Diffraction of water waves by a finite circular hollow cylinder in water of infinite depth

M. Hassan, S. N. Bora

Abstract. Within the framework of linear water wave theory, the diffraction of water waves by a hollow circular cylinder in an infinite ocean depth is considered. The whole fluid domain is divided into two regions: interior and exterior regions. Using separation of variables technique, Fourier sine transform and Havelock's expansion theorem, the diffracted potentials in each region are obtained, and consequently the exciting forces and their phase angles are evaluated for different draft and different radius of the cylinder. It is observed that the forces attain higher values in the lower frequency range for higher draft and also for higher radius values. The free surface elevation for both exterior and interior regions is also discussed. As the distance from the cylinder surface increases, the elevation of waves reduces. All the observations are validated through suitable graphs.

M.S.C. 2010: 35J25, 35J05, 35L05.
Key words: Wave diffraction; wave structure interaction; infinite depth; exciting force; Fourier sine transform; Havelock's expansion.

1 Introduction

Ocean waves are a huge but largely untapped energy resource and therefore efforts have been undertaken to explore the potential for extracting energy from these waves. Recently, significant progress has been accomplished on the front of interaction of surface waves with floating structures. A great deal of effort has been made in studying and modelling the wave energy converters for utilization of ocean waves. The present investigation is related to the diffraction of water waves by a vertical hollow cylinder in water of infinite depth. From a practical point of view, emphasis must be laid on the proper positioning of the device so that it is possible to capture waves as large as possible. The efficiency of wave energy and estimation of exciting forces for the wave energy device (oscillating water column) receive considerable attention from the designer of such devices.

A reasonable number of theoretical studies have been performed to analyze the wave motion and wave forces/moments on a structure in finite or infinite water depth. Most of the investigations of water wave diffraction problem are found to be concerned

Applied Sciences, Vol. 21, 2019, pp. 119-135.

mainly with one structure, usually a circular cylinder. The problem for a long floating horizontal cylinder oscillating with small amplitudes about a mean position in water of infinite depth was solved in [17]. By using potential or stream function, he deduced the wave-amplitude at a distance from the cylinder and the added mass of the cylinder due to the fluid motion. Yeung [20] presented a set of added mass and damping coefficients for heave, sway and roll motions of a circular cylinder in finite water depth. He used the method of matched eigenfunctions to solve the hydrodynamic problem in the exterior and the interior regions. Finnegan et al. [4] derived an analytical solution for heave, surge and pitch wave excitation on a floating cylinder in water of infinite depth. They used the method of separation of variables and Havelock's expansion theorem to derive the velocity potentials throughout the fluid domain and presented the wave excitation forces with respect to incident wave frequencies for various draft to radius ratios. Bhatta and Rahman [1] calculated the wave loading due to scattering and radiation for a floating cylinder in water of finite depth. They decomposed the total velocity potential into four: one due to scattering and the other three due to radiation. For each case, they derived the velocity potential by considering interior and exterior regions. Zhu and Mitchell [23] derived a first order analytical solution for the diffraction problem around a hollow cylinder in finite water depth. They used a new approach to analyze the dependence of the solution upon various parameters, as well as the rate of convergence of the series solution. Garrett [5] presented the results for the horizontal and vertical forces and torque on a circular dock. The problem was formulated in terms of potential on the cylindrical surface containing the dock. He used Galerkin's method to solve the problem numerically. Karmakar and Sahoo [10] investigated wave scattering by an articulated floating elastic plate in infinite water depth within the framework of linearized theory of water waves. They presented phase and group velocities, reflection and transmission coefficients and the vertical displacement response of the plate. Martins-Rivas and Mei [15] described linear theory of an oscillating water column installed on a straight coast and calculated the added mass and damping coefficients and chamber pressure. Lovas et al. [12] considered a large circular oscillating water column installed at the tip of a coastal corner in water of uniform depth and solved the diffraction and radiation problems by eigenfunction expansions for an arbitrary apex angle. Chakrabarti and Sahoo [3] solved the problem of obliquely incident surface water waves by a vertical cliff for both the cases of infinite and finite depth. The method of solution was based on the exploitation of logarithmic singularity of the velocity potential at the corner points, where the water surface met the wall in a rather natural manner, while applying Havelock's expansion theorem. Zhang et al. [21] used the method of separation of variables and eigenfunction expansion matching technique to derive the diffracted velocity potentials when linear waves were scattered by an infinitely long rectangular structure parallel to a vertical wall in oblique seas. They presented the influences of the various parameters on the wave force of the structure. The extension of this work, which includes the radiation and diffraction problems of infinitely long submerged rectangular structure parallel to a vertical wall, was also investigated by Zhang et al. [22]. The radiation problem for a floating half immersed sphere in water of infinite depth was solved in [9].

Various works have also been accomplished by researchers for multiple structures. Sahoo [16] investigated the generation of cylindrical surface wave in water of infinite depth of an impermeable circular cylinder surrounded by a coaxial permeable cylinder immersed vertically in the fluid region. He employed Havelock's expansion theorem and various properties of Bessel functions. He also presented the scattering problem when both the cylinders were fixed. Wu et al. [18, 19] investigated the problem of diffraction and radiation for two solid cylinders under different considerations. They obtained the expression for the velocity potential by using the eigenfunction expansion method and investigated the effect of the caisson, approximated by a solid cylinder on the floating cylinder. Hassan and Bora [6, 7] investigated the diffraction problem of water waves by a pair of a coaxial hollow cylinder and a solid cylinder below it in water of finite depth for two cases - when the lower cylinder is bottom-mounted and when it is raised to a finite height. They used the method of matched eigenfunction expansions and separation of variables; and presented sets of exciting forces for different radii of the cylinders and for different gaps between the cylinders. Hassan and Bora [8] also discussed the roll motion for a radiating hollow cylinder placed above a solid cylinder. The dependence of the hydrodynamic coefficients on various parameters was investigated. Chakrabarti [2] solved the mixed boundary value problem of scattering of two dimensional time-harmonic surface water waves in the case of infinite depth and derived the expression for reflection and transmission coefficients. The absolute value of reflection and transmission coefficients were presented graphically. Mandal and Chakrabarti [14] generalised the hybrid Fourier transform for discontinuous but integrable functions. Manam et al. [13] derived mode-coupling relations by using the Fourier integral theorem for the solution of Laplace's equation with higher order derivatives in the boundary conditions for both the cases of finite and infinite water depth. They investigated the problem of wavemaker by oblique water wave scattering caused by cracks in an ice-sheet in the case of infinite depth. As far as the diffraction problem involving a vertical hollow cylinder in water of infinite depth is concerned, the current authors have not come across any investigation that has employed an analytical approach such as Havelock's expansion.

In the present work, the diffraction problem for a semi-submerged vertical hollow cylinder in water of infinite depth is solved. This problem is subsequently modelled based on the assumption of the linearized water wave theory. The method of solution employs the separation of variables technique, Fourier sine transform and a suitable application of Havelock's expansion theorem. The analytical expressions for the diffracted potentials are derived. Using the analytical solutions for the diffraction problem, the effect of radius and draft of the cylinder on the wave force of the cylinder is investigated. Further, the surface wave elevation and its effects are also studied. The results that are obtained are expected to provide some useful information to the designer/engineer of such devices. To produce the maximum benefit, it is important to design structures with proper parameters. The forces and free surface elevation due to diffraction are evaluated and it is hoped that the results will be helpful in designing a suitable device and also in finding an appropriate position for the device so as to extract the maximum energy.

2 Mathematical formulation of the problem

We consider a vertical hollow cylinder of radius R and draft e_1 in water of infinite depth. It is to be noted that some part of the cylinder is above the free surface as shown in Fig. 1. The fluid is assumed to be inviscid, incompressible and homogenous, and the motion irrotational. A right-handed Cartesian coordinate system is defined with $z = 0$ coincident with the centre of the cylinder on undisturbed free surface, $x-$direction pointing in the direction of incoming wave and the $z-$direction pointing vertically downwards. We follow the technique of Yeung [20], Bhatta and Rahman [1] and Zhu and Mitchell [23] of dividing the whole region into two regions. Therefore the problem consists of two regions: an interior region defined by $r \leq R,\ \ 0 \leq \theta \leq 2\pi,\ \ 0 \leq z < \infty$ and an exterior region defined by $r \geq R,\ \ 0 \leq \theta \leq 2\pi,\ \ 0 \leq z < \infty$. The motion is described by the velocity potential $\Phi(r, \theta, z, t) = Re[\phi(r, \theta, z)e^{-i\omega t}]$ where Re denotes the real part, ω the angular wave frequency and $\phi(r, \theta, z)$ the spatial part of the velocity potential. The incident velocity potential of amplitude A and angular frequency ω propagating along the positive $x-$direction in deep water is given by [Kim [11]]

$$\phi_{\text{inc}} = -\frac{gA}{\omega}\ e^{-Kz} \sum_{m=0}^{\infty} \epsilon_m\ i^{m+1} J_m(Kr) \cos m\theta, \tag{2.1}$$

where $i = \sqrt{-1}$, g the gravitational acceleration, K the infinite depth wave number defined by $K = \omega^2/g$, $J_m(.)$ the Bessel function of first kind of order m, and ϵ_m, which is Neumann symbol, defined by

$$\epsilon_m = \begin{cases} 1 & m = 0, \\ 2 & m \geq 1. \end{cases} \tag{2.2}$$

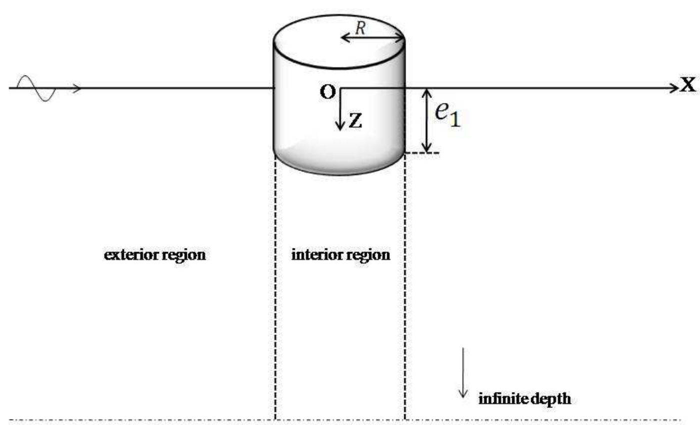

Figure 1: Schematic diagram and definition of fluid subdomains

2.1 The boundary value problem

The boundary value problem is to be set up with appropriate conditions in order to find the diffracted potentials.

2.1.1 The governing equation and boundary conditions

The diffracted velocity potential Φ_d can be written as $\Phi_d = Re[\phi_d(r,\theta,z)e^{-i\omega t}]$, where the spatial part ϕ_d satisfies the following boundary value problem:
(i) governing equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi_d}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi_d}{\partial \theta^2} + \frac{\partial^2 \phi_d}{\partial z^2} = 0, \quad \text{in the fluid region,} \tag{2.3}$$

(ii) linearized free surface condition:

$$\frac{\partial \phi_d}{\partial z} + K\phi_d = 0, \quad (z = 0), \tag{2.4}$$

(iv) bottom condition:

$$\phi_d,\ |\nabla \phi_d| \to 0, \ \ \text{as} \ \ z \to \infty, \tag{2.5}$$

(v) radiation condition:

$$\lim_{r\to\infty} \sqrt{r}\left(\frac{\partial \phi_d}{\partial r} - iK\phi_d\right) = 0. \tag{2.6}$$

3 Method of solution

In order to find the solutions for the boundary value problem, we divide the whole region into two regions, one is the interior region and the other is the exterior region as indicated in Fig. 1. The solutions for the boundary value problem are obtained in these two regions. Therefore the velocity potential ϕ is decomposed into two potentials defined on $r \le R$ and $r \ge R$, respectively, [Zhu and Mitchell, [23]]:

$$\phi = \begin{cases} \phi^{int}, & r \le R, \\ \phi^{ext}, & r \ge R, \end{cases} \tag{3.1}$$

where ϕ^{int} and ϕ^{ext} denote the velocity potential in the interior and exterior regions, respectively. For the continuity of the flow, appropriate matching conditions along the interface between the regions are to be considered.

3.1 Diffracted potentials

The velocity potential in the exterior region is the sum of the incident and diffracted velocity potentials, i.e, $\phi^{ext} = \phi_{\text{inc}} + \phi_d^{ext}$. We apply the method separation of variables and Fourier sine transform to derive the expressions of diffracted velocity potentials. The Fourier sine transform of $\phi(r,z)$ with respect to z is defined by

$$\mathcal{F}_s\{\phi(r,z)\} = \bar{\phi}(r,\xi) = -\int_0^\infty \phi(r,z)\sin\xi z\, dz, \tag{3.2}$$

and its inverse transform by

$$\phi(r,z) = -\frac{2}{\pi}\int_0^\infty \bar{\phi}(r,\xi)\sin\xi z\, d\xi. \tag{3.3}$$

Application of Fourier sine transform to the BVP is justified since the diffracted velocity potential ϕ_d vanishes as $z \to \infty$ (Condition (2.5)). Applying Fourier sine transform to the boundary value problem for $\phi(r,z)$, it is transformed to one for $\bar{\phi}(r,\xi)$. Inverting appropriately, we obtain the expressions for the diffracted potentials in interior and exterior regions, based on the result of Manam et al. [13], as

$$\phi^{int} = \sum_{m=0}^{\infty} \left[a_m \frac{J_m(Kr)}{J_m(KR)} e^{-Kz} + \frac{2}{\pi} \int_0^{\infty} \frac{A_m(\xi) I_m(r\xi) L(\xi,z)\, d\xi}{I_m(\xi R)\, (\xi^2+K^2)} \right] \cos m\theta, \tag{3.4}$$

$$\begin{aligned}\phi^{ext} = \sum_{m=0}^{\infty} \frac{-gA}{\omega} \epsilon_m i^{m+1} &\left[\left(J_m(Kr) + b_m \frac{H_m^{(1)}(Kr)}{H_m^{(1)}(KR)} \right) e^{-Kz} \right. \\ &\left. + \frac{2}{\pi} \int_0^{\infty} \frac{B_m(\xi) K_m(r\xi) L(\xi,z)\, d\xi}{K_m(\xi R)\, (\xi^2+K^2)} \right] \cos m\theta,\end{aligned} \tag{3.5}$$

where $L(\xi,z) = \xi \cos \xi z - K \sin \xi z$, $H_m^{(1)}(.)$ the first kind Hankel function of order m, $I_m(.)$ and $K_m(.)$ are, respectively, the first and second kind modified Bessel functions of order m. Here a_m and b_m are unknown constant coefficients while $A_m(\xi)$ and $B_m(\xi)$ are unknown functions to be determined.

4 Solution for the unknown coefficients

We introduce the appropriate matching conditions by means of continuity of pressure and velocity potentials along the vertical boundary $r = R$ as well as the body surface condition. These conditions are used to find the unknown coefficients. At $r = R$, the conditions which are to be satisfied at physical boundary and interface between the regions are:

$$\frac{\partial \phi^{ext}}{\partial r} = 0, \qquad 0 \le z \le e_1, \tag{4.1}$$

$$\phi^{ext} = \phi^{int}, \qquad e_1 \le z < \infty, \tag{4.2}$$

$$\frac{\partial \phi^{ext}}{\partial r} = \frac{\partial \phi^{int}}{\partial r}, \qquad e_1 \le z < \infty. \tag{4.3}$$

We use the following procedure [Finnegan et al., [4]] to derive an analytical approximation for exterior and interior velocity potentials. In order to derive expressions for b_m and $B_m(\xi)$, large draft values of the cylinder are considered, i.e., $e_1 \to \infty$ in the boundary condition (4.1), which corresponds to evaluating the velocity potential in the exterior region for a cylinder of infinite draft. Now from the boundary condition (4.1) and (3.5), we have

$$K \left[J_m'(KR) + b_m \frac{H_m^{(1)'}(KR)}{H_m^{(1)}(KR)} \right] e^{-kz} + \frac{2}{\pi} \int_0^{\infty} B_m(\xi) \frac{\xi K_m'(\xi R) L(\xi,z)}{K_m(\xi R)(\xi^2+K^2)} d\xi = 0, \tag{4.4}$$

or,

$$b_m \frac{KH_m^{(1)'}(KR)}{H_m^{(1)}(KR)} e^{-kz} + \frac{2}{\pi}\int_0^\infty B_m(\xi)\frac{\xi K_m'(\xi R)L(\xi,z)}{K_m(\xi R)(\xi^2+K^2)}d\xi = -KJ_m'(KR)e^{-kz}, \tag{4.5}$$

where $'$ denotes differentiation with respect to r. Note that the Havelock's expansion theorem [Chakrabarti and Sahoo [3], Chakrabarti [2]], states that if

$$g(z) = C_0 e^{-Kz} + \frac{2}{\pi}\int_0^\infty \frac{C(\xi)[\xi\cos(\xi z) - K\sin(\xi z)]d\xi}{\xi^2+K^2}, \qquad 0<\xi<\infty, \tag{4.6}$$

then,

$$C_0 = 2K\int_0^\infty g(z)e^{-Kz}dz, \tag{4.7}$$

$$C(\xi) = \int_0^\infty g(z)[\xi\cos(\xi z) - K\sin(\xi z)]dz, \tag{4.8}$$

where C_0 and K are constants, and $g(z)$ and its derivative are continuous and integrable in the range $(0,\infty)$. Utilizing this result, we have from (4.5):

$$\frac{b_m\, H_m^{(1)'}(KR)}{H_m^{(1)}(KR)} = -2KJ_m'(KR)\int_0^\infty e^{-2Kz}dz, \tag{4.9}$$

which implies

$$b_m = -J_m'(KR)\frac{H_m^{(1)}(KR)}{H_m^{(1)'}(KR)}, \tag{4.10}$$

and,

$$B_m(\xi) = -\frac{KJ_m'(KR)K_m(\xi R)}{\xi\, K_m'(\xi R)}\int_0^\infty e^{-Kz}(\xi\cos\xi z - K\sin\xi z)dz \tag{4.11}$$

$$\text{giving} \qquad B_m(\xi) = 0. \tag{4.12}$$

The result (4.12) is due to

$$\int_0^\infty e^{-Kz}(\xi\cos\xi z - K\sin\xi z)dz = 0. \tag{4.13}$$

Now using these values of b_m and $B_m(\xi)$ in (3.5) and then applying (4.2),

$$\frac{-gA\epsilon_m i^{m+1}}{\omega}\left[J_m(KR) - J_m'(KR)\frac{H_m^{(1)}(KR)}{H_m^{(1)'}(KR)}\right]e^{-Kz} = a_m e^{-Kz} + \frac{2}{\pi}\int_{e_1}^\infty A_m(\xi)\frac{L(\xi,z)}{\xi^2+K^2}d\xi. \tag{4.14}$$

From the Wronskian of the Bessel's function,

$$J_m(KR)H_m^{(1)'}(KR) - J'_m(KR)H_m^{(1)}(KR) = -\frac{2}{i\pi KR}. \tag{4.15}$$

Therefore from Eq. (4.14),

$$\frac{2gA\epsilon_m i^{m+1}}{\omega i\pi KRH_m^{(1)'}(KR)}\ e^{-Kz} = a_m e^{-Kz} + \frac{2}{\pi}\int_{e_1}^{\infty} A_m(\xi)\frac{L(\xi,z)}{\xi^2+K^2}d\xi. \tag{4.16}$$

From the Havelock's expansion theorem,

$$A_m(\xi) = \frac{2gA\epsilon_m i^{m+1}}{\omega i\pi KRH_m^{(1)'}(KR)}\int_{e_1}^{\infty} e^{-Kz}(\xi\cos(\xi z) - K\sin(\xi z))dz, \tag{4.17}$$

since

$$\int_{e_1}^{\infty} e^{-Kz}(\xi\cos(\xi z) - K\sin(\xi z))dz = -e^{-Ke_1}\sin(\xi e_1). \tag{4.18}$$

Therefore,

$$A_m(\xi) = \frac{-2gA\epsilon_m i^{m+1}}{i\omega\pi KRH_m^{(1)'}(KR)}e^{-Ke_1}\sin(\xi e_1), \tag{4.19}$$

and

$$a_m = \frac{4KgA\epsilon_m i^{m+1}}{\omega i\pi KRH_m^{(1)'}(KR)}\int_{e_1}^{\infty} e^{-2Kz}dz. \tag{4.20}$$

After simplifying and using Wronskian of the Bessel's function from (4.15),

$$a_m = \frac{2\ \epsilon_m\ i^{m+1}\ gA\ \ e^{-2Ke_1}}{\omega i\pi KR\ H_m^{(1)'}(KR)}. \tag{4.21}$$

Therefore, from (3.4), (4.19) and (4.21), the velocity potential in the interior region is obtained as

$$\begin{aligned}\phi^{int} = \sum_{m=0}^{\infty} \frac{2\epsilon_m i^{m+1} gAe^{-Ke_1}}{i\omega\pi KRH_m^{(1)'}(KR)}\Bigg[&\frac{J_m(Kr)e^{-Ke_1}e^{-kz}}{J_m(KR)} \\ &- \frac{2}{\pi}\int_0^{\infty}\frac{I_m(r\xi)L(\xi,z)\sin(\xi e_1)\ d\xi}{I_m(\xi R)\ (\xi^2+K^2)}\Bigg]\cos m\theta.\end{aligned} \tag{4.22}$$

From (3.5), (4.10) and (4.12), the velocity potential in the exterior region is obtained as

$$\phi^{ext} = \sum_{m=0}^{\infty}\frac{-gA}{\omega}\epsilon_m i^{m+1}\left[\left(J_m(Kr) - \frac{J'_m(KR)H_m^{(1)}(Kr)}{H_m^{(1)'}(KR)}\right)e^{-Kz}\right]\cos m\theta. \tag{4.23}$$

5 Wave force

The horizontal exciting force acting on the cylinder can be expressed as $\mathbf{F} = Re[F_c\, e^{-i\omega t}]$, where F_c is the horizontal exciting force independent of time and can be calculated by

$$F_c = i\rho\omega \int_S (\phi^{ext}(R,\theta,z) - \phi^{int}(R,\theta,z))\ n_x ds, \tag{5.1}$$

where S is wetted surface of the cylinder, ρ is the uniform density of water and n_x is the x-component of the unit normal to the surface of the cylinder. The expression of F_c in brackets in (5.1) indicates that by considering this expression, the effect of the internal fluid domain is appropriately taken into account. For the horizontal exciting force, $n_x = -\cos\theta$ and by using the orthogonality of cosine function, the only non-zero value of the integral $\int_0^{2\pi} \cos m\theta \cos\theta d\theta$ is the one corresponding to $m = 1$. Therefore, the horizontal exciting force F_c from the (5.1) with the use of the Wronskian of Bessel's function from (4.15), is given by

$$F_c = \frac{4\rho g A}{K H_1^{(1)'}(KR)} \left[\frac{(1 - e^{-Ke_1})}{K} - e^{-Ke_1} \left\{ \frac{e^{-Ke_1}(1 - e^{-Ke_1})}{K} \right.\right. \\ \left.\left. - \frac{2}{\pi} \int_0^\infty \frac{\sin(\xi e_1)(\xi \sin(\xi e_1) + K\cos(\xi e_1) - K)d\xi}{\xi^3 + \xi K} \right\} \right]. \tag{5.2}$$

The dimensionless horizontal exciting force F_c/μ, where $\mu = \rho g \pi R^2 A$, is given by

$$F_c/\mu = \frac{4}{\pi R^2 K H_1^{(1)'}(KR)} \left[\frac{(1 - e^{-Ke_1})}{K} - e^{-Ke_1} \left\{ \frac{e^{-Ke_1}(1 - e^{-Ke_1})}{K} \right.\right. \\ \left.\left. - \frac{2}{\pi} \int_0^\infty \frac{\sin(\xi e_1)(\xi \sin(\xi e_1) + K\cos(\xi e_1) - K)d\xi}{\xi^3 + \xi K} \right\} \right]. \tag{5.3}$$

However, when forces are calculated for a fixed draft and varying radius, the non-dimensionalizing parameter is taken as $\lambda = \rho g \pi e_1^2 A$.
Free surface elevation in the exterior region is given by

$$\eta = Re\left[\frac{i\omega}{g} \phi^{ext} e^{-i\omega t} \right]. \tag{5.4}$$

Therefore the non-dimensional free surface elevation for $m = 1$, in the exterior region, can be obtained as

$$\eta/A = Re\left[2i \left(J_1(Kr) - \frac{J_1'(KR)}{H_1^{(1)'}(KR)} H_1^{(1)}(Kr) \right) e^{-i\omega t} \right] \cos\theta. \tag{5.5}$$

In a similar way, free surface elevation in the interior region corresponding to $m = 1$ is given by

$$\eta/A = Re\left[\frac{-4e^{-Ke_1}}{\pi K R H_1^{(1)'}(KR)} \left(\frac{J_1(Kr)e^{-Ke_1}}{J_1(KR)} \right.\right. \\ \left.\left. - \frac{2}{\pi} \int_0^\infty \frac{I_1(r\xi) L(\xi,z) \sin(\xi e_1)\, d\xi}{I_1(\xi R)\ (\xi^2 + K^2)} \right) e^{-i\omega t} \right] \cos\theta. \tag{5.6}$$

6 Numerical results and discussions

Having obtained the unknown coefficients $a_m, b_m, A_m(\xi)$ and $B_m(\xi)$ appearing in (3.4)-(3.5) and consequently in (5.1), we proceed to study the horizontal exciting forces acting on the cylinder and the corresponding phase angle and free surface elevation. We investigate the various effects of different parameters on the force and wave elevation for this infinite depth problem.
Figure 2 shows the non-dimensional horizontal exciting force F_c/μ on the cylinder against KR with different values of the draft of the cylinder, to be precise for $e_1/R = 0.5,\ 1.0,\ 3.0$. It is observed that the exciting force increases within the lower values of wave numbers as the draft of the cylinder increases and that for higher values of wave numbers, the values of the exciting force steadily diminish. Consequently, the peak value occurs for the highest draft value, $e_1/R = 3.0$ to be precise.

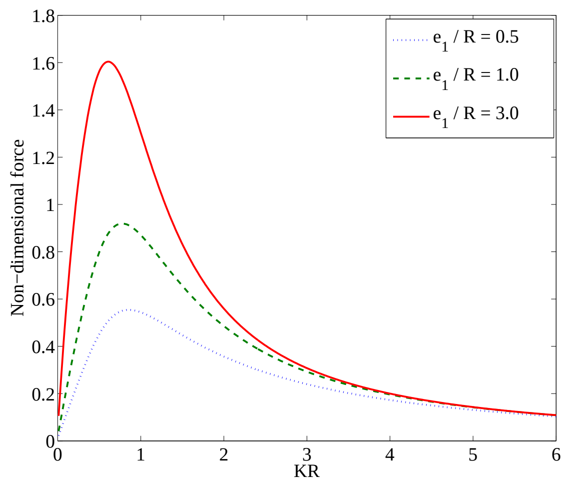

Figure 2: Non-dimensional horizontal exciting forces (F_c/μ) versus KR for different values of draft of the cylinder with fixed radius $R = 1.2$ metre.

Figure 3 shows the phase angle of the exciting forces F_c/μ for different drafts of the cylinder for a fixed radius, $R = 1.2$ metre, corresponding to the exciting forces in Figure 2. It is observed that, irrespective of the draft values of the cylinder, the phase angle reduces rapidly as wave number increases. Moreover, the phase angle has the same values for all drafts considered. Figures 4 and 5 show the non-dimensional exciting forces F_c/λ and their corresponding phase angles against the wave number for different values of the radius of the cylinder for a fixed draft $e_1 = 0.8$ metre. In Figure 4, it is observed that the exciting forces increase as the radius of the cylinder increases in the lower range of wave number. For higher range, the forces decrease steadily and each force tends to a fixed value later on. For higher radius values, the exciting force attains higher values. Also, as the values of the radius increases, the peak value shifts towards left. Figure 5 shows that the absolute value of the phase angle is higher for higher radius value. As the wave number increases, the phase angle

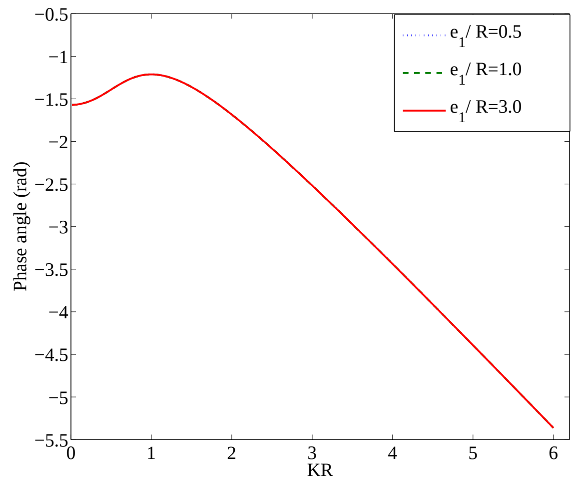

Figure 3: Phase angle of the force F_c/μ, versus wave number for different values of draft of the cylinder with fixed radius of the cylinder $R = 1.2$ metre.

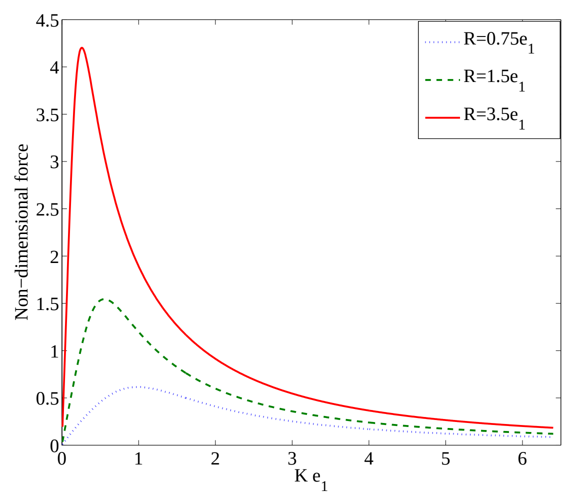

Figure 4: Non-dimensional force F_c/λ, versus wave number for different values of the radius of the cylinder with fixed draft $e_1 = 0.8$ metre.

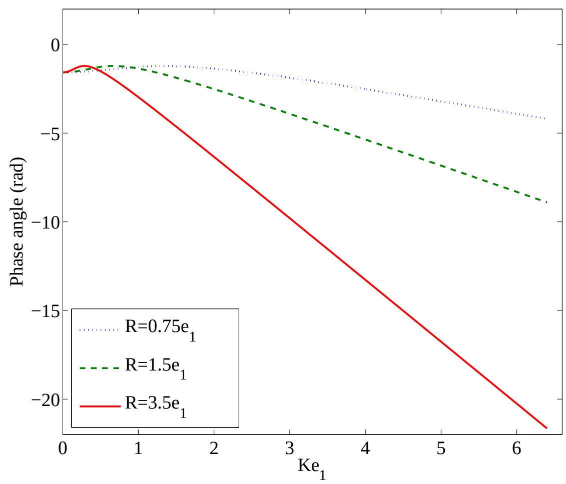

Figure 5: Phase angle of the force F_c/λ, versus wave number for different values of the radius of the cylinder with fixed draft $e_1 = 0.8$ metre.

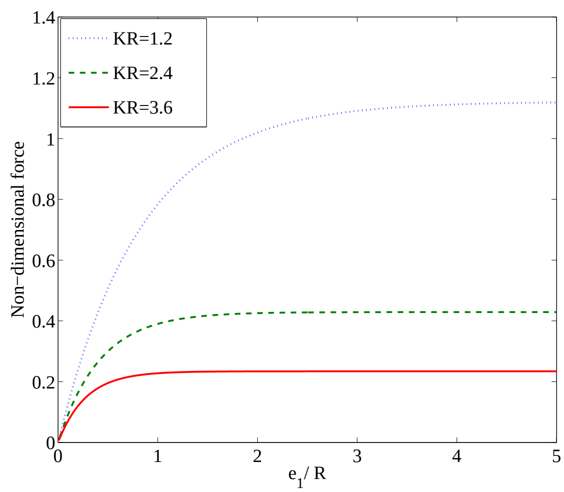

Figure 6: Non-dimensional exciting forces versus draft of the cylinder for different values of wave numbers with fixed radius of the cylinder $R = 1.2$ metre.

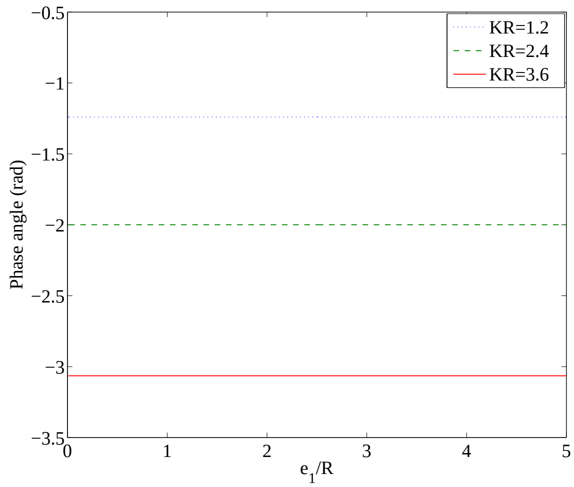

Figure 7: Phase angle of the force F_c/μ, against e_1/R for different values of wave number with fixed $R = 1.2$ metre.

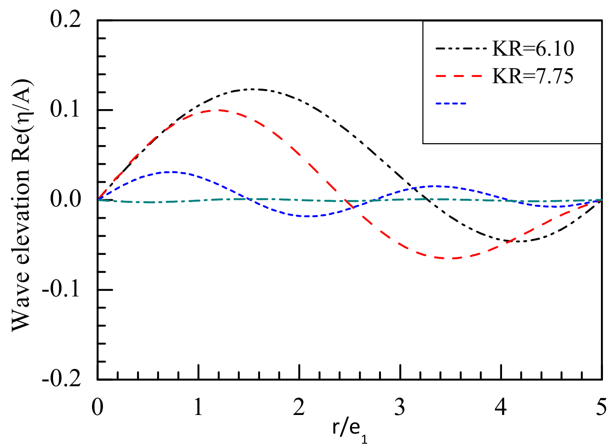

Figure 8: Free surface elevation in the interior region for different wave numbers with $R/e_1 = 6.25, e_1 = 0.8$ metre, $\theta = 0, m = 1$.

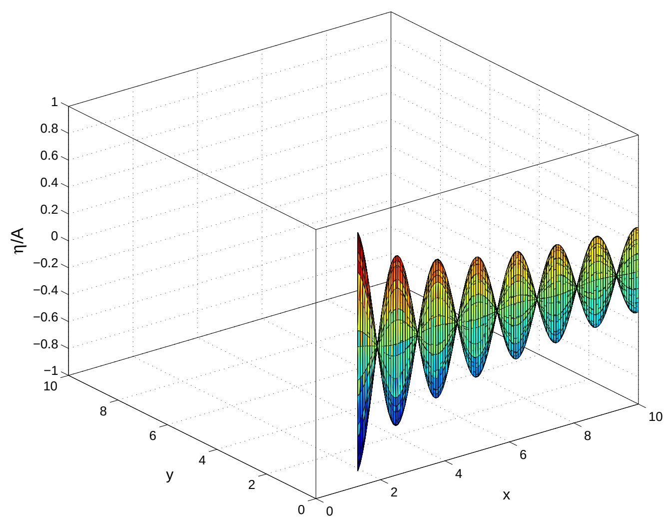

Figure 9: Surface plot for wave elevation in exterior region with $K = 2.5493\, metre^{-1}, \theta = 0, m = 1$.

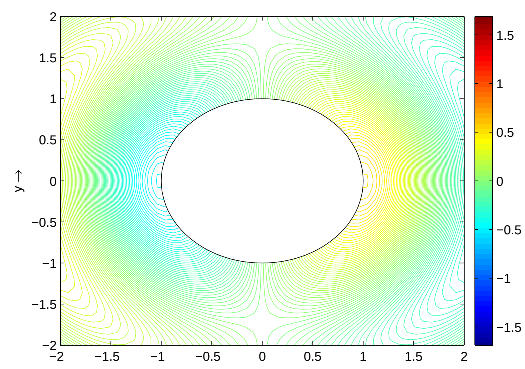

Figure 10: Contour plot for wave elevation in exterior region with $R = 1.0$ metre, $K = 2.5493\, metre^{-1}, t = 1.2566s, \theta = 0, m = 1$.

decreases and there is no appreciable change in the values of the phase angle for very low wave numbers, i.e., for long waves.

Figures 6 and 7 show the non-dimensional exciting forces F_c/μ and their phase angles against draft of the cylinder for different values of wave numbers for a fixed radius $R = 1.2$ metre. It is observed from Figure 6 that the exciting force increases as the draft increases and each force attains a constant value for very high draft values. It is also seen that the force values are higher corresponding to the lower values of wave number. It is also interesting to note that the greater variation of increase takes place for smaller values of e_1/R. Further, this increase is much more noticeable for long waves. For shallow water, the change in force values is much more significant than those for intermediate and deep water depth. Figure 7 shows that the phase angle is constant for each wave number. It does not depend on the draft of the cylinder and it is further observed that the absolute value of the phase angle is higher for higher wave numbers. Figure 8 shows the plots of wave elevation inside the cylinder for a fixed draft and a fixed radius. It is observed that for higher wave numbers, the elevation is less and the highest elevation corresponds to the wave number $KR = 6.10$. Figures 9 and 10 give us the surface plot and contour plot, respectively, for the free surface elevation in the exterior region.

7 Conclusion

The diffraction problem of water waves by a semi-submerged hollow cylinder is formulated and solved to analyze wave motion/force in water of infinite depth under the assumptions of linearized theory of water waves. The whole fluid domain is divided into two regions: interior and exterior ones. We derive the expressions for the diffracted potentials in each region by using the method of separation of variables and Fourier sine transform. In order to determine the unknown coefficients, which are present in the expressions of diffracted potentials, Havelock's expansion theorem and matching conditions along the virtual boundary are utilized. The hydrodynamic influences of various parameters on exciting forces, their phase angles and free surface elevation are discussed. With regard to the effect of the draft of the cylinder, it is observed that the exciting force increases as the draft of the cylinder increases and the peak value occurs only for the lower values of wave numbers. The same observation is noticed for various values of the radius of the cylinder for a fixed draft. With regard to the phase angle of the exciting forces, it decreases as the wave number increases for a fixed radius. We also graphically present the free surface elevation in the interior and exterior regions. It is observed that as the distance from the surface of the cylinder increases in the exterior region, the amplitude of the oscillation decreases.

Acknowledgements. The first author wishes to thank North Eastern Regional Institute of Science and Technology, Nirjuli, Arunachal Pradesh, India for providing necessary facilities and Department of Science and Technology SERB (through approval YSS/14/000884), Govt. of India for the support provided for completing the work. Both authors express their deep gratitude to the Editor-in-Chief Prof. Dr. Constantin Udriste and the Managing Editor Prof. Dr. Vladimir Balan for accepting the manuscript.

References

[1] D. D. Bhatta and M. Rahman, *On the scattering and radiation problem for a cylinder in water of finite depth*, Int. J. of Engg. Sci., 41 (2003), 931-967.

[2] A. Chakrabarti, *On the solution of the problem of scattering of surface-water waves by the edge of an ice cover*, Proceedings of the Royal Society of London A, 456 (2000), 1087-1099.

[3] A. Chakrabarti and T. Sahoo *On the incoming water waves against a vertical cliff*, Proceeding Indian Acad. of Sci. (Math. Sci.), 107(1), (1997), 89-93.

[4] W. Finnegan, M. Meere and J. Goggins, *The wave excitation forces on a truncated vertical cylinder in water of infinite depth*, J. Fluid Structure, 40 (2013), 201-213.

[5] C. J. R. Garrett, *Wave forces on a circular dock*, J. of Fluid Mech., 46 (1971), 129-139.

[6] M. Hassan and S. N. Bora, *Exciting forces for a pair of coaxial hollow cylinder and bottom-mounted cylinder in water of finite depth*, Ocean Engg., 50 (2012) 38-43.

[7] M. Hassan and S. N. Bora, *Exciting Forces for a wave energy device consisting of a pair of coaxial cylinders in water of finite depth*, J. of Marine Sci. and Appl., 12 (2013), 315-324.

[8] M. Hassan and S. N. Bora, *Rotational motion due to the interaction of water waves with a pair of coaxial cylinders in water of finite depth*, J. of Adv. Res. in Appl. Math., 6(1), (2014), 48-65.

[9] T. Havelock, *Waves due to floating sphere making periodic heaving oscillation*, Proceedings of the Royal Society of London A, 231 (1955), 1-7.

[10] D. Karmakar and T. Sahoo, *Scattering of waves by articulated floating elastic plates in water of infinite depth*, Marine Structure, 18 (2005), 451-471.

[11] C. H. Kim, *Nonlinear Wave and Offshore Structure, Advanced Series on Ocean Engineering*, World Scientific Publishing Co. Pvt. Ltd., Singapore, (2008).

[12] S. Lovas, C. C. Mei and Y. Liu, *Oscillating water column at a coastal corner for wave power extraction*, Appl. Ocean Res., 32 (2010), 267-283.

[13] S. R. Manam, J. Bhattacharjee and T. Sahoo, *Expansion formula in wave structure interaction problems*, Proceedings of the Royal Society of London A, 462 (2006), 263-287.

[14] B. N. Mandal and A. Chakrabarti, *A generalisation to the hybrid Fourier transform and its application*, Appl. Math. Letter, 16 (2003), 703-708.

[15] H. Martins-Rivas and C. C. Mei, *Wave power extraction from an oscillating water column along a straight coast*, Ocean Engg., 36 (2009), 426-433.

[16] T. Sahoo, *On the generation of water waves by cylindrical porous wave maker*, Acta Mechanica, 126 (1998), 231-239.

[17] F. Ursell, *On the heaving motion of a circular cylinder on the surface of a fluid*, Quarterly J. of Mech. and Appl. Math., 2(2), (1949), 218-231.

[18] B. J. Wu, Y. H. Zhang, Y. G. You, D. S. Jie and Y. Chen, *On diffraction and radiation problem for two cylinders in water of finite depth*, Ocean Engg., 33 (2006), 679-704.

[19] B. J. Wu, Y. H. Zhang, Y. G. You, X. Y. Sun and Y. Chen, *On diffraction and radiation problem for a cylinder over a caisson in water of finite depth*, Int. J. of Engg. Sci., 42 (2004), 1193-1213.

[20] R. W Yeung, *Added mass and damping of a vertical cylinder in finite-depth water*, Appl. Ocean Res., 3(3), (1981), 119-133.

[21] Y. H. Zhang, Y. M. Shen, Y. G. You, B. J. Wu and L. Rong, *Scattering of linear water waves by a fixed and infinitely long rectangular structure parallel to a vertical wall in oblique seas*, Journal of Waterway Port Coastal and Ocean Engg., 132(2), (2006), 106-113.

[22] Y. H. Zhang, Y. M. Shen and J. Tang, *Radiation and diffraction of linear water waves by an infinitely long submerged rectangular structure parallel to a vertical wall*, Ocean Engg., 34 (2007), 69-82.

[23] S. P. Zhu and L. Mitchell, *Diffraction of ocean waves around a hollow cylindrical shell structure*, Wave Motion, 46, (2009), 78-88.

Authors' addresses:

Mohammad Hassan
Department of Mathematics,
North Eastern Regional Institute of Science and Technology,
Itanagar, Arunachal Pradesh, 791 109, India.
E-mail: mdhassan000@gmail.com

Swaroop Nandan Bora (corresponding author)
Department of Mathematics,
Indian Institute of Technology Guwahati,
Guwahati, Assam, 781 039, India.
E-mail: swaroop@iitg.ac.in