the vector \( H \).

Gaussian curvature of a surface \( M : F(s, t) \) can be calculated by using the shape operator matrices as

\[ K = \frac{-\det(A_{\eta_1}) + \det(A_{\eta_2})}{W^2} = \frac{-c^1_{11} c^2_{22} + c^1_{12} c^2_{12} + (c^1_{12})^2 - (c^2_{12})^2}{eg - f^2}. \]

A surface is said to be minimal (flat) if its mean curvature vector (Gaussian curvature) vanishes [4].

Factorable surfaces (also known as homothetical surfaces) in Euclidean and Minkowski 3–spaces can be parametrized locally as \( F(s, t) = (s, t, f(s)g(t)) \), where \( f \) and \( g \) are differentiable functions [10] [11]. Some authors have considered factorable surfaces in Euclidean space and in semi-Euclidean spaces [8] [9] [11] [12].
Van de Woestyne showed that minimal factorable surfaces in \(\mathbb{L}^3\) are helicoids and planes.

In [1], Yu. A. Aminov introduced the surface \(M\) in \(\mathbb{E}^4\) given by
\[
F(s,t) = (s,t,z(s,t),w(s,t)),
\]
where \(z\) and \(w\) are differentiable functions. The representation (1.6) is called a Monge patch. Also, in [2], the authors investigated the curvature properties of these type of surfaces.

In the present study, we consider a spacelike factorable surface in Minkowski 4-space, which can locally be written as a monge patch
\[
F(s,t) = (s,t,f_1(s)g_1(t),f_2(s)g_2(t)),
\]
for some differentiable functions, \(f_i(s), g_i(t), i = 1, 2\). We characterize such surfaces in terms of their Gaussian curvature and mean curvature functions.

### 2. Spacelike factorable surfaces in \(\mathbb{E}_1^4\)

**Definition 2.1.** Let \(M\) be a surface in 4-dimensional Minkowski space \(\mathbb{E}_1^4\). If the surface is given by an explicit form \(z(s,t) = f_1(s)g_1(t)\) and \(w(s,t) = f_2(s)g_2(t)\) where \(s, t, z, w\) are Cartezian coordinates in \(\mathbb{E}_1^4\) and \(f_i, g_i, i \in \{1, 2\}\) are smooth functions, then the surface is called a factorable surface in \(\mathbb{E}_1^4\). Thus, the factorable surface can be written as a monge patch
\[
F(s,t) = (s,t,f_1(s)g_1(t),f_2(s)g_2(t)).
\]

Let \(M\) be a spacelike factorable surface with the parametrization (2.1). We determine a normal frame \(\{\eta_1, \eta_2\}\) such that \(\langle \eta_1, \eta_1 \rangle = -1, \langle \eta_2, \eta_2 \rangle = 1\), and \(\{F_s, F_t, \eta_1, \eta_2\}\) is positively oriented frame in \(\mathbb{E}_1^4\).

The tangent space of \(M\) is spanned by the vector fields
\[
F_s = (1, 0, f_1'(s)g_1(t), f_2'(s)g_2(t)),
F_t = (0, 1, f_1(s)g_1'(t), f_2(s)g_2'(t)).
\]

Thus the coefficients of the first fundamental form of the surface can be expressed as
\[
e = \langle F_s, F_s \rangle = -1 + (f'_1g_1)^2 + (f'_2g_2)^2,
f = \langle F_s, F_t \rangle = f'_1f'_1g_1g_1 + f'_2f'_2g_2g_2,
g = \langle F_t, F_t \rangle = 1 + (f_1g_1')^2 + (f_2g_2')^2,
\]
where \(\langle, \rangle\) is the Lorentzian inner product in \(\mathbb{E}_1^4\). As the surface \(M\) is spacelike, then \(W = \sqrt{eg - f^2}\).

The second partial derivatives of \(F(s,t)\) are
\[
F_{ss} = (0, 0, f''_1(s)g_1(t), f''_2(s)g_2(t)),
F_{st} = (0, 0, f'_1(s)g_1'(t), f'_2(s)g_2'(t)),
F_{tt} = (0, 0, f_1(s)g_1''(t), f_2(s)g_2''(t)).
\]
Further, the normal space of $M : F(s, t)$ is spanned by the orthonormal vector fields

$$\eta_1 = \frac{1}{\sqrt{|A|}}(f'_1(s)g_1(t), -f_1(s)g'_1(t), 1, 0), \quad (2.4)$$

$$\eta_2 = \frac{1}{\sqrt{|AD|}}(Af'_2(s)g_2(t) - Bf'_1(s)g_1(t), Bf_1(s)g'_1(t) - Af_2(s)g'_2(t), -B, A),$$

where

$$A = 1 - \left(f'_1g_1\right)^2 + \left(f_1g'_1\right)^2,$$

$$B = -f'_1f_2g_1g_2 + f_1f'_2g'_1,$$

$$C = 1 - \left(f'_2g_2\right)^2 + \left(f_2g'_2\right)^2,$$

$$D = AC - B^2. \quad (2.5)$$

Since $M$ is spacelike surface in $\mathbb{E}^4$ with respect to chosen orthonormal frame, $A$ and $D$ are negative definite. Using $(2.3)$ and $(2.4)$, one can find the coefficient functions of the second fundamental form as follows;

$$c_{11}^1 = \frac{f''_1g_1}{\sqrt{|A|}}, \quad c_{22}^1 = \frac{f'_1g''_1}{\sqrt{|A|}}$$

$$c_{12}^1 = \frac{f'_1g'_1}{\sqrt{|A|}}, \quad c_{12}^2 = \frac{Af'_2g'_2 - Bf'_1g'_1}{\sqrt{|AD|}}$$

$$c_{11}^2 = \frac{Af''_2g_2 - Bf''_1g_1}{\sqrt{|AD|}}, \quad c_{22}^2 = \frac{Af''_2g_2 - Bf''_1g_1}{\sqrt{|AD|}}. \quad (2.6)$$

Using Gram-Schmidt orthonormalization method for the spacelike vector fields $F_s$ and $F_t$, we get orthonormal tangent vectors

$$X_1 = \frac{F_s}{\sqrt{\epsilon}},$$

$$X_2 = \frac{\sqrt{\epsilon}}{W} \left(F_t - \frac{f}{\epsilon} F_s\right). \quad (2.7)$$

By the use of $(1.3)$, $(1.4)$, $(1.5)$ and $(2.7)$ the second fundamental tensors $A_{\eta k}$ become

$$A_{\eta 1} = \frac{1}{\epsilon \sqrt{|A|}} \begin{pmatrix} f''_1g_1 & f'_1g'_1e - f_1g_1f' \\ f'_1g'_1e - f_1g_1f' & f_1g''_1e^2 - 2f'_1g'_1ef + f_1g_1f^2 \end{pmatrix},$$

and

$$A_{\eta 2} = \frac{1}{\epsilon \sqrt{|AD|}} \begin{pmatrix} \lambda & \frac{\mu e - M}{W} \\ \frac{\mu e - M}{W} & \frac{\delta^2 - 2\mu ef + \lambda f^2}{W^2} \end{pmatrix},$$
where

\[\begin{align*}
\lambda &= A f''_2 g_2 - B f'_2 g_3, \\
\mu &= A f'_2 g'_2 - B f'_1 g'_3, \\
\delta &= A f_2 g''_2 - B f_1 g''_1.
\end{align*}\]

2.1. Flat factorable surfaces.

**Theorem 2.2.** Let \( M \) be a spacelike factorable surface in \( E^4 \). Then the Gaussian curvature of the surface is given by

\[
K = \frac{(f'_1 f_3 g'''_1 - f''_1 g''_1) C - (f'_2 f_3 g'''_1 + f''_1 f_3 g''_1 - 2 f'_1 f'_2 g''_1) B + (f''_1 f_2 g''_2 - f''_2 g''_2) A}{D W^2}.
\]

**Corollary 2.3.** Let \( M \) be a spacelike factorable surface in Minkowski 4-space. If \( M \) is given by one of the following parametrizations, then it is a flat surface:

1. \( F(s, t) = (s, t, a_1 g_1(t), a_2 g_2(t)) \),
2. \( F(s, t) = (s, t, b_1 f_1(s), b_2 f_2(s)) \),
3. \( F(s, t) = (s, t, a_1 g_1(t), a_2 f_2(s)) \),
4. \( F(s, t) = (s, t, b_1 f_1(s), b_2 g_2(t)) \),
5. \( F(s, t) = (s, t, a_1 b_1, \exp(a_2 s + b_2) \exp(a_3 t + b_3)) \),
6. \( F(s, t) = \left(s, t, a_1 b_1, (a_2 s + b_2) \frac{1}{a_1} (a_3 t + b_3) \right) \),
7. \( F(s, t) = \left(s, t, \exp(a_1 s + b_1) \exp(a_2 t + b_2), \exp(a_3 s + b_3) \exp(a_4 a_3 t + b_4) \right) \),
8. \( F(s, t) = (s, t, f_1(s) \cos t, f_1(s) \sin t) \),

the function \( f_1(s) \) satisfies

\[
s = \pm \int \sqrt{\frac{a_1 f_1^2(s) + 1}{f_1^2(s) + 1}} df_1(s)
\]

where \( i, j = 1, 2, i \neq j \) and \( a_k, b_k, k = 1, ..., 4 \) are real constants.

**Proof.** Let \( M \) be a spacelike factorable surface given with the parametrization (2.1) in \( E^4 \).

If \( f'_1(s) = 0 \), \( f'_2(s) = 0 \) or \( g'_1(t) = 0 \), \( g'_2(t) = 0 \) or \( f'_1(s) = 0 \), \( g'_1(t) = 0 \), then we obtain the cases (1), (2), (3) and (4).

If \( f'_1(s) = 0 \), \( g'_1(t) = 0 \), then we have

\[
f''_1 f_2 g''_2 g_2 - f''_2 g''_2 = 0. \tag{2.8}
\]

Let \( p(s) = \frac{df_2}{ds} \) and \( q(t) = \frac{dp}{dt} \). By the use of (2.8), we can write

\[
f_2(s) p(s) \frac{dp}{df_2} g_2(t) q(t) \frac{dq}{dg_2} - (p(s) q(t))^2 = 0. \tag{2.9}
\]

If \( p(s) \neq 0 \), \( q(t) \neq 0 \), from (2.9), we get

\[
f_2(s) \frac{dp}{df_2} g_2(t) \frac{dq}{dg_2} = p(s) q(t).
\]

Then we have differential equation

\[
\frac{f_2(s)}{p(s)} \frac{dp}{df_2} = \frac{q(t)}{g_2(t) \frac{dq}{dg_2}} = \lambda, \tag{2.10}
\]
where \( \lambda \) is constant.

(1) If \( \lambda = 1 \), from (2.10) we have
\[
\begin{align*}
f_2(s) &= \exp(a_2s + b_2), \\
g_2(t) &= \exp(a_3t + b_3),
\end{align*}
\]
which gives the case (5).

(2) If \( \lambda \neq 1 \) from (2.10) we have
\[
\begin{align*}
f_2(s) &= \frac{a_2s + b_2}{\lambda}, \\
g_2(t) &= \frac{a_3t + b_3}{\lambda},
\end{align*}
\]
which gives the case (6).

Further, we assume \( f_1'' f_1 g''_i g_i - f_1^2 g''_2 = 0 \) holds for \( i = 1 \) and \( i = 2 \). Then we get
\[
\begin{align*}
f_1(s) &= \exp(a_1s + b_1), \\
g_1(t) &= \exp(a_2t + b_2),
\end{align*}
\]
Substituting these functions into \( B = 0 \) and \( f_1'' f_2 g_1 g_2' + f_1 f_2' g_1' g_2' - 2 f_1' f_2' g_1' g_2 = 0 \), we have \( a_4 = \frac{a_2a_1}{a_1} \), \( i, j = 1, 2 \) (\( i \neq j \)) which vanish the Gaussian curvature of the surface. Thus, we obtain the case (7).

Also, if \( f_1(s) = f_2(s) \) and \( g_1(t) = \cos t, g_2(t) = \sin t \), then by the use of the previous theorem, for a flat surface we get
\[
-f_i''(s)f_i(s) \left( f_i''(s) + 1 \right) + (f_i')(s)^2 \left( (f_i'(s))^2 - 1 \right) = 0.
\]
By the solution of this differential equation we obtain the case (8). \( \square \)

2.2. Minimal factorable surfaces.

**Theorem 2.4.** Let \( M \) be a spacelike factorable surface in \( \mathbb{E}_4^1 \). Then the mean curvature vector of the surface is given by
\[
\overline{H} = \frac{f_1'' g_1 g + f_2'' g_2 e - 2f_1' g_1' f}{\sqrt{|A|w^2}},
\]
where \( A = f_1'' g_1 g + f_2'' g_2 e - 2f_1' g_1' f \), \( H = -H_1, \) and \( H_2, \) for a minimal surface, \( H_1 = 0, H_2 = 0. \) By the use of the previous theorem, we get (2.14). The converse statement is trivial. \( \square \)

**Corollary 2.6.** Let \( M \) be a spacelike factorable surface in Minkowski 4-space. If \( M \) is given by one of the following parametrizations, then it is a minimal surface:

1. \( F(s,t) = (s, t, (a_1s + a_2) b_1, (a_3s + a_4) b_2), \)
2. \( F(s,t) = (s, t, a_1 (b_1t + b_2), a_2 (b_3t + b_4)), \)
3. \( F(s,t) = (s, t, (a_1s + a_2) b_1, a_3 (b_3t + b_4)), \)
4. \( F(s,t) = \left( s, t, a_1b_1, (s + a_2) \frac{1-\exp(b_2t+b_3)}{1+\exp(b_2t+b_3)} \right), \)
5. \( F(s,t) = (s, t, a_1b_1, \tan(a_2s + a_3) (t + b_2)), \)
(6) \( F(s, t) = \left( s, t, \frac{-1 - a_1^2 + \exp(\pm 2a_1(s + a_2))}{\exp(\pm 2a_1(s + a_2))} \cos t, \frac{-1 - a_1^2 + \exp(\pm 2a_1(s + a_2))}{\exp(\pm 2a_1(s + a_2))} \sin t \right) \).

(7) \( F(s, t) = \left( s, t, (s + a_1) \frac{-1 - \exp(b_1 t + b_2)}{-1 + \exp(b_1 t + b_2)}, (s + a_1) \frac{-1 - \exp(b_1 t + b_2)}{-1 + \exp(b_1 t + b_2)} \right) \).

(8) \( F(s, t) = (s, t, \tan(a_1 s + a_2) (t + b_1), \tan(a_1 s + a_2) (t + b_1)) \).

(9) \( F(s, t) = (s, t, a_1 b_1, f_2(s) g_2(t)) \).

(10) \( F(s, t) = (s, t, f_1(s) g_1(t), f_1(s) g_1(t)) \),
the functions \( f_i(s), g_i(t), i = 1, 2 \) satisfy the equations

\[
\begin{align*}
\frac{df_i(s)}{\sqrt{2m \ln f_i(s) + a_1}}, \quad & t = \frac{dg_i(t)}{\sqrt{a_2 g_i^4(t) - \frac{n}{2}}} , \\
\text{or} \quad \frac{df_i(s)}{\sqrt{a_1 f_i^4(s) - \frac{m}{2}}}, \quad & t = \frac{dg_i(t)}{\sqrt{2 \ln g_i(t) + a_2}} , \\
\text{or} \quad \frac{df_i(s)}{\sqrt{a_1 f_i^2(1+c)(s) - a_2}}, \quad & t = \frac{dg_i(t)}{\sqrt{a_3 g_i^{2(1-c)}(t) - a_4}} ,
\end{align*}
\]

where \( c, m, n, a_k, b_k, k = 1, \ldots, 4 \) are real constants and \( c \neq \pm 1 \).

**Proof.** Let \( M \) be a spacelike factorable surface with the parametrization (2.1) in \( E^4_1 \). By the use of (2.14) with (2.2),

\[
f_i' g_i \left( 1 + f_i^2 g_i^2 + f_2 g_2^2 \right) + f_i g_i' \left( -1 + f_i^2 g_i^2 + f_2 g_2^2 \right) - 2 f_i g_i' (f_i' f_1 g_1 + f_2' f_2 g_2) = 0, \quad (2.15)
\]

holds for \( i = 1, 2 \). If \( g_1^2(t) = 0, g_2^2(t) = 0 \) or \( f_1^2(s) = 0, f_2^2(s) = 0 \), we obtain the cases (1) and (2), respectively.

If \( f_2(t) = 0, g_1(t) = 0, i, j = 1, 2, i \neq j \), then

\[
\begin{align*}
f_1' g_1 \left( 1 + f_1^2 g_1^2 + f_2^2 g_2^2 \right) &= 0, \quad (2.16) \\
f_2 g_2' \left( -1 + f_1^2 g_1^2 + f_2^2 g_2^2 \right) &= 0. \quad (2.17)
\end{align*}
\]

Since the first fundamental forms \( e \) and \( g \) are positive, then we get \( f_1''(s) = 0 \) and \( g_2'(t) = 0 \) which congruent the case (3).

If \( f_1'(s) = 0, g_1'(t) = 0 \), from the equality (2.15) for \( i = 2 \), we get

\[
\frac{f_2''(s)}{f_2(s)} - \frac{g_2'(t)}{g_2(t)} + (f_2''(s) f_2(s) - f_2'(s)) g_2'(t) + (g_2'(t) g_2(t) - g_2'(t)) f_2''(s) = 0. \quad (2.18)
\]

If \( f_2''(s) = 0 \) or \( g_2'(t) = 0 \) in (2.18), we obtain the cases (4) and (5).

If \( f_2''(s) g_2'(t) \neq 0 \) in (2.18), differentiating (2.18) with respect to \( s \) and \( t \), we have

\[
\frac{\left( f_2''(s) f_2(s) - f_2'(s) \right)'}{(f_2'(s))^2} = - \frac{(g_2'(t) g_2(t) - g_2'(t))'}{(g_2'(t))^2} = c. \quad (2.19)
\]

Thus, we can write

\[
\begin{align*}
f_2''(s) f_2(s) - (1 + c) f_2'(s) = m, \\
g_2'(t) g_2(t) - (1 - c) g_2'(t) = n. \quad (2.20)
\end{align*}
\]
If \( c = 1, c = -1 \) and \( c \neq \pm 1 \), then from the solution of (2.20), we obtain the case (9).

If \( f_1(s) = f_2(s) \) and \( g_1(t) = \cos t, g_2(t) = \sin t \), then we get

\[
f_i''(s) \left( 1 + f_i^2(s) \right) - f_i(s) \left( 1 + (f'_i(s))^2 \right) = 0.
\]

By the solution of this differential equation we obtain the case (6).

If \( f_1(s) = f_2(s), g_1(t) = g_2(t) \) in (2.15), then for \( i = 1 \) or \( i = 2 \), we find

\[
\frac{f''_i(s)}{f_i(s)} - \frac{g''_i(t)}{g_i(t)} + (f''_i(s)f_i(s) - f'^2_i(s)) 2g'^2_i(t) + (g''_i(t)g_i(t) - g'^2_i(t)) 2f'^2_i(s) = 0. \tag{2.21}
\]

If \( f''_i(s) = 0 \) or \( g''_i(t) = 0 \) in (2.21), we obtain the cases (7) and (8). Also, if \( f''_i(s)g''_i(t) \neq 0 \), we obtain the case (10), which completes the proof. \( \square \)

**Example 2.7.** By selecting \( a_1 = 1, b_1 = 2, b_2 = 0 \) for the case (7) in Corollary 2.6, we can plot the projection of this surface with mapple command:

\[
\text{plot3d}([s, t, z + w], s = a..b, t = c..d). \tag{2.22}
\]

![Figure 1. 3D Model of the surface given by the case (7) in Corollary 2.6](image)
References


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