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COEFFICIENT ESTIMATES FOR SOME SUBCLASSES OF M-FOLD SYMMETRIC BI-UNIVALENT FUNCTIONS

H. M. Srivastava, S. Gaboury, F. Ghanim

Abstract. In the present investigation, we consider two new general subclasses $\mathcal{H}_{\Sigma_m}(\tau,\gamma;\alpha)$ and $\mathcal{H}_{\Sigma_m}(\tau,\gamma;\beta)$ of Σ_m consisting of analytic and m-fold symmetric bi-univalent functions in the open unit disk $\mathbb{U}$. For functions belonging to the two classes introduced here, we derive estimates on the initial coefficients $|a_{m+1}|$ and $|a_{2m+1}|$. Several related classes are also considered and connections to earlier known results are made.

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1. Introduction, Definitions and Preliminaries

Let $\mathcal{A}$ denote the class of functions $f(z)$ normalized by

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1}$$

which are analytic in the open unit disk

$$\mathbb{U} = \{z: \ z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

We denote by $\mathcal{S}$ the class of all functions $f(z) \in \mathcal{A}$ which are univalent in $\mathbb{U}$ [3, 11, 16]. Some of the important and well-investigated subclasses of the univalent function class $\mathcal{S}$ include the class $\mathcal{S}^*(\alpha)$ of starlike functions of order α in $\mathbb{U}$ and the class $\mathcal{K}(\alpha)$ of convex functions of order α in $\mathbb{U}$.

It is well-known that every function $f(z) \in \mathcal{S}$ has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U})$$

and

$$f^{-1}(f(w)) = w \qquad \left(|w| < r_0(f);\ r_0(f) \geqq \frac{1}{4}\right).$$

The inverse function f^{-1} may analytically continued to $\mathbb{U}$ as follows:

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots . \tag{2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{U}$. We denote by Σ the class of bi-univalent functions in $\mathbb{U}$ given by (1).

For each function $f \in \mathcal{S}$, the function

$$h(z) = \sqrt[m]{f(z^m)} \qquad (z \in \mathbb{U};\ m \in \mathbb{N}) \tag{3}$$

is univalent and maps the unit disk $\mathbb{U}$ into a region with m-fold symmetry. A function is said to be m-fold symmetric (see [7, 10]) if it has the following normalized form:

$$f(z) = z + \sum_{k=1}^{\infty} a_{mk+1} z^{mk+1} \qquad (z \in \mathbb{U};\ m \in \mathbb{N}). \tag{4}$$

We denote by S_m the class of m-fold symmetric univalent functions in $\mathbb{U}$, which are normalized by the series expansion (4). The functions in the class $\mathcal{S}$ are said to be *one*-fold symmetric.

Each bi-univalent function generates an m-fold symmetric bi-univalent function for each integer $m \in \mathbb{N}$. The normalized form of f is given as in (4) and the series expansion for f^{-1}, which has been recently proven by Srivastava *et al.* [17], is given as follows:

$$\begin{aligned} g(w) =& w - a_{m+1} w^{m+1} + \left[(m+1)a_{m+1}^2 - a_{2m+1}\right] w^{2m+1} \\ & - \left[\frac{1}{2}(m+1)(3m+2)a_{m+1}^3 - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}\right] w^{3m+1} + \cdots , \end{aligned} \tag{5}$$

where $f^{-1} = g$. We denote by Σ_m the class of m-fold symmetric bi-univalent functions in $\mathbb{U}$. It is easily seen that for $m = 1$, the formula (5) coincides with the formula (2). Here are some examples of m-fold symmetric bi-univalent functions.

$$\left(\frac{z^m}{1 - z^m}\right)^{\frac{1}{m}}, \quad \left[\frac{1}{2}\log\left(\frac{1 + z^m}{1 - z^m}\right)\right]^{\frac{1}{m}} \quad \text{and} \quad \left[-\log(1 - z^m)\right]^{\frac{1}{m}}$$

with the corresponding inverse functions

$$\left(\frac{w^m}{1-w^m}\right)^{\frac{1}{m}}, \quad \left(\frac{\mathrm{e}^{2w^m}-1}{\mathrm{e}^{2w^m}+1}\right)^{\frac{1}{m}} \quad \text{and} \quad \left(\frac{\mathrm{e}^{w^m}-1}{\mathrm{e}^{w^m}}\right)^{\frac{1}{m}},$$

respectively.

In 1967, Lewin [8] investigated the class Σ and showed that $|a_2| < 1.51$. Subsequently, Brannan and Clunie [1] conjectured that $|a_2| \leqq \sqrt{2}$. Afterwards in 1981, Styer and Wright [18] showed that there exist functions $f(z) \in \Sigma$ for which $|a_2| > \frac{4}{3}$. The best known estimate for functions in Σ has been obtained in 1984 by Tan [19], that is, $|a_2| \leqq 1.485$. The coefficient estimate problem involving the bound of $|a_n|$ $(n \in \mathbb{N} \setminus \{1,2\})$ for each $f \in \Sigma$ given by (4) is still an open problem.

Recently, many researchers [5, 6, 9, 13, 14, 15, 17, 20, 21], following the work of Brannan and Taha [2], introduced and investigated a lot of interesting subclasses of the bi-univalent function class Σ and they obtained non-sharp estimates of the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$.

In this paper, we derive estimates on the initial coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ for functions belonging to the new general subclasses $\mathcal{H}_{\Sigma_m}(\tau,\gamma;\alpha)$ and $\mathcal{H}_{\Sigma_m}(\tau,\gamma;\beta)$ of Σ_m. Several related classes are also considered and connections to earlier known results are made. These two new subclasses $\mathcal{H}_{\Sigma_m}(\tau,\gamma;\alpha)$ and $\mathcal{H}_{\Sigma_m}(\tau,\gamma;\beta)$ are defined as follows:

Definition 1. *A function $f(z) \in \Sigma_m$ given by* (4) *is said to be in the class $\mathcal{H}_{\Sigma_m}(\tau,\gamma;\alpha)$ if the following conditions are satisfied:*

$$\left|\arg\left(1+\frac{1}{\tau}\left[f'(z)+\gamma z f''(z)-1\right]\right)\right| < \frac{\alpha\pi}{2} \qquad (z \in \mathbb{U}) \tag{6}$$

and

$$\left|\arg\left(1+\frac{1}{\tau}\left[g'(w)+\gamma w g''(w)-1\right]\right)\right| < \frac{\alpha\pi}{2} \qquad (w \in \mathbb{U}) \tag{7}$$

$$\left(0<\alpha \leqq 1;\ \tau \in \mathbb{C}\setminus\{0\};\ 0 \leqq \gamma \leqq 1\right),$$

and where the function $g = f^{-1}$ is given by (5).

Definition 2. *A function $f(z) \in \Sigma_m$ given by* (4) *is said to be in the class $\mathcal{H}_{\Sigma_m}(\tau,\gamma;\beta)$ if the following conditions are satisfied:*

$$\Re\left(1+\frac{1}{\tau}\left[f'(z)+\gamma z f''(z)-1\right]\right) > \beta \qquad (z \in \mathbb{U}) \tag{8}$$

and

$$\Re\left(1+\frac{1}{\tau}\left[g'(w)+\gamma w g''(w)-1\right]\right)>\beta \qquad (w\in\mathbb{U}) \tag{9}$$

$$\left(0\leqq\beta<1;\ \tau\in\mathbb{C}\setminus\{0\};\ 0\leqq\gamma\leqq 1\right),$$

and where the function $g=f^{-1}$ *is given by* (5).

The following lemma [3] will be required in order to derive our main results.

Lemma 1. *If* $h\in\mathcal{P}$*, then* $|c_k|\leqq 2$ *for each* $k\in\mathbb{N}$*, where* $\mathcal{P}$ *is the family of all functions* h*, analytic in* $\mathbb{U}$*, for which*

$$\Re\left(h(z)\right)>0, \qquad (z\in\mathbb{U}),$$

where

$$h(z)=1+c_1z+c_2z^2+\cdots \qquad (z\in\mathbb{U}).$$

2. Coefficient Bounds for the functions class $\mathcal{H}_{\Sigma_m}(\tau,\gamma;\alpha)$

We begin this section by finding the estimates on the coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ for functions in the class $\mathcal{H}_{\Sigma_m}(\tau,\gamma;\alpha)$.

Theorem 2. *Let* $f(z)\in\mathcal{H}_{\Sigma_m}(\tau,\gamma;\alpha)$ $(0<\alpha\leqq 1;\ \tau\in\mathbb{C}\setminus\{0\};\ 0\leqq\gamma\leqq 1)$ *be of the form* (4)*. Then*

$$|a_{m+1}|\leqq\frac{2\alpha|\tau|}{\sqrt{\left|\tau\alpha(m+1)(2m+1)(2\gamma m+1)+(1-\alpha)(m+1)^2(\gamma m+1)^2\right|}} \tag{10}$$

and

$$|a_{2m+1}|\leqq\frac{2\alpha^2|\tau|^2}{(m+1)(\gamma m+1)^2}+\frac{2\alpha|\tau|}{(2m+1)(2\gamma m+1)}. \tag{11}$$

Proof. It follows from (6) and (7) that

$$1+\frac{1}{\tau}\left[f'(z)+\gamma z f''(z)-1\right]=[p(z)]^{\alpha} \tag{12}$$

and

$$1+\frac{1}{\tau}\left[g'(w)+\gamma w g''(w)-1\right]=[q(w)]^{\alpha}, \tag{13}$$

where the functions $p(z)$ and $q(w)$ are in $\mathcal{P}$ and have the following series representations:

$$p(z)=1+p_m z^m+p_{2m}z^{2m}+p_{3m}z^{3m}+\cdots \tag{14}$$

and

$$q(w)=1+q_m w^m+q_{2m}w^{2m}+q_{3m}w^{3m}+\cdots. \tag{15}$$

Now, equating the coefficients in (12) and (13), we obtain

$$\frac{(m+1)(\gamma m+1)}{\tau}\,a_{m+1}=\alpha p_m, \tag{16}$$

$$\frac{(2m+1)(2\gamma m+1)}{\tau}\,a_{2m+1}=\alpha p_{2m}+\frac{1}{2}\alpha(\alpha-1)p_m^2, \tag{17}$$

$$-\frac{(m+1)(\gamma m+1)}{\tau}\,a_{m+1}=\alpha q_m, \tag{18}$$

and

$$\frac{(2m+1)(2\gamma m+1)}{\tau}\,\left[(m+1)a_{m+1}^2-a_{2m+1}\right]=\alpha q_{2m}+\frac{1}{2}\alpha(\alpha-1)q_m^2. \tag{19}$$

From (16) and (18), we find

$$p_m=-q_m \tag{20}$$

and

$$2\,\frac{(m+1)^2(\gamma m+1)^2}{\tau^2}\,a_{m+1}^2=\alpha^2(p_m^2+q_m^2). \tag{21}$$

From (17), (19) and (21), we get

$$\begin{aligned}\frac{(2m+1)(2\gamma m+1)}{\tau}\,(m+1)\,a_{m+1}^2\\ =\alpha(p_{2m}+q_{2m})+\frac{\alpha(\alpha-1)}{2}\,(p_m^2+q_m^2)\end{aligned}$$

$$= \alpha(p_{2m} + q_{2m}) + \frac{(\alpha - 1)}{\alpha} \frac{(m+1)^2(\gamma m + 1)^2}{\tau^2}\, a_{m+1}^2. \qquad (22)$$

Therefore, we have

$$a_{m+1}^2 = \frac{\tau^2 \alpha^2 (p_{2m} + q_{2m})}{[\tau\alpha(m+1)(2m+1)(2\gamma m+1) + (1-\alpha)(m+1)^2(\gamma m+1)^2]}. \qquad (23)$$

Applying Lemma 1 for the coefficients p_{2m} and q_{2m}, we have

$$|a_{m+1}| \leqq \frac{2\alpha|\tau|}{\sqrt{\left|\tau\alpha(m+1)(2m+1)(2\gamma m+1) + (1-\alpha)(m+1)^2(\gamma m+1)^2\right|}}. \qquad (24)$$

This gives the desired bound for $|a_{m+1}|$ as asserted in (10).

In order to find the bound on $|a_{2m+1}|$, by subtracting (19) from (17), we get

$$2\, \frac{(2m+1)(2\gamma m+1)}{\tau}\, a_{2m+1} - \frac{(2m+1)(2\gamma m+1)}{\tau}\, (m+1)\, a_{m+1}^2$$

$$= \alpha(p_{2m} - q_{2m}) + \frac{\alpha(\alpha-1)}{2} \left(p_m^2 - q_m^2\right). \qquad (25)$$

It follows from (20) and (25) that

$$a_{2m+1} = \frac{\alpha^2\tau^2(p_m^2 + q_m^2)}{4(m+1)(\gamma m+1)^2} + \frac{\alpha\tau(p_{2m} - q_{2m})}{2(2m+1)(2\gamma m+1)}. \qquad (26)$$

Applying Lemma 1 once again for the coefficients p_m, p_{2m}, q_m and q_{2m}, we readily obtain

$$|a_{2m+1}| \leqq \frac{2\alpha^2|\tau|^2}{(m+1)(\gamma m+1)^2} + \frac{2\alpha|\tau|}{(2m+1)(2\gamma m+1)}. \qquad (27)$$

3. Coefficient Bounds for the functions class $\mathcal{H}_{\Sigma_m}(\tau, \gamma; \beta)$

This section is devoted to find the estimates on the coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ for functions in the class $\mathcal{H}_{\Sigma_m}(\tau, \gamma; \beta)$.

Theorem 3. *Let* $f(z) \in \mathcal{H}_{\Sigma_m}(\tau,\gamma;\beta)$ $(0 \leqq \beta \leqq 1;\ \tau \in \mathbb{C}\setminus\{0\};\ 0 \leqq \gamma \leqq 1)$ *be of the form* (4)*. Then*

$$|a_{m+1}| \leqq \sqrt{\frac{4|\tau|(1-\beta)}{(m+1)(2m+1)(2\gamma m+1)}} \tag{28}$$

and

$$|a_{2m+1}| \leqq \frac{2|\tau|^2(1-\beta)^2}{(m+1)(\gamma m+1)^2} + \frac{2|\tau|(1-\beta)}{(2m+1)(2\gamma m+1)}. \tag{29}$$

Proof. It follows from (8) and (9) that there exist $p, q \in \mathcal{P}$ such that

$$1 + \frac{1}{\tau}\left[f'(z) + \gamma z f''(z) - 1\right] = \beta + (1-\beta)p(z) \tag{30}$$

and

$$1 + \frac{1}{\tau}\left[g'(w) + \gamma w g''(w) - 1\right] = \beta + (1-\beta)q(w), \tag{31}$$

where $p(z)$ and $q(w)$ have the forms (14) and (15), respectively. By suitably comparing coefficients in (30) and (31), we get

$$\frac{(m+1)(\gamma m+1)}{\tau}\, a_{m+1} = (1-\beta)p_m, \tag{32}$$

$$\frac{(2m+1)(2\gamma m+1)}{\tau}\, a_{2m+1} = (1-\beta)p_{2m}, \tag{33}$$

$$-\frac{(m+1)(\gamma m+1)}{\tau}\, a_{m+1} = (1-\beta)q_m \tag{34}$$

and

$$\frac{(2m+1)(2\gamma m+1)}{\tau}\, \left[(m+1)a_{m+1}^2 - a_{2m+1}\right] = (1-\beta)q_{2m}. \tag{35}$$

From (32) and (34), we find

$$p_m = -q_m \tag{36}$$

and

$$2\,\frac{(m+1)^2(\gamma m+1)^2}{\tau^2}\, a_{m+1}^2 = (1-\beta)^2(p_m^2 + q_m^2). \tag{37}$$

Adding (33) and (35), we have

$$\frac{(2m+1)(2\gamma m+1)}{\tau}\,(m+1)\,a_{m+1}^2=(1-\beta)(p_{2m}+q_{2m}). \tag{38}$$

Applying Lemma 1, we obtain

$$|a_{m+1}|\leqq\sqrt{\frac{4|\tau|(1-\beta)}{(m+1)(2m+1)(2\gamma m+1)}}. \tag{39}$$

This is the bound on $|a_{m+1}|$ asserted in (28).

In order to find the bound on $|a_{2m+1}|$, by subtracting (35) from (33), we get

$$2\,\frac{(2m+1)(2\gamma m+1)}{\tau}\,a_{2m+1}-\frac{(2m+1)(2\gamma m+1)}{\tau}\,(m+1)\,a_{m+1}^2$$

$$=(1-\beta)(p_{2m}-q_{2m})$$

or, equivalently,

$$a_{2m+1}=\frac{(m+1)}{2}\,a_{m+1}^2+\frac{\tau(1-\beta)(p_{2m}-q_{2m})}{2(2m+1)(2\gamma m+1)}. \tag{40}$$

It follows from (36) and (37) that

$$a_{2m+1}=\frac{\tau^2(1-\beta)^2(p_m^2+q_m^2)}{4(m+1)(\gamma m+1)^2}+\frac{\tau(1-\beta)(p_{2m}-q_{2m})}{2(2m+1)(2\gamma m+1)}. \tag{41}$$

Applying Lemma 1 once again for the coefficients p_m, p_{2m}, q_m and q_{2m}, we easily obtain

$$|a_{2m+1}|\leqq\frac{2|\tau|^2(1-\beta)^2}{(m+1)(\gamma m+1)^2}+\frac{2|\tau|(1-\beta)}{(2m+1)(2\gamma m+1)}. \tag{42}$$

4. Applications of the main results

For *one*-fold symmetric bi-univalent functions and for $\tau=1$, Theorem 1 and Theorem 2 reduce to Corollary 1 and Corollary 2, respectively, which were proven very recently by Frasin [4] (see also [12]).

Corollary 4. *Let* $f(z) \in \mathcal{H}_\Sigma(\alpha,\gamma)$ $(0 < \alpha \leqq 1;\ 0 \leqq \gamma \leqq 1)$ *be of the form* (1). *Then*

$$|a_2| \leqq \frac{2\alpha}{\sqrt{2(\alpha+2)+4\gamma(\alpha+\gamma+2-\alpha\gamma)}} \tag{43}$$

and

$$|a_3| \leqq \frac{\alpha^2}{(\gamma+1)^2} + \frac{2\alpha}{3(2\gamma+1)}. \tag{44}$$

Corollary 5. *Let* $f(z) \in \mathcal{H}_\Sigma(\beta,\gamma)$ $(0 < \alpha \leqq 1;\ 0 \leqq \gamma \leqq 1)$ *be of the form* (1). *Then*

$$|a_2| \leqq \sqrt{\frac{2(1-\beta)}{3(2\gamma+1)}} \tag{45}$$

and

$$|a_3| \leqq \frac{(1-\beta)^2}{(\gamma+1)^2} + \frac{2(1-\beta)}{3(2\gamma+1)}. \tag{46}$$

The classes $\mathcal{H}_\Sigma(\alpha,\gamma)$ and $\mathcal{H}_\Sigma(\beta,\gamma)$ are defined in the following way:

Definition 3. *A function* $f(z) \in \Sigma$ *given by* (1) *is said to be in the class* $\mathcal{H}_\Sigma(\alpha,\gamma)$ *if the following conditions are satisfied:*

$$\left|\arg\left(f'(z)+\gamma z f''(z)\right)\right| < \frac{\alpha\pi}{2} \qquad (z \in \mathbb{U}) \tag{47}$$

and

$$\left|\arg\left(g'(w)+\gamma w g''(w)\right)\right| < \frac{\alpha\pi}{2} \qquad (w \in \mathbb{U}) \tag{48}$$

$$\left(0 < \alpha \leqq 1;\ 0 \leqq \gamma \leqq 1\right),$$

and where the function $g = f^{-1}$ *is given by* (2).

Definition 4. *A function* $f(z) \in \Sigma$ *given by* (1) *is said to be in the class* $\mathcal{H}_\Sigma(\beta,\gamma)$ *if the following conditions are satisfied:*

$$\Re\left(f'(z)+\gamma z f''(z)\right) > \beta \qquad (z \in \mathbb{U}) \tag{49}$$

and

$$\Re\left(g'(w)+\gamma w g''(w)\right) > \beta \qquad (w \in \mathbb{U}) \tag{50}$$

$$\left(0 \leqq \beta < 1;\ 0 \leqq \gamma \leqq 1\right),$$

and where the function $g = f^{-1}$ *is given by* (2).

If we set $\gamma = 0$ and $\tau = 1$ in Theorem 1 and Theorem 2, then the classes $\mathcal{H}_{\Sigma_m}(\tau, \gamma; \alpha)$ and $\mathcal{H}_{\Sigma_m}(\tau, \gamma; \beta)$ reduce to the classes $\mathcal{H}^{\alpha}_{\Sigma_m}$ and $\mathcal{H}^{\beta}_{\Sigma_m}$ investigated recently by Srivastava *et al.* [17] and thus, we obtain the following corollaries:

Corollary 6. *Let* $f(z) \in \mathcal{H}^{\alpha}_{\Sigma_m}$ $(0 < \alpha \leqq 1)$ *be of the form* (4). *Then*

$$|a_{m+1}| \leqq \frac{2\alpha}{\sqrt{(m+1)(\alpha m + m + 1)}} \tag{51}$$

and

$$|a_{2m+1}| \leqq \frac{2\alpha(2\alpha m + \alpha + m + 1)}{(m+1)(2m+1)}. \tag{52}$$

Corollary 7. *Let* $f(z) \in \mathcal{H}^{\beta}_{\Sigma_m}$ $(0 \leqq \beta \leqq 1)$ *be of the form* (4). *Then*

$$|a_{m+1}| \leqq 2\sqrt{\frac{(1-\beta)}{(m+1)(2m+1)}} \tag{53}$$

and

$$|a_{2m+1}| \leqq 2(1-\beta)\left(\frac{(1-\beta)(2m+1) + m + 1}{(m+1)(2m+1)}\right). \tag{54}$$

The classes $\mathcal{H}^{\alpha}_{\Sigma_m}$ and $\mathcal{H}^{\beta}_{\Sigma_m}$ are respectively defined as follows:

Definition 5. *A function* $f(z) \in \Sigma_m$ *given by* (4) *is said to be in the class* $\mathcal{H}^{\alpha}_{\Sigma_m}$ *if the following conditions are satisfied:*

$$\left|\arg\left\{f'(z)\right\}\right| < \frac{\alpha\pi}{2} \qquad (z \in \mathbb{U}) \tag{55}$$

and

$$\left|\arg\left\{g'(w)\right\}\right| < \frac{\alpha\pi}{2} \qquad (w \in \mathbb{U}) \tag{56}$$

$$(0 < \alpha \leqq 1),$$

and where the function g *is given by* (5).

Definition 6. *A function* $f(z) \in \Sigma_m$ *given by* (4) *is said to be in the class* $\mathcal{H}^{\beta}_{\Sigma_m}$ *if the following conditions are satisfied:*

$$\Re\left(f'(z)\right) > \beta \qquad (z \in \mathbb{U}) \tag{57}$$

and

$$\Re\left(g'(w)\right) > \beta \qquad (w \in \mathbb{U}) \tag{58}$$

$$(0 \leqq \beta < 1),$$

and where the function g *is given by* (5).

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Hari M. Srivastava
Department of Mathematics and Statistics,
University of Victoria,
Victoria, British Columbia V8W 3R4, Canada
email: *harimsri@math.uvic.ca*

Sébastien Gaboury
Department of Mathematics and Computer Science,
University of Québec at Chicoutimi,
Chicoutimi, Québec G7H 2B1, Canada
email: *s1gabour@uqac.ca*

Firas Ghanim
Department of Mathematics, College of Sciences,
University of Sharjah,
Sharjah, United Arab Emirates
email: *fgahmed@sharjah.ac.ae*