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S_1-PARACOMPACTNESS WITH RESPECT TO AN IDEAL

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Abstract. In this paper, we study S_1-paracompact spaces in ideal topological spaces and give new characterizations of such spaces. Also, we generalize some of its properties in ideal topological spaces. We study subsets and subspaces of $S_1\mathcal{I}-$paracompact spaces and discuss their properties. Also, we investigate the invariants of $S_1\mathcal{I}-$paracompact spaces by functions.

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1. Introduction and Preliminaries

In 2011, Al-Zoubi and Rawashdeh introduced and studied the concept of S_1-paracompact spaces. A space (X, τ) is said to be S_1-*paracompact space* [2] if every semiopen cover of X has a locally finite open refinement. In this paper, we introduce a new class of spaces, called $S_1\mathcal{I}-$*paracompact spaces*. We give some characterizations of these spaces and investigate the relation between $S_1\mathcal{I}-$paracompact spaces and $\mathcal{I}-$paracompact spaces.

The subject of ideals in topological spaces has been studied by Kuratowski [15] and Vaidyanathaswamy [22]. An *ideal* $\mathcal{I}$ on a set X is a nonempty collection of subsets of X which satisfies (i) $A \in \mathcal{I}$ and $B \subset A$ implies $B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\wp(X)$ is the set of all subsets of X, a set operator $()^\star : \wp(X) \to \wp(X)$, called a *local function* [13] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A \subset X$, $A^\star(\mathcal{I}, \tau) = \{x \in X \mid U \cap A \notin \mathcal{I} \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau \mid x \in U\}$. A Kuratowski closure operator $cl^\star()$ for a topology $\tau^\star(\mathcal{I}, \tau)$, called $\star-$*topology*, finer than τ is defined by $cl^\star(A) = A \cup A^\star(\mathcal{I}, \tau)$ [13] and $\beta = \{U - I \mid U \in \tau \text{ and } I \in \mathcal{I}\}$ is a basis for $\tau^\star$ [13]. We simply write $\tau^\star$ for $\tau^\star(\mathcal{I}, \tau)$. If $\mathcal{I}$ is an ideal on X, then $(X, \tau, \mathcal{I})$ is called an ideal space. If $\beta = \tau^\star$, then we say $\mathcal{I}$ is $\tau-$*simple* [13]. A sufficient condition for $\mathcal{I}$ to be simple is the following: for $A \subseteq X$, if for every $a \in A$ there exists $U \in \tau(a)$ such that $U \cap A \in \mathcal{I}$, then $A \in \mathcal{I}$. If $(X, \tau, \mathcal{I})$ satisfies this condition, then τ is said to be *compatible with respect to* $\mathcal{I}$ [13] or $\mathcal{I}$ is said to be $\tau-$*local*, denoted by $\mathcal{I} \sim \tau$. Given a space $(X, \tau, \mathcal{I})$, we say $\mathcal{I}$ is $\tau-$*boundary* [13] or $\tau-$*codense* if $\mathcal{I} \cap \tau = \{\emptyset\}$, that is, each member of $\mathcal{I}$ has empty $\tau-$interior. An ideal $\mathcal{I}$ is *completely codense* [10] if $\mathcal{I} \subset \mathcal{N}$ where $\mathcal{N}$ is the ideal of nowhere dense subsets in (X, τ). An ideal $\mathcal{I}$ is said to be *weakly* $\tau-$*local* [14] if $A^\star = \emptyset$ implies $A \in \mathcal{I}$. $\mathcal{I}$ is called $\tau-$*locally finite* [12] if the union of each $\tau-$locally finite family contained in $\mathcal{I}$ belongs to $\mathcal{I}$.

We always mean a topological space (X, τ) with no separation properties assumed. A subset A is said to be *semiopen* [16], (resp. *regular open*, $\alpha-$*open* [17], *preopen* [7], *semipreopen* [8]) in (X, τ) if $A \subset cl(int(A))$ (resp. $A = int(cl(A))$, $A \subset int(cl(int(A)))$, $A \subset int(cl(A))$, $A \subset cl(int(cl(A)))$). The union of any family of semiopen subsets of (X, τ) is semiopen [16]. The complement of a *semiopen* (resp. *regular open*) set is said to be *semiclosed* [6] (resp. *regular closed*). The *semiclosure* of A, denoted by $scl(A)$ [7] is defined by the intersection of all semiclosed sets containing A. A subset A is said to be *semiregular* [8] if it is both semiopen and semiclosed. The family of all *semiopen* (resp. *semiclosed*, *semiregular*, *regular open*, *regular closed*, *preopen*) sets is denoted by $SO(X)$ (resp. $SC(X)$, $SR(X)$, $RO(X)$, $RC(X)$, $PO(X)$). A space (X, τ) is said to be *extremally disconnected*(E.D) if the closure of every open set in (X, τ) is open. A space (X, τ) is said to be *semiregular* [9] if for each semiclosed set F and each point $x \notin F$, there exist disjoint semiopen sets U and V such that $x \in U$ and $F \subseteq V$. This is equivalent to for each $U \in SO(X)$ and for each $x \in U$, there exists $V \in SO(X)$ such that $x \in V \subseteq scl(V) \subseteq U$. The family of α-sets of a space (X, τ), denoted by τ^α, forms a topology on X, finer than τ. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *irresolute* [7] (resp. *semicontinuous* [16], *strongly semicontinuous* [1]) if the inverse image of every semiopen (resp. open, semiopen) set is semiopen (resp. semiopen, open). A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *presemiopen* [6] if $f(U) \in SO(Y)$ for every $U \in SO(X)$. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *almost open* [23] if $f^{-1}(cl(V)) \subset cl(f^{-1}(V))$ for every open subset V of Y. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *almost-closed* [21] if $f(F)$ is closed in Y for every regular closed set F of X. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *semi-closed* [20] if $f(F)$ is semiclosed in Y for every closed set F of X. A subset S of a space X is said to be $N-$*closed relative to* X *(*$N-$*closed)* [5] if for every cover $\{U_\alpha \mid \alpha \in \triangle\}$ of S by open sets of X, there exists a finite subfamily $\triangle_0$ of $\triangle$ such that $S \subset \bigcup\{int(cl(U_\alpha)) \mid \alpha \in \triangle_0\}$. The following lemmas will be useful in the sequel.

Lemma 1. [2] *Let* (X, τ) *be an E.D. semiregular space. Then*
(a) $SO(X, \tau) = \tau$.

(b) (X,τ) *is regular.*

Lemma 2. (a) *If* A *is an open set in* (X,τ) *and* $B \in SO(X,\tau)$, *then* $A \cap B \in SO(X,\tau)$. [18]
(b) *Let* (A,τ_A) *be a subspace of a space* (X,τ) *and* $B \subset A$. *If* $A \in \tau$ *and* $B \in SO(A,\tau_A)$, *then* $B \in SO(X,\tau)$. [7]

Lemma 3. [17] *For any space* (X,τ), $SO(X,\tau^\alpha) = SO(X,\tau)$.

Lemma 4. [18] *Let* A *and* X_0 *be subsets of* X *such that* $A \subset X_0$ *and* $X_0 \in SO(X)$. *Then* $A \in SO(X)$ *if and only if* $A \in SO(X_0)$.

Lemma 5. [4] *If* $\{A_\alpha \mid \alpha \in \triangle\}$ *is a locally finite family of subsets in a space* X, *and if* $B_\alpha \subset A_\alpha$ *for each* $\alpha \in \triangle$, *then the family* $\{B_\alpha \mid \alpha \in \triangle\}$ *is a locally finite in* X.

Lemma 6. [3] *The union of a finite family of locally finite collection of sets in a space is locally finite family of sets.*

Lemma 7. [19] *Let* $f : X \to Y$ *be almost closed surjection with* $N-$*closed point inverses. If* $\{U_\alpha \mid \alpha \in \triangle\}$ *is a locally finite open cover of* X, *then* $\{f(U_\alpha) \mid \alpha \in \triangle\}$ *is a locally finite cover of* Y.

Lemma 8. [11] *If* $f : X \to Y$ *is a continuous function and* $\mathcal{U} = \{V_\beta \mid \beta \in \triangle\}$ *is locally finite in* Y, *then* $f^{-1}(\mathcal{U}) = \{f^{-1}(V_\beta) \mid \beta \in \triangle\}$ *is locally finite in* X.

Lemma 9. [13] *Let* (X,τ) *be a space with* $\mathcal{I}$ *an ideal on* X. *Then the following are equivalent*
(a) $X = X^\star$,
(b) $\tau \cap \mathcal{I} = \{\emptyset\}$,
(c) *If* $I \in \mathcal{I}$, *then* $int(I) = \emptyset$, *and*
(d) *For every* $U \in \tau$, $U \subset U^\star$.

Lemma 10. [12] $\mathcal{I}$ *is weakly* $\tau-$*local implies* $\mathcal{I}$ *is* $\tau-$*locally finite.*

2. $S_1\mathcal{I}-$PARACOMPACT SPACES

A space $(X,\tau,\mathcal{I})$ is said to be $S_1\mathcal{I}-$*paracompact* (S_1-paracompact modulo $\mathcal{I}$) if for every semiopen cover $\mathcal{U}$ of X, there exist $I \in \mathcal{I}$ and $X-$locally finite $X-$open refinement $\mathcal{V}$ such that $X = \bigcup\{V \mid V \in \mathcal{V}\} \cup I$. A space (X,τ) is said to be S_1-*almost paracompact* if for every semiopen cover $\mathcal{U}$ of X, there exists a $X-$locally finite open refinement $\mathcal{V}$ such that $X = cl(\bigcup\{V \mid V \in \mathcal{V}\})$. A space $(X,\tau,\mathcal{I})$ is said to be $\mathcal{I}-$*paracompact* (*paracompact modulo* $\mathcal{I}$) [12] if and only if every open cover $\mathcal{U}$ of X has a locally finite open refinement $\mathcal{V}$ (not necessarily a cover) such that

$X - \cup\mathcal{V} \in \mathcal{I}$. A collection $\mathcal{V}$ of subsets of X is said to be an $\mathcal{I}-$*cover* [24] of X if $X - \cup\mathcal{V} \in \mathcal{I}$. A space is S_1-paracompact if and only if it is S_1-paracompact modulo $\{\emptyset\}$. Since $\tau \subset SO(X,\tau)$, $S_1\mathcal{I}-$ paracompact implies $\mathcal{I}-$paracompact. Theorem 11 shows that the converse holds only if the space X is E.D and semiregular, the proof of which follows from Lemma 1. In this section, we characterize $S_1\mathcal{I}-$paracompact spaces and investigate the relation between $S_1\mathcal{I}-$paracompact spaces and $\mathcal{I}-$paracompact spaces.

Theorem 11. *Let (X,τ) be an E.D semiregular space. If $(X,\tau,\mathcal{I})$ is $\mathcal{I}-$paracompact, then $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact.*

Proof. By lemma 1, the theorem follows.

Theorem 12. *Let $(X,\tau,\mathcal{I})$ be $S_1\mathcal{I}-$ paracompact space. If $\mathcal{J}$ is an ideal on X with $\mathcal{I} \subset \mathcal{J}$, then $(X,\tau,\mathcal{J})$ is $S_1\mathcal{J}-$paracompact.*

Theorem 13. *Let $(X,\tau,\mathcal{I})$ be an ideal space and $\mathcal{N} \subset \mathcal{I}$. If (X,τ) is S_1-almost paracompact, then $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact.*

Proof. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in \triangle_0\}$ be a semiopen cover of X. By hypothesis, there exists an $X-$locally finite $X-$open family $\mathcal{V} = \{V_\beta \mid \beta \in \triangle_1\}$ which refines $\mathcal{U}$ such that $X = cl(\bigcup\{V_\beta \mid \beta \in \triangle_1\})$. Now $X = cl(\bigcup\{V_\beta \mid \beta \in \triangle_1\})$ implies $X - cl(\bigcup\{V_\beta \mid \beta \in \triangle_1\}) = \emptyset$ which implies $int(X - \bigcup\{V_\beta \mid \beta \in \triangle_1\}) = \emptyset$ which in turn implies that $int(cl(X - \bigcup\{V_\beta \mid \beta \in \triangle_1\})) = \emptyset$ and so $X - \bigcup\{V_\beta \mid \beta \in \triangle_1\} \in \mathcal{N}$. Since $\mathcal{N} \subset \mathcal{I}$, $X - \bigcup\{V_\beta \mid \beta \in \triangle_1\} \in \mathcal{I}$. Therefore, $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$ paracompact.

Theorem 14. *Let $(X,\tau,\mathcal{I})$ be an ideal space. If $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact and $\mathcal{I}$ is codense, then (X,τ) is S_1-almost paracompact.*

Proof. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in \triangle_0\}$ be a semiopen cover of X. By hypothesis, there exist $I \in \mathcal{I}$ and $X-$locally finite $X-$open family $\mathcal{V} = \{V_\beta \mid \beta \in \triangle_1\}$ which refines $\mathcal{U}$ such that $X - \bigcup\{V_\beta \mid \beta \in \triangle_1\} \in \mathcal{I}$. Since $\mathcal{I}$ is codense, $int(X - \bigcup\{V_\beta \mid \beta \in \triangle_1\}) = \emptyset$ which implies $X - cl(\bigcup\{V_\beta \mid \beta \in \triangle_1\}) = \emptyset$ which in turn implies that $X \subset cl(\bigcup\{V_\beta \mid \beta \in \triangle_1\})$. So $X = cl(\bigcup\{V_\beta \mid \beta \in \triangle_1\})$. Hence (X,τ) is S_1-almost paracompact.

Corollary 15. *Let $(X,\tau,\mathcal{I})$ be an ideal space. If $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact and $\mathcal{I}$ is completely codense, then (X,τ) is S_1-almost paracompact*

Corollary 16. *Let $(X,\tau,\mathcal{I})$ be an ideal space with $\mathcal{I} = \mathcal{N}$. Then (X,τ) is S_1-almost paracompact if and only if $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact.*

Theorem 17. *Let (X,τ) be an E.D semiregular space with an ideal $\mathcal{I}$. Then $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact if and only if $(X,\tau^\alpha,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact.*

Proof. Suppose $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact. Let $\mathcal{U}=\{U_\alpha \mid \alpha \in\triangle_0\}$ be a $\tau^\alpha-$semiopen cover of X. Then $\mathcal{U}=\{U_\alpha \mid \alpha \in\triangle_0\}$ is a $\tau-$semiopen cover of X, by Lemma 3. By hypothesis, there exist $I\in\mathcal{I}$ and $\tau-$locally finite $\tau-$open family $\mathcal{V}=\{V_\beta \mid \beta \in\triangle_1\}$ which refines $\mathcal{U}$ such that $X=\bigcup\{V_\beta \mid \beta \in\triangle_1\}\cup I$. Let $x\in X$. Since $\mathcal{V}$ is $\tau-$locally finite, there exists $W\in\tau(x)$ such that $V_\beta\cap W\neq\emptyset$ for all $\beta=\beta_1,\beta_2,...,\beta_n$. Since $\tau\subset\tau^\alpha$, the family $\mathcal{V}=\{V_\beta \mid \beta \in\triangle_1\}$ is $\tau^\alpha-$locally finite which refines $\mathcal{U}$. Therefore, $(X,\tau^\alpha,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact. Conversely, let $\mathcal{U}=\{U_\alpha \mid \alpha \in\triangle_0\}$ be a $\tau-$semiopen cover of X. Then $\mathcal{U}=\{U_\alpha \mid \alpha \in\triangle_0\}$ is a $\tau^\alpha-$semiopen cover of X, by Lemma 3. By hypothesis, there exist $I\in\mathcal{I}$ and $\tau^\alpha-$locally finite $\tau^\alpha-$open family $\mathcal{V}=\{V_\beta \mid \beta \in\triangle_1\}$ which refines $\mathcal{U}$ such that $X=\bigcup\{V_\beta \mid \beta \in\triangle_1\}\cup I$. Let $x\in X$. Since $\mathcal{V}$ is $\tau^\alpha-$locally finite, there exists $W\in\tau^\alpha(x)$ such that $V_\beta\cap W\neq\emptyset$ for all $\beta=\beta_1,\beta_2,...,\beta_n$. Since $W\in\tau^\alpha(x)$, $W\subset int(cl(int(W)))$. Then $int(cl(int(W)))\in\tau(x)$ such that $V_\beta\cap(int(cl(int(W))))\neq\emptyset$ for all $\beta=\beta_1,\beta_2,...,\beta_n$. Thus, by Lemma 1, the family $\mathcal{V}=\{V_\beta \mid \beta \in\triangle_1\}$ is $\tau-$locally finite $\tau-$open which refines $\mathcal{U}$. Therefore, $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact.

Theorem 18. *If $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then for every cover $\mathcal{U}$ of regular closed sets of X, there exists an open $X-$locally finite $\mathcal{I}-$cover refinement.*

Proof. Since regular closed sets are semiopen, the theorem follows.

Theorem 19. *Let (X,τ) be a semiregular space. If $(X,\tau,\mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then each semiopen cover of X has $X-$locally finite semiclosed $\mathcal{I}-$cover refinement.*

Proof. Let $\mathcal{U}$ be a semiopen cover of X. For each $x\in X$, pick $U_x\in\mathcal{U}$ such that $x\in U_x$. Since (X,τ) is semiregular, there exists $V_x\in SO(X,\tau)$ such that $x\in V_x\subset scl(V_x)\subset U_x$. Then the family $\mathcal{V}=\{V_x \mid x\in X\}$ is a semiopen cover of X. By hypothesis, there exist $I\in\mathcal{I}$ and $X-$locally finite $X-$open family $\mathcal{W}=\{W_\alpha \mid \alpha \in\triangle\}$ which refines $\mathcal{V}$ such that $X\subset\bigcup\{W_\alpha \mid \alpha \in\triangle\}\cup I$. Since $\bigcup W_\alpha\subset\bigcup scl(W_\alpha)$, $X-\bigcup\{scl(W_\alpha) \mid \alpha \in\triangle\}\subset X-\bigcup\{W_\alpha \mid \alpha \in\triangle\}$. Thus, $X-\bigcup\{scl(W_\alpha) \mid \alpha \in\triangle\}\in\mathcal{I}$. Let $x\in X$. Since $\mathcal{W}$ is $X-$locally finite, there exists $P\in\tau(x)$ such that $W_\alpha\cap P\neq\emptyset$ for $\alpha=\alpha_1,\alpha_2,...,\alpha_n$. Since $W_\alpha\subset scl(W_\alpha)$, $W_\alpha\cap P\subset scl(W_\alpha)\cap P$. Then $scl(W_\alpha)\cap P\neq\emptyset$ for $\alpha=\alpha_1,\alpha_2,...,\alpha_n$. Thus, the collection $\mathcal{W}'=\{scl(W_\alpha) \mid \alpha \in\triangle\}$ is $X-$locally finite. Let $scl(W_\alpha)\in\mathcal{W}'$. Then $W_\alpha\in\mathcal{W}$. Since $\mathcal{W}$ refines $\mathcal{V}$, there exists $V_x\in\mathcal{V}$ such that $W_\alpha\subset V_x$ so that $scl(W_\alpha)\subset scl(V_x)\subset U_x$. Hence $\mathcal{W}'$ refines $\mathcal{U}$. Therefore, the family $\mathcal{W}'$ is an $X-$locally finite semiclosed refinement of $\mathcal{U}$. Hence each semiopen cover of X has $X-$locally finite semiclosed $\mathcal{I}-$cover refinement.

If $\mathcal{I}=\{\emptyset\}$ in the above Theorem 19, we have the Corollary 20.

Corollary 20. [2, Theorem 2.13] *Let (X, τ) be a semiregular space. If each semiopen cover of a space X has a locally finite refinement, then each semiopen cover of X has locally finite semiclosed refinement*

Theorem 21. *Let $(X, \tau, \mathcal{I})$ be an ideal space with a codense ideal $\mathcal{I}$. If $(X, \tau^\star)$ is $S_1\mathcal{I}-$paracompact and $\mathcal{I}$ is $\tau-$simple, then every semiopen cover of $(X, \tau, \mathcal{I})$ has $X-$locally finite $X-$semiopen $\mathcal{I}-$cover refinement.*

Proof. Let $\mathcal{U} = \{U_\beta \mid \beta \in \triangle_0\}$ be a $\tau-$semiopen cover of X. By Lemma 9, $SO(X, \tau) \subset SO(X, \tau^\star)$. Then $\mathcal{U}$ is a $\tau^\star-$semiopen cover of X. By hypothesis, there exist $I \in \mathcal{I}$ and $\tau^\star-$locally finite $\tau^\star-$open refinement $\mathcal{V} = \{V_\alpha - I_\alpha \mid \alpha \in \triangle_1, V_\alpha \in \tau, I_\alpha \in \mathcal{I}\}$ such that $X = \bigcup\{V_\alpha - I_\alpha \mid \alpha \in \triangle_1\} \cup I$. Let $x \in X$. Then there exists a $\tau^\star-$open set V containing x such that $V \cap (V_\alpha - I_\alpha) = \emptyset$ for $\alpha \neq \alpha_1, \alpha_2, ..., \alpha_n$. Since $\mathcal{I}$ is $\tau-$simple, $V = U - J$ for some $U \in \tau$ and $J \in \mathcal{I}$. Thus, $(U - J) \cap (V_\alpha - I_\alpha) = \emptyset$ for $\alpha \neq \alpha_1, \alpha_2, ..., \alpha_n$ which implies $(U \cap V_\alpha) - (J \cup I_\alpha) = \emptyset$ for $\alpha \neq \alpha_1, \alpha_2, ..., \alpha_n$ which in turn implies that $U \cap V_\alpha = \emptyset$ for $\alpha \neq \alpha_1, \alpha_2, ..., \alpha_n$, since $\mathcal{I}$ is codense. Then $U \cap (V_\alpha \cap U_\beta) = \emptyset$ for $\alpha \neq \alpha_1, \alpha_2, ..., \alpha_n$. Therefore, the family $\mathcal{V}_1 = \{V_\alpha \cap U_\beta \mid \alpha \in \triangle_1\}$ is $\tau-$locally finite. Also, the family $\mathcal{V}_1$ is $X-$semiopen which refines $\mathcal{U}$, by Lemma 2(a). Since $\mathcal{V}$ refines $\mathcal{U}$, for every $V_\alpha - I_\alpha \in \mathcal{V}$, there exists $U_\beta \in \mathcal{U}$ such that $V_\alpha - I_\alpha \subset U_\beta$. Thus, $V_\alpha - I_\alpha = (V_\alpha - I_\alpha) \cap U_\beta \subset (V_\alpha \cap U_\beta) - I_\alpha \subset V_\alpha \cap U_\beta$ so that $X - \bigcup(V_\alpha \cap U_\beta) \subset X - \bigcup(V_\alpha - I_\alpha) \in \mathcal{I}$. Therefore, $X - \bigcup(V_\alpha \cap U_\beta) \in \mathcal{I}$ which completes the proof.

Theorem 22. *Let $(X, \tau, \mathcal{I})$ be an ideal space and $\mathcal{I}$ is weakly $\tau-$local. If $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then $(X, \tau^\star)$ is $S_1\mathcal{I}-$paracompact.*

Proof. Let $\mathcal{U} = \{U_\alpha - I_\alpha \mid U_\alpha \in \tau, I_\alpha \in \mathcal{I}, \alpha \in \triangle_0\}$ be an $\tau^\star-$semiopen cover of X. Then $\mathcal{U}_1 = \{U_\alpha \mid \alpha \in \triangle_0\}$ is a $\tau-$semiopen cover of X. By hypothesis, there exist $I \in \mathcal{I}$ and $\tau-$locally finite $\tau-$open refinement $\mathcal{V}_1 = \{V_\beta \mid \beta \in \triangle_1\}$ such that $X = \bigcup\{V_\beta \mid \beta \in \triangle_1\} \cup I$. Now $\{V_\beta \cap I_\alpha \mid \beta \in \triangle_1\}$ is a $\tau-$locally finite subset of $\mathcal{I}$ and $\mathcal{I}$ is weakly $\tau-$local, $\bigcup(V_\beta \cap I_\alpha) \in \mathcal{I}$, by Lemma 10. Then $X - \bigcup(V_\beta - I_\alpha) \subset (X - \bigcup V_\beta) \cup (\bigcup(V_\beta \cap I_\alpha)) \in \mathcal{I}$. Therefore, $X - \bigcup(V_\beta - I_\alpha) \in \mathcal{I}$. Since $\mathcal{V}_1 = \{V_\beta \mid \beta \in \triangle_1\}$ is $\tau-$locally finite, $\mathcal{V} = \{V_\beta - I_\alpha \mid \beta \in \triangle_1\}$ is $\tau-$locally finite. Since $\tau \subset \tau^\star$, $\mathcal{V} = \{V_\beta - I_\alpha \mid \beta \in \triangle_1\}$ is τ^*-locally finite which refines $\mathcal{U}$. Hence $(X, \tau^\star)$ is $S_1\mathcal{I}-$paracompact.

3. $S_1\mathcal{I}-$ paracompact subsets

In this section, we define the subsets and subspaces of $S_1\mathcal{I}-$paracompact spaces and discuss some of its properties. A subset A of an ideal space $(X, \tau, \mathcal{I})$ is said to be $S_1\mathcal{I}-$*paracompact relative to* X if for every $X-$semiopen cover $\mathcal{U}$ of A, there exist $I \in \mathcal{I}$ and $X-$locally finite family $\mathcal{V}$ of $X-$open sets which refines $\mathcal{U}$ such that $A \subset \bigcup\{V \mid V \in \mathcal{V}\} \cup I$. A is *$S_1\mathcal{I}-$paracompact* if $(A, \tau_A, \mathcal{I}_A)$ is $S_1\mathcal{I}_A-$paracompact as a subspace where τ_A is the usual subspace topology.

Theorem 23. *Every regular open subspace of an $S_1\mathcal{I}-$paracompact space is $S_1\mathcal{I}-$paracompact.*

Proof. Let A be a regular open subspace of $(X, \tau, \mathcal{I})$. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in \triangle_0\}$ be a τ_A-semiopen cover of A. Since A is an open subset of X, $U_\alpha \in SO(X, \tau)$ for each $\alpha \in \triangle_0$, by Lemma 2(b). Then $\mathcal{U}_1 = \{U_\alpha \mid \alpha \in \triangle_0\} \cup \{X - A\}$ is a semiopen cover of X. By hypothesis, there exist $I \in \mathcal{I}$ and $X-$locally finite $X-$open refinement $\mathcal{V}_1 = \{V_\beta \mid \beta \in \triangle_1\}$ such that $X = \bigcup\{V_\beta \mid \beta \in \triangle_1\} \cup I$ which implies $A \subset \bigcup\{V_\beta \cap A \mid \beta \in \triangle_1\} \cup I_A$ where $I_A = I \cap A$. Let $x \in A$. Since $\mathcal{V}_1 = \{V_\beta \mid \beta \in \triangle_1\}$ is $X-$locally finite, there exists $W \in \tau(x)$ such that $V_\beta \cap W = \emptyset$ for $\beta \neq \beta_1, \beta_2, ..., \beta_n$. For $\beta \neq \beta_1, \beta_2, ..., \beta_n$, $V_\beta \cap W = \emptyset$ implies that $(V_\beta \cap W) \cap A = \emptyset$ which implies $(V_\beta \cap A) \cap (W \cap A) = \emptyset$. Therefore, $\mathcal{V} = \{V_\beta \cap A \mid \beta \in \triangle_1\}$ is τ_A-locally finite. Let $V_\beta \cap A \in \mathcal{V}$. Then $V_\beta \in \mathcal{V}_1$. Since $\mathcal{V}_1$ refines $\mathcal{U}_1$, there exists $U_\alpha \in \mathcal{U}_1$ such that $V_\beta \subset U_\alpha$ and so $V_\beta \cap A \subset U_\alpha \cap A \subset U_\alpha$. Hence $\mathcal{V}$ refines $\mathcal{U}$. The family $\mathcal{V} = \{V_\beta \cap A \mid \beta \in \triangle_1\}$ is τ_A-locally finite τ_A-open refinement of $\mathcal{U}$. Therefore, A is $S_1\mathcal{I}-$paracompact.

Corollary 24. *Every clopen subspace of an $S_1\mathcal{I}-$paracompact space is $S_1\mathcal{I}-$paracompact.*

If $\mathcal{I} = \{\emptyset\}$ in the above Theorem 23, we have the Corollary 25.

Corollary 25. [2, Theorem 3.1] *Every regular open subspace of an S_1-paracompact space is S_1-paracompact.*

Theorem 26. *Let $(X, \tau, \mathcal{I})$ be an ideal space and let $A \in \tau^\alpha$. If A is $S_1\mathcal{I}-$paracompact relative to X, then A is $S_1\mathcal{I}-$paracompact.*

Proof. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in \triangle_0\}$ be a cover of A by semiopen sets of A. Since $A \in \tau^\alpha$, $A \in SO(X, \tau)$ and so by Lemma 4, $\mathcal{U}$ is a $\tau-$semiopen cover of A. By hypothesis, there exist $I \in \mathcal{I}$ and $\tau-$locally finite $\tau-$open refinement $\mathcal{V}_1 = \{V_\beta \mid \beta \in \triangle_1\}$ such that $A \subset \bigcup\{V_\beta \mid \beta \in \triangle_1\} \cup I$ which implies $A \subset \bigcup\{V_\beta \cap A \mid \beta \in \triangle_1\} \cup (I \cap A)$. Let $x \in A$. Since $\mathcal{V}_1 = \{V_\beta \mid \beta \in \triangle_1\}$ is $\tau-$locally finite, there exists $W \in \tau(x)$ such that $V_\beta \cap W = \emptyset$ for $\beta \neq \beta_1, \beta_2, ..., \beta_n$. Then $(V_\beta \cap W) \cap A = \emptyset$ for $\beta \neq \beta_1, \beta_2, ..., \beta_n$ which implies $(V_\beta \cap A) \cap (W \cap A) = \emptyset$ for $\beta \neq \beta_1, \beta_2, ..., \beta_n$. Thus, the family $\mathcal{V} = \{V_\beta \cap A \mid \beta \in \triangle_1\}$ is τ_A-open τ_A-locally finite refinement of $\mathcal{U}$. Therefore, A is $S_1\mathcal{I}-$paracompact.

Theorem 27. *If A and B are $S_1\mathcal{I}-$paracompact relative to an ideal space $(X, \tau, \mathcal{I})$, then $A \cup B$ is $S_1\mathcal{I}-$paracompact relative to X.*

Proof. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in\triangle_0\}$ be a $X-$semiopen cover of $A \cup B$. Then $\mathcal{U} = \{U_\alpha \mid \alpha \in\triangle_0\}$ is a $X-$semiopen cover of A and B. By hypothesis, there exist $I_A, I_B \in \mathcal{I}$ and $X-$open $X-$locally finite families $\mathcal{V}_A = \{V_\alpha \mid \alpha \in\triangle_1\}$ of A and $\mathcal{V}_B = \{V_\beta \mid \beta \in\triangle_1\}$ of B which refines $\mathcal{U}$ such that $A \subset \bigcup\{V_\alpha \mid \alpha \in\triangle_1\} \cup I_A$ and $B \subset \bigcup\{V_\beta \mid \beta \in\triangle_1\} \cup I_B$. Now $A \cup B \subset \big(\bigcup\{V_\alpha \mid \alpha \in\triangle_1\} \cup I_A\big) \cup \big(\bigcup\{V_\beta \mid \beta \in\triangle_1\} \cup I_B\big)$ implies that $A \cup B \subset \bigcup\{V_\alpha \cup V_\beta \mid \alpha, \beta \in\triangle_1\} \cup (I_A \cup I_B)$ which implies $A \cup B \subset \bigcup\{V_\alpha \cup V_\beta \mid \alpha, \beta \in\triangle_1\} \cup I$ where $I = I_A \cup I_B$. Since the families $\mathcal{V}_A$ and $\mathcal{V}_B$ are $X-$locally finite, the family $\mathcal{V} = \{V_\alpha \cup V_\beta \mid \alpha, \beta \in\triangle_1\}$ is $X-$locally finite, by Lemma 6, which refines $\mathcal{U}$. Therefore, $A \cup B$ is $\mathcal{I}-$paracompact relative to X.

Theorem 28. *Let A and B be subsets of an ideal space $(X, \tau, \mathcal{I})$. If A is $S_1\mathcal{I}-$paracompact relative to X and B is semiclosed in X, then $A \cap B$ is $S_1\mathcal{I}-$paracompact relative to X.*

Proof. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in\triangle_0\}$ be a cover of $A \cap B$ such that $U_\alpha \in SO(X, \tau)$. Since $X - B$ is semiopen in X, $\mathcal{U}_1 = \{U_\alpha \mid \alpha \in\triangle_0\} \cup \{X - B\}$ is a $X-$semiopen cover of A. By hypothesis, there exist $I \in \mathcal{I}$ and $X-$locally finite $X-$open family $\mathcal{V}_1 = \{V_\beta \mid \beta \in\triangle_1\} \cup \{V\}$ ($V_\beta \subset U_\alpha$ and $V \subset X - B$) which refines $\mathcal{U}_1$ such that $A \subset \bigcup_\beta\{V_\beta \mid \beta \in\triangle_1\} \cup V \cup I$. Now $A \subset \bigcup_\beta\{V_\beta \mid \beta \in\triangle_1\} \cup V \cup I$ implies that $A \cap B \subset \bigcup_\beta(\{V_\beta \mid \beta \in\triangle_1\} \cup V \cup I) \cap B$ which implies $A \cap B \subset \bigcup_\beta\{V_\beta \mid \beta \in\triangle_1\} \cup V \cup I$. Thus, $A \cap B - \bigcup_\beta V_\beta = A \cap B - (V \cup (\bigcup_\beta V_\beta))$ and so $A \cap B - \bigcup_\beta V_\beta \in \mathcal{I}$. Since $V_\beta \subset V_\beta \cup V$, $\mathcal{V} = \{V_\beta \mid \beta \in\triangle_1\}$ is $X-$locally finite, by Lemma 5. Therefore, $\mathcal{V} = \{V_\beta \mid \beta \in\triangle_1\}$ is $X-$locally finite $X-$open family which refines $\mathcal{U}$. Hence $A \cap B$ is $S_1\mathcal{I}-$paracompact relative to X.

Corollary 29. *If $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact and B is semiclosed, then B is $S_1\mathcal{I}-$paracompact relative to X.*

Corollary 30. *If A and B are semiclosed sets of an $S_1\mathcal{I}-$paracompact space $(X, \tau, \mathcal{I})$, then $A \cap B$ is $S_1\mathcal{I}-$paracompact relative to X.*

Theorem 31. *In an ideal space $(X, \tau, \mathcal{I})$, if A is $S_1\mathcal{I}-$paracompact relative to X, then every cover of A by semiregular sets of X has locally finite open $\mathcal{I}-$cover refinement.*

Proof. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in\triangle_0\}$ be a cover of A such that $U_\alpha \in SR(X, \tau)$. Then $\mathcal{U}$ is an $X-$semiopen cover of A. By hypothesis, there exist $I \in \mathcal{I}$ and $X-$locally finite $X-$open family $\mathcal{V} = \{V_\beta \mid \beta \in\triangle_1\}$ which refines $\mathcal{U}$ such that $A \subset \bigcup\{V_\beta \mid \beta \in\triangle_1\} \cup I$. This completes the proof.

Theorem 32. *Let A and B be subsets of an ideal space $(X, \tau, \mathcal{I})$ such that $A \subset B \subset X$ and $B \in SO(X, \tau)$. If A is $S_1\mathcal{I}-$paracompact relative to X, then A is $S_1\mathcal{I}-$paracompact relative to B.*

Proof. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in \triangle_0\}$ be a cover of A such that $U_\alpha \in SO(B)$. Since $B \in SO(X, \tau)$, by Lemma 4, $\mathcal{U}$ is an $X-$semiopen cover of A. By hypothesis, there exist $I \in \mathcal{I}$ and $X-$locally finite $X-$open family $\mathcal{V} = \{V_\beta \mid \beta \in \triangle_1\}$ which refines $\mathcal{U}$ such that $A \subset \bigcup\{V_\beta \mid \beta \in \triangle_1\} \cup I$. Then $A \cap B \subset \bigcup\{V_\beta \cap B \mid \beta \in \triangle_1\} \cup (I \cap B)$ implies $A \subset \bigcup\{V_\beta \cap B \mid \beta \in \triangle_1\} \cup I$. Let $x \in B$. Since $\mathcal{V} = \{V_\beta \mid \beta \in \triangle_1\}$ is $X-$locally finite, there exists $W \in \tau(x)$ such that $W \cap V_\beta = \emptyset$ for $\beta \neq \beta_1, \beta_2, ..., \beta_n$ which implies $(W \cap V_\beta) \cap B = \emptyset$ for $\beta \neq \beta_1, \beta_2, ..., \beta_n$ which implies $(V_\beta \cap B) \cap (W \cap B) = \emptyset$ for $\beta \neq \beta_1, \beta_2, ..., \beta_n$. Therefore, the family $\mathcal{V}_1 = \{V_\beta \cap B \mid \beta \in \triangle_1\}$ is $B-$locally finite. Let $V_\beta \cap B \in \mathcal{V}_1$. Since $\mathcal{V}$ refines $\mathcal{U}$, there exists $U_\alpha \in \mathcal{U}$ such that $V_\beta \subset U_\alpha$ and so $V_\beta \cap B \subset U_\alpha$. Hence $\mathcal{V}_1$ refines $\mathcal{U}$. Therefore, A is $S_1\mathcal{I}-$paracompact relative to B.

Corollary 33. *If A is $S_1\mathcal{I}-$paracompact relative to X, then the following hold.*
(a) *$A \cap B$ is $S_1\mathcal{I}-$paracompact relative to B for each $B \in SR(X, \tau)$.*
(b) *If $B \in SR(X, \tau)$ and $B \subset A$, then B is $S_1\mathcal{I}-$paracompact relative to X.*

Proof. (a) Let A be $S_1\mathcal{I}-$paracompact relative to X. Since $B \in SR(X, \tau)$, $B \in SC(X, \tau)$. By Theorem 28, $A \cap B$ is $S_1\mathcal{I}-$paracompact relative to X. Since $A \cap B \subset B$ and $B \in SO(X, \tau)$, by Theorem 32, $A \cap B$ is $S_1\mathcal{I}-$paracompact relative to B.
(b) Since $B \subset A$ and $B \in SR(X, \tau)$, by Theorem 28, B is $S_1\mathcal{I}-$paracompact relative to X.

4. Invariants of $S_1\mathcal{I}-$paracompact space under mappings

In this section, we discuss that if a function is open irresolute almostclosed surjection with $N-$closed point inverses, then it preserves $S_1\mathcal{I}-$paracompact spaces. If a function is presemiopen, continuous and bijective, then it inverse preserves $S_1\mathcal{I}-$paracompact spaces.

Theorem 34. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be an open, irresolute, almostclosed, surjective mapping with $N-$closed point inverses and $\mathcal{J} = f(\mathcal{I})$. If $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then $(Y, \sigma, \mathcal{J})$ is $S_1\mathcal{J}-$paracompact.*

Proof. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in \triangle_0\}$ be a semiopen cover of Y. Since f is irresolute, $\mathcal{U}_1 = \{f^{-1}(U_\alpha) \mid \alpha \in \triangle_0\}$ is a semiopen cover of X. By hypothesis, there exist $I \in \mathcal{I}$ and $X-$locally finite $X-$open family $\mathcal{V}_1 = \{V_\beta \mid \beta \in \triangle_1\}$ which refines $\mathcal{U}_1$ such that $X = \bigcup\{V_\beta \mid \beta \in \triangle_1\} \cup I$. Then $f(X) = f(\bigcup\{V_\beta \mid \beta \in \triangle_1\} \cup I)$ which implies that

$Y = \bigcup\{f(V_\beta) \mid \beta \in\triangle_1\} \cup f(I)$ which implies $Y = \bigcup\{f(V_\beta) \mid \beta \in\triangle_1\} \cup J$, where $J = f(I)$. Since $\mathcal{V}_1$ is $X-$locally finite, $\mathcal{V} = \{f(V_\beta) \mid \beta \in\triangle_1\}$ is $Y-$locally finite, by Lemma 7. Since f is open, $f(V_\beta)$ is open in Y. Let $f(V_\beta) \in \mathcal{V}$. Then $V_\beta \in \mathcal{V}_1$. Since $\mathcal{V}_1$ refines $\mathcal{U}_1$, there exists $f^{-1}(U_\alpha) \in \mathcal{U}_1$ such that $V_\beta \subset f^{-1}(U_\alpha)$. Thus, $f(V_\beta) \subset f(f^{-1}(U_\alpha))$ implies that $f(V_\beta) \subset U_\alpha$ for some $U_\alpha \in \mathcal{U}$. Hence $\mathcal{V}$ refines $\mathcal{U}$. Therefore, $(Y, \sigma, \mathcal{J})$ is $S_1\mathcal{J}-$paracompact.

Since compact sets are $N-$closed and closed maps are almostclosed, the proof of the following Corollary 35 follows from Theorem 34.

Corollary 35. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be an open, irresolute, closed, surjective mapping with compact point inverses and $\mathcal{J} = f(\mathcal{I})$. If $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then $(Y, \sigma, \mathcal{J})$ is $S_1\mathcal{J}-$paracompact.*

Corollary 36. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be an open, semicontinuous, almost-closed, surjective mapping with $N-$closed point inverses and $\mathcal{J} = f(\mathcal{I})$. If $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then $(Y, \sigma, \mathcal{J})$ is $\mathcal{J}-$paracompact.*

Corollary 37. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be an open, semicontinuous, closed, surjective mapping with compact point inverses and $\mathcal{J} = f(\mathcal{I})$. If $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then $(Y, \sigma, \mathcal{J})$ is $\mathcal{J}-$paracompact.*

If $\mathcal{I} = \{\emptyset\}$ in the above Corollary 35, we have the Corollary 38.

Corollary 38. [2, Theorem 3.5] *Let $f : (X, T) \to (Y, M)$ be a continuous, open and closed surjective function such that $f^{-1}(y)$ is compact for each $y \in Y$. If (X, T) is S_1-paracompact, then so is (Y, M).*

Theorem 39. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be an open, strongly semicontinuous, almostclosed, surjective mapping with $N-$closed point inverses and $\mathcal{J} = f(\mathcal{I})$. If $(X, \tau, \mathcal{I})$ is $\mathcal{I}-$paracompact, then $(Y, \sigma, \mathcal{J})$ is $S_1\mathcal{J}-$paracompact.*

Proof. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in\triangle_0\}$ be a semiopen cover of Y. Since f is strongly semicontinuous, $\mathcal{U}_1 = \{f^{-1}(U_\alpha) \mid \alpha \in\triangle_0\}$ is an open cover of X. By hypothesis, there exist $I \in \mathcal{I}$ and $X-$locally finite $X-$open family $\mathcal{V}_1 = \{V_\beta \mid \beta \in\triangle_1\}$ which refines $\mathcal{U}_1$ such that $X = \bigcup\{V_\beta \mid \beta \in\triangle_1\} \cup I$. Then $f(X) = f(\bigcup\{V_\beta \mid \beta \in\triangle_1\} \cup I)$ which implies $Y = \bigcup\{f(V_\beta) \mid \beta \in\triangle_1\} \cup f(I)$ which implies $Y = \bigcup\{f(V_\beta) \mid \beta \in\triangle_1\} \cup J$, where $J = f(I)$. Since $\mathcal{V}_1$ is $X-$locally finite, $\mathcal{V} = \{f(V_\beta) \mid \beta \in\triangle_1\}$ is $Y-$locally finite, by Lemma 7. Since f is open, $f(V_\beta)$ is open in Y. Let $f(V_\beta) \in \mathcal{V}$. Then $V_\beta \in \mathcal{V}_1$. Since $\mathcal{V}_1$ refines $\mathcal{U}_1$, there exists $f^{-1}(U_\alpha) \in \mathcal{U}_1$ such that $V_\beta \subset f^{-1}(U_\alpha)$. Thus, $f(V_\beta) \subset f(f^{-1}(U_\alpha))$ implies that $f(V_\beta) \subset U_\alpha$ for some $U_\alpha \in \mathcal{U}$. Hence $\mathcal{V}$ refines $\mathcal{U}$. Therefore, $(Y, \sigma, \mathcal{J})$ is $S_1\mathcal{J}-$paracompact.

Corollary 40. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be an open, strongly semicontinuous, closed, surjective mapping with compact point inverses and $\mathcal{J} = f(\mathcal{I})$. If $(X, \tau, \mathcal{I})$ is $\mathcal{I}-$paracompact, then $(Y, \sigma, \mathcal{J})$ is $S_1\mathcal{J}-$paracompact.*

Theorem 41. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be a presemiopen, continuous, bijective mapping and $\mathcal{I} = f^{-1}(\mathcal{J})$. If A is $S_1\mathcal{J}-$paracompact relative to Y, then $f^{-1}(A)$ is $S_1\mathcal{I}-$paracompact relative to X.*

Proof. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in \triangle_0\}$ be a $X-$semiopen cover of $f^{-1}(A)$. Since f is presemiopen, $\mathcal{U}_1 = \{f(U_\alpha) \mid \alpha \in \triangle_0\}$ is a $Y-$semiopen cover of A. By hypothesis, there exist $J \in \mathcal{J}$ and $Y-$locally finite $Y-$open family $\mathcal{V}_1 = \{V_\beta \mid \beta \in \triangle_1\}$ which refines $\mathcal{U}_1$ such that $A \subset \bigcup\{V_\beta \mid \beta \in \triangle_1\} \cup J$. Now $A \subset \bigcup\{V_\beta \mid \beta \in \triangle_1\} \cup J$ implies that $f^{-1}(A) \subset \bigcup\{f^{-1}(V_\beta) \mid \beta \in \triangle_1\} \cup f^{-1}(J)$ which implies $f^{-1}(A) \subset \bigcup\{f^{-1}(V_\beta) \mid \beta \in \triangle_1\} \cup I$, where $I = f^{-1}(J)$. Since f is continuous, by Lemma 8, $\mathcal{V} = \{f^{-1}(V_\beta) \mid \beta \in \triangle_1\}$ is $X-$open, $X-$locally finite. Let $f^{-1}(V_\beta) \in \mathcal{V}$. Then $V_\beta \in \mathcal{V}_1$. Since $\mathcal{V}_1$ refines $\mathcal{U}_1$, there exists $f(U_\alpha) \in \mathcal{U}_1$ such that $V_\beta \subset f(U_\alpha)$. Then $f^{-1}(V_\beta) \subset f^{-1}(f(U_\alpha))$ implies $f^{-1}(V_\beta) \subset U_\alpha$ for some $U_\alpha \in \mathcal{U}$. Hence $\mathcal{V}$ refines $\mathcal{U}$. Therefore, $f^{-1}(A)$ is $S_1\mathcal{I}-$paracompact relative to X.

Corollary 42. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be a presemiopen, continuous, bijective mapping and $\mathcal{I} = f^{-1}(\mathcal{J})$. If $(Y, \sigma, \mathcal{J})$ is $S_1\mathcal{J}-$paracompact, then $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact.*

Corollary 43. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be a semiopen, continuous, bijective mapping and $\mathcal{I} = f^{-1}(\mathcal{J})$. If $(Y, \sigma, \mathcal{J})$ is $S_1\mathcal{J}-$paracompact, then $(X, \tau, \mathcal{I})$ is $\mathcal{I}-$paracompact.*

Corollary 44. [2, Theorem 3.8] *Let $f : (X, T) \to (Y, M)$ be a continuous, semi-closed, surjection and $f^{-1}(y)$ is compact for each $y \in Y$. If (Y, M) is S_1-paracompact space, then (X, T) is paracompact.*

Theorem 45. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be an open, irresolute, almost closed, surjective mapping with $N-$closed point inverses and $\mathcal{J} = f(\mathcal{I})$ is codense. If $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then (Y, σ) is S_1-almost paracompact.*

Proof. By Theorem 34, $(Y, \sigma, \mathcal{J})$ is $S_1\mathcal{I}-$paracompact. Since $\mathcal{J} = f(\mathcal{I})$ is codense, by Theorem 14, (Y, σ) is S_1-almost paracompact.

Corollary 46. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be an open, irresolute, closed, surjective mapping with compact point inverses and $\mathcal{J} = f(\mathcal{I})$ is codense. If $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then (Y, σ) is S_1-almost paracompact.*

Corollary 47. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be an open, irresolute, almost closed, surjective mapping with $N-$closed point inverses and $\mathcal{J} = f(\mathcal{I})$ is completely codense. If $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then (Y, σ) is S_1-almost paracompact.*

Corollary 48. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be an open, irresolute, closed, surjective mapping with compact point inverses and $\mathcal{J} = f(\mathcal{I})$ is completely codense. If $(X, \tau, \mathcal{I})$ is $S_1\mathcal{I}-$paracompact, then (Y, σ) is S_1-almost paracompact.*

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