Mathematical model of the drag coefficient using three passive controls on a circular cylinder

C. Imron, L. Mardianto, B. Widodo, T. Yuwono

Abstract. This research deals with calculating drag coefficient on a circular cylinder numerically by SIMPLE algorithm. In order to reduce drag coefficient, three passive controls are used. It is a smaller object than a circular cylinder used as the main object. Circular, elliptical and type-I cylinder are kind of shape used as passive control. A passive control in front of the main object is type-I cylinder while two passive controls in rear are various with same shape. An angle is formed because of the existence of two passive controls in rear. The passive control distance to the main object is various. Mathematical model is developed from modified Lagrange polynomial to obtain the effective shape, angle and distance of the passive control.

M.S.C. 2010: 35C11, 35Q30, 65D05, 65M06, 75D05,75D55.
Key words: Drag coefficient; mathematical model; passive control; SIMPLE algorithm.

1 Introduction

Drag coefficient is a parameter to determine the value of drag force. It will appear when fluid flow past an object. It also harms for the existence of the object. In order to control it, passive control can be used to reduce it. This application can be found in offshore structure, bridge structure and industrial chimney.

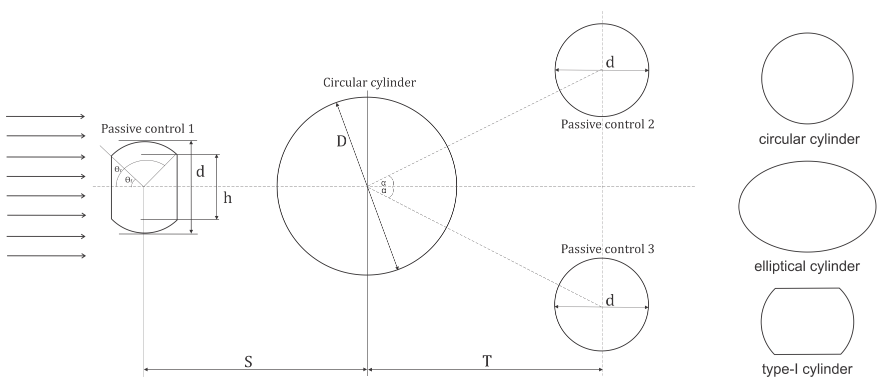
Passive control is a small object used to control the fluid flow in around the main object. It can be a circular cylinder [1], a type I cylinder [2] or rod which is placed in front of the main object or permeable and inclined short plate which is placed in rear [3]. It is also modified with number and configuration. An elliptical cylinder was used as an additional passive control in rear by Imron et al. [4]. Kuo and Chen used two small circular cylinder behind the main object [5]. A screen was set up around a circular cylinder by Oruc [6].

In the research before, the passive control used was type-I cylinder which gave small impact on drag reduction [2]. An additional passive control was used to reduce drag coefficient up to 39.38% at a circular cylinder by using smaller elliptical cylinder

Applied Sciences, Vol. 21, 2019, pp. 136-145.

[4]. In this research, three passive control is used to reduce drag coefficient at a circular cylinder.

The problem in this research is about two-dimensional incompressible fluid flow around a circular cylinder using three passive control as it seen in Figure 1 at $Re = 1000$. A passive control in front of the main object, circular cylinder with diameter D, is type-I cylinder and two passive controls in rear are various with the same shape as in Figure 1 and landscape position. Type-I cylinder is cut of 53° in front and 127° in rear with diameter d. Elliptical cylinder has major axis $1.5d$ and minor axis d. The ratio of D and d is 8:1. Both two passive controls invent same angle α to the horizontal axis. The distance between a passive control in front and the main object is S/D and the distance between a passive control in rear and it is T/D. The distance S/D and T/D, and angle α are various



(a) (b)

Figure 1: (a) Experimental design of the research (b) Passive control shape

2 Numerical method

In this research, the problem is about the incompressible Navier-Stokes equations written as follows

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \mathbf{p} + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u} \tag{2.1}$$

It is solved by using Semi Implicit Method for Pressure Linked Equation or SIMPLE algorithm. First, pressure component is ignored in (2.1) such that

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u}$$

The pressure component is expressed as

$$\frac{1}{\Delta t}\nabla \cdot \mathbf{u}^* = \nabla^2 \mathbf{p} \tag{2.2}$$

where $\mathbf{u}^*$ is defined as temporary velocity. Successive Over Relaxation (SOR) is used to result faster convergence formulated as

$$(p_n)_{i,j} = (1-\epsilon)(p_{n-1})_{i,j} + \epsilon(p_n^*)_{i,j}$$

where p_n^* is a temporary pressure component obtained from (2.2). The last is velocity component correction written as

$$\frac{\partial \mathbf{u}}{\partial t} = -\nabla \mathbf{p}$$

3 Results and discussion

The result of this research is drag coefficient calculation on a circular cylinder using three passive control. It is solved by SIMPLE algorithm. After drag coefficient is obtained at each value of S/D, T/D and angle, mathematical model is created. It is developed by modifying Lagrange polynomial which usually use a single variable to be Lagrange polynomial using three variables.

3.1 Validation

Validation is needed to check whether the program simulation is working or not. To check the program, drag coefficient is calculated at a circular cylinder without passive control with $Re = 100$. In this research, it is obtained $C_d = 1.358$ which is closed value with the results of other researchers, as seen in Table 1. Thus, it can be continue for other result and discussions.

Table 1: Comparison of drag coefficient with other researches

Re	100
Imron (Actual)	1.358
Ding (2007) [7]	1.356
Noor (2009) [8]	1.4
Silva (2003) [9]	1.39

3.2 Drag coefficient

The use of three passive controls to reduct drag coefficient is very effective. It uses two circular cylinders in rear and a type-I cylinder in front. The drag coefficient decreased until 42.06% with configuration $S/D = 1.8$, $T/D = 1.8$ and angle 30° as it seen in Table 2.

From Table 3, the least drag coefficient is 0.834 with two elliptical cylinders in rear and type-I cylinder in front. It has configuration $S/D = 1.8$, $T/D = 1.8$ and angle 30°. It is still higher than that using two circular cylinder in rear.

Table 2: Drag coefficient using three passive controls with circular cylinder in rear

C_d		Angle				
S/D	T/D	15°	30°	45°	60°	90°
1.8	1.8	0.783	0.701	0.742	0.800	0.902
	2.1	0.741	0.710	0.758	0.824	0.908
2.4	1.8	0.784	0.702	0.742	0.797	0.898
	2.1	0.742	0.711	0.756	0.821	0.905

Table 3: Drag coefficient using three passive controls with elliptical cylinder in rear

C_d		Angle				
S/D	T/D	15°	30°	45°	60°	90°
1.8	1.8	0.859	0.834	0.884	0.932	0.991
	2.1	0.851	0.842	0.891	0.934	0.979
2.4	1.8	0.872	0.846	0.894	0.942	1.000
	2.1	0.864	0.854	0.901	0.945	0.990

The Table 4 represents drag coefficient by using a type-I cylinder in front and two type-I cylinders in rear. It yields drag coefficient $C_d = 0.703$ as the least with configuration $S/D = 1.8$, $T/D = 1.8$ and angle 30°. It is fairly higher than that using two circular cylinders in rear, but is fewer than that using two elliptical cylinders in rear. Three type passive controls, i.e. circular, elliptical and type-I cylinder, in rear have same configuration to reach the least drag coefficient.

Table 4: Drag coefficient using three passive controls with type-I cylinder in rear

C_d		Angle				
S/D	T/D	15°	30°	45°	60°	90°
1.8	1.8	0.789	0.703	0.758	0.809	0.892
	2.1	0.748	0.715	0.777	0.833	0.899
2.4	1.8	0.790	0.705	0.756	0.806	0.889
	2.1	0.750	0.715	0.775	0.830	0.897

3.3 Mathematical model

Mathematical model is an equation or more which represents a real physical system. In this research, the real data about drag coefficient are modeled to get the other in each point. The drag coefficient model is obtained by modifying Lagrange polynomial interpolation which use a single variable to be Lagrange trilinear interpolation which use three variables. The formula can be seen as follows.

$$\hat{f}(x,y,z) = \sum_{k=1}^{r} \left(\sum_{j=1}^{q} \left(\sum_{i=1}^{p} f(x_i, y_j, z_k) L_{p,a}(x) \right) H_{q,b}(y) \right) G_{r,c}(z) \tag{3.1}$$

where, for each $a = 1, 2, \ldots, p$, $b = 1, 2, \ldots, q$ and $c = 1, 2, \ldots r$

$$L_{p,a} = \prod_{\substack{u=1\\u\neq a}}^{p} \frac{x - x_u}{x_a - x_u}, \quad H_{q,b} = \prod_{\substack{v=1\\v\neq b}}^{q} \frac{y - y_v}{y_b - y_v}, \quad G_{r,c} = \prod_{\substack{w=1\\w\neq c}}^{r} \frac{z - z_w}{z_c - z_w}$$

with remainder

$$R(x,y,z) = \frac{f(\xi(x,y,z))^{p+q+r}}{p!q!r!}(x - x_1)(x - x_2)\ldots(x - x_p)(y - y_1)(y - y_2)\ldots(y - y_q)$$
$$(z - z_1)(z - z_2)\ldots(z - z_r)$$

where p is the number of point in x-component, q is the number of point in y-component and r is the number of point in z-component.

The variables used are S/D as x-component, T/D as y-component and angle z-component. Those variables is substituted in (3.1). By using data in Table 2, the drag coefficient model for passive controls using circular cylinder in rear is

$$\begin{aligned}
\tilde{f}(x,y,z) = {} & (\frac{z}{30} - 2)(\frac{z}{45} - 1)(\frac{z}{60} - \frac{1}{2})(\frac{z}{75} - \frac{1}{5})((\frac{x}{150} - \frac{457}{500})(\frac{10y}{3} - 7) \\
& - (\frac{x}{200} - \frac{917}{1000})(\frac{10y}{3} - 6)) - (\frac{z}{15} - 2)(\frac{z}{30} - \frac{3}{2})(\frac{z}{45} - \frac{4}{3})(\frac{z}{75} - \frac{6}{5}) \\
& ((\frac{x}{600} + \frac{39}{50})(\frac{10y}{3} - 7) - (\frac{x}{600} + \frac{369}{500})(\frac{10y}{3} - 6)) + (\frac{z}{15} - 1)(\frac{z}{15} - 3) \\
& (\frac{z}{30} - 2)(\frac{z}{60} - \frac{3}{2})((\frac{x}{600} + \frac{349}{500})(\frac{10y}{3} - 7) - (\frac{x}{600} + \frac{707}{1000})(\frac{10y}{3} - 6)) \\
& - (\frac{z}{15} - 3)(\frac{z}{30} - 1)(\frac{z}{30} - 3)(\frac{z}{45} - \frac{1}{3})((\frac{x}{200} - \frac{809}{1000})(\frac{10y}{3} - 7) \\
& - (\frac{x}{200} - \frac{833}{1000})(\frac{10y}{3} - 6)) - (\frac{z}{15} - 2)(\frac{z}{15} - 4)(\frac{z}{30} - \frac{1}{2})(\frac{z}{45} - 2) \\
& (\frac{371y}{150} + (\frac{x}{300} - \frac{191}{250})(\frac{10y}{3} - 6) - \frac{2597}{500})
\end{aligned} \tag{3.2}$$

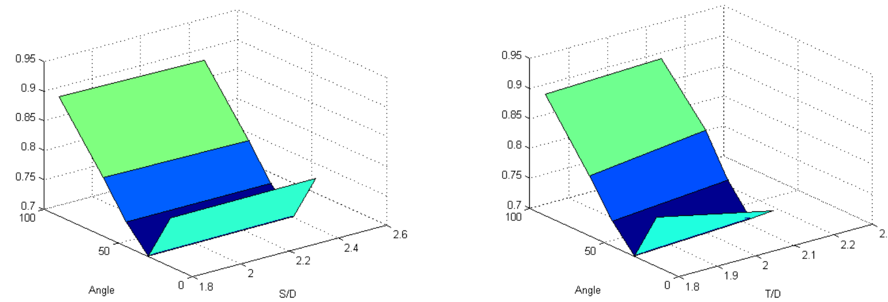
and remainder

$$R(x,y,z) = \frac{f(\xi(x,y,z))^{2+2+5}}{2!2!5!}(x - 1.8)(x - 2.4)(y - 1.8)(y - 2.1)$$
$$(z - 15)(z - 30)\ldots(z - 75)$$

By using (3.2), the least drag coefficient is 0.701 with configuration $S/D = 1.8$, $T/D = 1.8$ and angle 30°. The result is same with the real data plot.

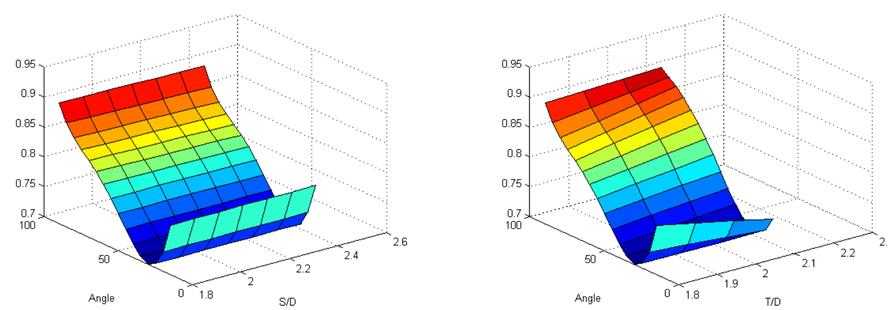
In order to compare the plot of drag coefficient using two circular cylinders in rear with the plot of model, the result can be seen in three dimensional plot. The real data plot can be seen in Figures 3(a) and 3(b), compared with the plots of Figures 2(a) and 2(b).

Data of drag coefficient using three passive controls with two elliptical cylinder in rear from Table 3 are plotted as in Figure 4(a) and 4(b). It is also substituted in (3.1) to obtained the mathematical model. The result is



(a) (b)

Figure 2: Real data plot of Table 2 with two circular cylinder in rear (a) $T/D = 1.5$, (b) $S/D = 1.8$



(a) (b)

Figure 3: Plot of drag coefficient model as in (3.2) with fixed (a) $T/D = 1.5$, (b) $S/D = 1.8$

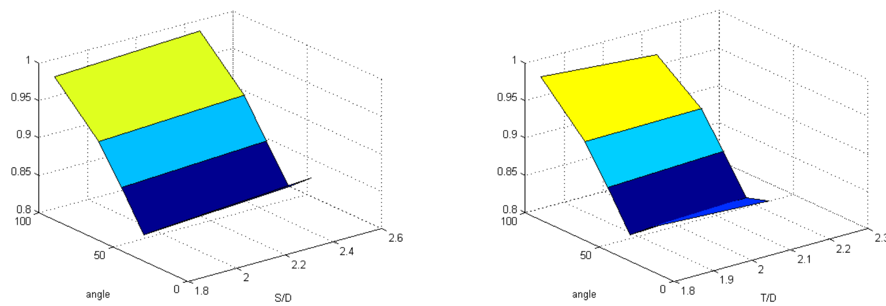
$$
\begin{aligned}
\tilde{f}(x,y,z) = {} & (\frac{z}{15}-1)(\frac{z}{15}-3)(\frac{z}{30}-2)(\frac{z}{60}-\frac{3}{2})((\frac{x}{50}+\frac{399}{500})(\frac{10y}{3}-7)-(\frac{x}{50}+\frac{403}{500}) \\
& (\frac{10y}{3}-6))-(\frac{z}{15}-2)(\frac{z}{30}-\frac{3}{2})(\frac{z}{45}-\frac{4}{3})(\frac{z}{75}-\frac{6}{5})((\frac{13x}{600}+\frac{41}{50})(\frac{10y}{3}-7) \\
& -(\frac{13x}{600}+\frac{203}{250})(\frac{10y}{3}-6))-(\frac{z}{30}-2)(\frac{z}{45}-1)(\frac{z}{60}-1/2)(\frac{z}{75}-\frac{1}{5}) \\
& ((\frac{3x}{200}+\frac{241}{250})(\frac{10y}{3}-7)-(\frac{11x}{600}+\frac{473}{500})(\frac{10y}{3}-6))-(\frac{z}{15}-2)(\frac{z}{15}-4) \\
& (\frac{z}{30}-\frac{1}{2})(\frac{z}{45}-2)((\frac{x}{60}+\frac{427}{500})(\frac{10y}{3}-7)-(\frac{x}{60}+\frac{861}{1000})(\frac{10y}{3}-6)) \\
& +(\frac{z}{15}-3)(\frac{z}{30}-1)(\frac{z}{30}-3)(\frac{z}{45}-\frac{1}{3})((\frac{x}{60}+\frac{451}{500})(\frac{10y}{3}-7)-(\frac{11x}{600}+\frac{901}{1000}) \\
& (\frac{10y}{3}-6))
\end{aligned} \tag{3.3}
$$

and remainder

$$R(x,y,z) = \frac{f(\xi(x,y,z))^{2+2+5}}{2!2!5!}(x-1.8)(x-2.4)(y-1.8)(y-2.1)$$
$$(z-15)(z-30)\dots(z-75)$$

The remainder of (3.3) is almost similar with (3.2), but it is different because the data used are also different.

From (3.3), the least drag using three passive controls with two elliptical cylinders in rear is 0.8286 with configuration $S/D = 1.8$, $T/D = 1.8$ and angle 25°. It is lower than the real data as in Table 3. The plot of this model can be seen in Figures 5(a) and 5(b). Those figures almost same with the real ones, as is Figure 4(a) and 4(b).

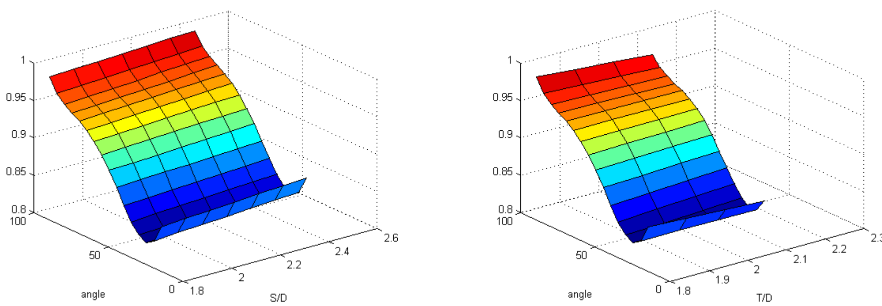


(a) (b)

Figure 4: Real data plot using two elliptical cylinder in rear with fixed (a) $T/D = 1.5$, (b) $S/D = 1.8$

The real data plot of drag coefficient using three passive controls with type-I cylinder in rear as in Table 4 can be seen in Figures 6(a) and 6(b). If it is substituted in (3.1), the result is

$$\begin{aligned}
\tilde{f}(x,y,z) = {} & (\frac{z}{15}-2)(\frac{z}{30}-\frac{3}{2})(\frac{z}{45}-\frac{4}{3})(\frac{z}{75}/75-\frac{6}{5})((\frac{x}{300}+\frac{371}{500})(\frac{10y}{3}-6) \\
& -(\frac{x}{600}+\frac{393}{500})(\frac{10y}{3}-7))-(\frac{z}{15}-3)(\frac{z}{30}-1)(\frac{z}{30}-3)(\frac{z}{45}-\frac{1}{3}) \\
& ((\frac{x}{200}-\frac{409}{500})(\frac{10y}{3}-7)-(\frac{x}{200}-\frac{421}{500})(\frac{10y}{3}-6))+(\frac{z}{15}-2)(\frac{z}{15}-4) \\
& (\frac{z}{30}-\frac{1}{2})(\frac{z}{45}-2)((\frac{x}{300}-\frac{191}{250})(\frac{10y}{3}-7)-(\frac{x}{300}-\frac{783}{1000})(\frac{10y}{3}-6)) \\
& -(\frac{z}{30}-2)(\frac{z}{45}-1)(\frac{z}{60}-1/2)(\frac{z}{75}-\frac{1}{5})((\frac{x}{300}-\frac{181}{200})(\frac{10y}{3}-6) \\
& -(\frac{x}{200}-\frac{901}{1000})(\frac{10y}{3}-7))+(\frac{z}{15}-1)(\frac{z}{15}-3)(\frac{z}{30}-2)(\frac{z}{60}-\frac{3}{2}) \\
& ((\frac{x}{300}+\frac{697}{1000})(\frac{10y}{3}-7)-\frac{143y}{60}+\frac{429}{100}) \qquad (3.4)
\end{aligned}$$



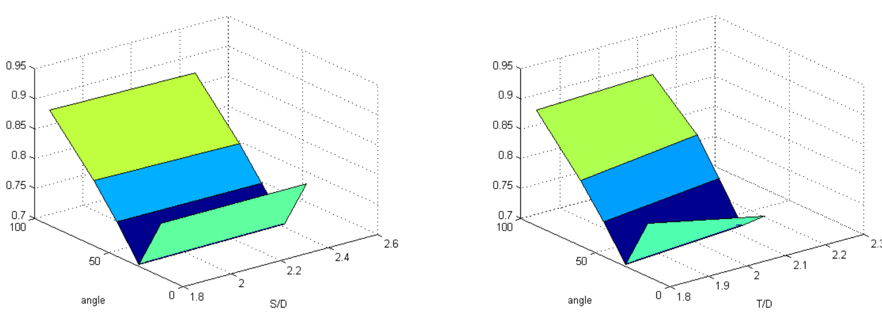
(a) (b)

Figure 5: Plot of drag coefficient model using three passive control with two elliptical cylinder at fixed (a) $T/D = 1.5$, (b) $S/D = 1.8$

and remainder

$$R(x, y, z) = \frac{f(\xi(x, y, z))^{2+2+5}}{2!2!5!}(x - 1.8)(x - 2.4)(y - 1.8)(y - 2.1)$$
$$(z - 15)(z - 30)\ldots(z - 75)$$

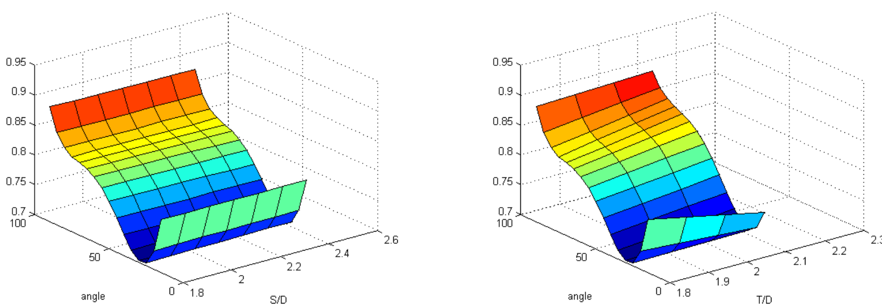
The least drag coefficient by using (3.4) is 0.703 with configuration $S/D = 1.8$, $T/D = 1.8$ and angle $30°$. It is the same result with the real data as in Table 4.



(a) (b)

Figure 6: Real data plot using three passive control with two circular cylinder in rear at (a) $T/D = 1.5$, (b) $S/D = 1.8$

The interval of S/D, T/D and angle should be $x \in [1.8, 2.4]$, $y \in [1.8, 2.1]$ and $z \in [15°, 90°]$ if those are substituted into the (3.2) - 3.4. If the value of S/D, T/D and angle are not in those interval, the error will increase.



(a) (b)

Figure 7: Plot of drag coefficient model at (a) $T/D = 1.5$, (b) $S/D = 1.8$ using three passive controls with two type-I cylinders in rear

4 Conclusion

Three passive control used to reduct the drag coefficient with different distance, angle and shape is studied. The least drag coefficient provided by using three different conditions, two passive controls in the rear, is at $S/D = 1.8$, $T/D = 1.8$ and angle $30°$. The shape passive control in rear which yields the least drag coefficient is a circular cylinder. By using the mathematical model, the least drag coefficient is 0.701 using two circular cylinders in rear at $S/D = 1.8$, $T/D = 1.8$ and angle $30°$.

Acknowledgements. The authors express their appreciation to Lembaga Penelitian dan Pengembangan Masyarakat (LPPM) Institut Teknologi Sepuluh Nopember (LPPM-ITS) Surabaya, Indonesia who, on behalf of the Ministry of Technology and Higher Education, provided the possibility of disseminating the present research based on the research grant under contract No. 868/PKS/ITS/2018 in an International Journal.

References

[1] F. Bouak, J. Lemay, *Passive control of the aerodynamic forces acting on a circular cylinder*, Exp. Therm. Fluid Sci. 16 (1998), 112-121.

[2] C. Imron, Suhariningsih, B. Widodo, T. Yuwono, *Numerical simulation of fluid flow around circular and i-shape cylinder in a tandem configuration*, Appl. Math. Sci. 7 (2013), 5657-5666.

[3] G. M. Ozkan, E. Firat, H. Akilli, *Passive flow control in the near wake of a circular cylinder using attached permeable and inclined short plates*, Ocean Eng. 134 (2017), 35-49.

[4] C. Imron, L. Mardianto, B. Widodo, T. Yuwono, *Mathematical modeling of drag coefficient reduction in circular cylinder using two passive controls at Re=1000*, Math. Comput. Appl. 23 (2018), 2.

[5] C.-H. Kuo, C.-C. Chen, *Passive control of wake flow by two small control cylinders at Reynolds number 80*, J. Fluid Struct. 25 (2009), 1021-1028.

[6] V. Oruc, *Passive control of flow structure around a circular cylinder by using screen* J. Fluid Struc. 33 (2012), 229-242.

[7] H. Ding, C. Shu, K. S. Yeo, D. Xu, *Numerical simulation of flows around two circular cylinders by mesh-free least square based finite different method*, Int. J. Numer. Meth. Fluid 37 (2007), 305-332.

[8] D. Z. Noor, M.-J. Chern, *An Immersed boundary method to solve fluid-solid interaction problems*, Comput. Mech. 44 (2009), 447-453.

[9] A.L.F.L.E. Silva, A. Silveira-Neto, J.J.R. Damasceno, *Numerical simulation of two-dimensional flows over a circular cylinder using immersed boundary method*, J. Comput. Phys. 189 (2003), 351370.

Authors' addresses:

Chairul Imron (correspondent author), Basuki Widodo
Department of Mathematics, Institut Teknologi Sepuluh Nopember,
Jl. Raya ITS, Keputih, Sukolilo, Surabaya, Indonesia.
E-mail: imron-its@matematika.its.ac.id , b_widodo@matematika.its.ac.id

Lutfi Mardianto
Department of Science, Institut Teknologi Sumatera,
Jl. Terusan Ryacudu, Way Hui, Jati Agung,
Lampung Selatan, Indonesia.
E-mail: lutfi.mardianto@ma.itera.ac.id

Triyogi Yuwono
Department of Mechanical Engineering, Institut Teknologi Sepuluh Nopember,
Jl. Raya ITS, Keputih, Sukolilo, Surabaya, Indonesia.
E-mail: triyogi@me.its.ac.id