Game theory application of Monti's proposal for European government bonds stabilization

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Abstract. The aim of this paper is to present an extension of Mario Monti's proposal for an anti-spread shield, presented at the European Council on 28-29 June 2012, about the stabilization of yield on European government bonds. In particular, by using Game Theory, we focus our attention on three players: a large bank with immediate access to the market of government bonds (hereinafter called Speculator, our first player), the European Financial Stability Facility (EFSF, the second player) and the State in economic troubles (our third player). We propose the introduction of a tax on financial transactions (cashed directly by the State in economic troubles), which hits only the speculative profits. We show that the above tax would be able to limit the speculation, and, even in case of speculation on its government bonds, the State would manage to pull itself out of the crisis. Finally, we also propose a cooperative solution that enables all economic actors involved (the Speculator, the EFSF and the State) to obtain a profit.

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1 Introduction

Lately, the global economic crisis has grown, affecting also States which are considered important in the European economic framework (such as, for example, Italy). One of the causes of the crisis is the exponential growth of government bonds yields, which has increased the public debt of the States, despite of the economic countermeasures adopted by the European Union.

1.1 The growth in European government bonds yields

Up to May 2011 the Italian 10-years and 3-years government bonds had offered a yield of approximately 4.80% and 3.15%, while in December 2011 their value grew over 7.50% (see [27]).

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In Fig. 1 (source: Commerzbank Research) we can see the trend upwards of Irish, Portuguese and Spanish 10-years government bonds from January 2010 to July 2011 (see [28]).

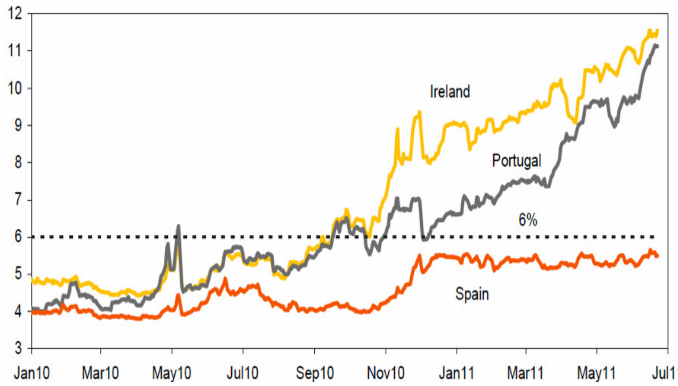


Figure 1: Trend of Irish, Portuguese and Spanish 10-years government bonds

1.2 Economic measures adopted by the European Union

In May 2010, the 27 EU States have created the European Financial Stability Facility (EFSF), also called saving-State fund (since July 2012 it will be replaced by the European Stability Mechanism), in order to financially support the State-members (when they are in troubles), in order to preserve the financial stability of the Eurozone.

The EFSF may issue bonds or other debt instruments in the market to raise funds in order to:

- provide loans to Euro Area members in financial troubles;
- recapitalize banks;
- buy government bonds.

The European countries that, at this time, are advantaging (or are going to advantage) from the aid by the EFSF are: Ireland, Portugal, Greece, Spain and Cyprus.

1.3 Monti's anti-spread proposal

In the European Council of 28-29 June 2012 in Brussels, Italy has fought for the approval of the "anti-spread shield": the States that respects certain budget parameters can ask to use the saving-States funds in order to buy government bonds and reduce interest rates about them. Moreover, the virtuous States should not be subjected to additional checks by the European Troika (composed by the EU Commission, European Central Bank and International Monetary Fund). The Italian Prime Minister Mario Monti said that, according to the agreement reached at the European Council, the "virtuous" States may require the help by the EFSF without the intervention by the Troika and without signing a new commitments program. In short, the instrument adopted for spread moderation is the use of the EFSF, and the main novelty is that the States with positive budget accounts will benefit from the help of EFSF without any EU external commissioner.

Limits of Monti's proposal. The major limit of the Monti's proposal is that:

- the use of the saving-States fund is a subsequent measure, which which tries to cure the system but it does't prevent the disease.

Is it possible to fight preventively the speculation on government bonds?

In our opinion the answer is yes.

1.4 Our model

By using game theory (for a complete study of a game see [3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 19]), we propose a possible method to stabilize the government bonds market of the States in economic troubles, without any losses of collective gain. In this way, by the introduction of a simple but effective tax, the market would be able - by itself - to reduce speculation and, consequently, the yields on government bonds, without further economic measures at global level: thus the States with financial troubles could finally begin (hopefully) a slow economic recovery. In [7], the authors assumed to introduce, by regulatory authorities, a tax that affects the gain obtained through speculative trades involving the government bonds and they proposed that the tax increased the reserves of the BCE. In this paper, we assume that the tax is cashed directly by the State in financial troubles and we study the effects of this normative measure.

1.5 Financial preliminaries

Here we clarify two financial concepts we shall consider in the present article.

1. Short-selling of bonds is a financial transaction involving the sale of bonds without having their property, hoping to buy them later at a lower price. So, the short-seller would realize a profits. About government bonds, the hope of short-sellers consists in an increase of yield of government bonds.

2. The government bonds are not usual goods with a purchase unit price and a sale price. The concept that characterizes them is the yield, which depends upon the interest given on the capitals "loaned" to the State.

2 Basic building blocks of our model

2.1 Description of the financial interaction

The normal-form 3-player game G, we propose to model our financial interaction, requires a construction which takes place on 3 times in a very short period, say time 0, time 1/2 and time 1.

- At time 0, the Speculator (first player) may decide:

1. to short-sell government bonds, in order to obtain greater profits betting on a greater future yield of the bonds;
2. to intervene not in the government bonds market.

- At time 1/2, the EFSF decides how much to intervene in the bonds market in order to limit the growth of the bonds yield. In this way, even in case of lack of demand of government bonds, the issuer State finds the funds necessary to the national financial requirement.
- At time 1 the Speculator close its position (opened at time 0) by buying government bonds.

Remark. During the game, we will refer to the yield on government bonds. When we pass from one time period to another one, we would have to discount or capitalize the values to "transfer in time", but, since the interest i (used in the capitalization and discount factors) is much lower than the yield of government bonds, we assume i equal to 0. Therefore, in this model, the values referred to different times are not capitalized or discounted. This choice is reinforced by the time-shortness of the period in which the players take action.

2.2 Strategies of our model: game theory perspective

- Our game $G = (f, >)$ is a three player game. Our three players are:
  1. a Speculator;
  2. the EFSF;
  3. a State.
- In what follows, M represents the quantity expressed in money of bonds issued by the States (our third player): the State decides to issue a quantity equal to M of its own government bonds, in order to face its financial commitments. The strategy set of our third player is formed by one only strategy, it is the set {M}.
- Our first player, the Speculator, may choose to short-sell government bonds, in order to cause an increase in their yield and so to obtain profits. The Speculator chooses among strategies in $E := [0, 1]$; each $x \in [0, 1]$ represents the percentage of the quantity of government bonds M that the Speculator decides to buy, depending on it wants:
  1. to make no financial transaction, i.e. strategy $x = 0$;
  2. to short-sell xM government bonds, i.e. any strategy $x \in]0, 1]$.
- On the other hand, the EFSF, our second player, operates in the bonds market in consequence of the operation of the first player. It chooses any strategy $y \in F := [0, 1]$, which represents the percentage of the quantity of government bonds M that the EFSF purchases; in this case:
  1. to intervene not in the government bonds market is represented by the strategy $y = 0$);

2. to buy government bonds of the State in economic troubles corresponds to a strategy $y > 0$;

- The strategy set of our game G is the square $E \times F \times \{M\}$, it is a square of the three dimensional space $\mathbb{R}^3$; nevertheless, since the third player has only one strategy, we can identify this square with the square $S := E \times F$. In Fig. 2 we see the bi-strategy space $E \times F$ of our game G.

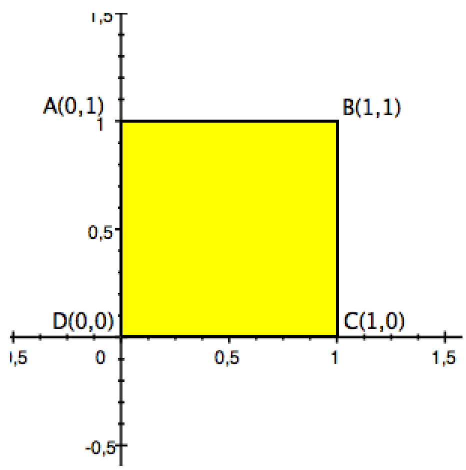


Figure 2: The bi-strategy space of the game

3 The no tax game

3.1 The payoff function of the Speculator in the no tax game

The payoff function of the Speculator, say f_1, which is the function representing the gain of the first player, is given, at any bi-strategy (x, y) of the game, by the quantity expressed in money of purchased bonds xM, multiplied by the difference

$$R_1(x, y) - R_0,$$

between the value at time 1 of the yield to be cashed (at time 1 the Speculator buys the same amount xM of securities that it has short-sold at time 0) and the value at time 0 of the yield to be paid (at time 0 the Speculator short-sells a certain amount of government bonds).

The mechanism could practically proceed as follows:

1. at 0, Speculator short-sells xM of State bonds in the secondary market to a certain purchaser, at rate R_0, receiving

$$xM - R_0 xM;$$

2. at 1/2, immediately after 0, EFSF sees a rate $R_{1/2}(x) > R_0$ of the State bonds and buys yM (at that interest rate), to contain the growth of the yield: after that the yield goes down to a rate

$$R_1(x, y) \in]R_0, R_{1/2}(x)[;$$

3. immediately after, at 1, Speculator buy xM from the State, paying

$$xM - R_1(x, y)xM.$$

At this time the total payoff of the Speculator is

$$f_1(x, y) = (xM - R_0 xM) - (xM - R_1(x, y)xM) = xM(R_1(x, y) - R_0);$$

4. at the maturity of the bonds, State pays xM to the Speculator and the Speculator pays xM to the purchaser: at the end of the day, the circle is closed.

So, the payoff function $f_1 : S \to \mathbb{R}$ of the Speculator, we are interest in, is defined by:

$$f_1(x, y) = xM(R_1(x, y) - R_0), \tag{3.1}$$

for every bi-strategy (x, y) in the square S, where:

1. xM is the amount of government bonds that the Speculator short-sells at time 0 (choosing to play x);

2. $R_1(x, y)$ is the value of the government bonds yield at time 1, after the players 1 and 2 have chosen the bi-strategy (x, y). Now, we have to assume a particular form of the function R_1, also in consideration of the time-shortness of the horizon $(0, 1/2, 1)$.

   Assumption 1. We assume an affine dependence of R_1 on the strategies, for sake of first approximation, it is given by

   $$R_1(x, y) = R_0 + mx - ny,$$

   where:

   - R_0 is the yield that remunerates the capitals "loaned" to the State at time 0;
   - $m > 0$ is a marginal coefficient indicating the incidence of x on $R_1(x, y)$;
   - $n > 0$ is a marginal coefficient indicating the incidence of y on $R_1(x, y)$.

   The value $R_1(x, y)$ depends on x and y, if the Speculator and the EFSF intervene in the government bonds market with strategies x and y different from 0, because:

   - behavioral finance suggests ([1, 14, 18]) the vertical diffusion of a behavior (the so-called "herd behavior" [[16, 24]]) similar to that adopted by the great investors;

- just this decrease (or increase) in demand has a positive (or negative) effect on the interest charged on the government bond ([2, 15, 17, 23]).

Note. We are assuming as a first approximation hypothesis a linear dependence on x and y,

3. R_0 is the value of the yield at time 0. It is a constant because our strategies x and y have no impact on it.

The payoff function of the Speculator. Therefore, recalling the definition of function R_1 and of function f_1, we have

$$f_1(x, y) = Mx(mx - ny). \tag{3.2}$$

3.2 The payoff function of the EFSF in the no tax game

The payoff function $f_2 : S \to \mathbb{R}$ of the EFSF, representing the algebraic gain of the EFSF, is defined, at any (x, y), by the multiplication of the quantity expressed in money of government bonds yM (that the EFSF buys at time $1/2$) by the bonds yield at time $1/2$, $R_{1/2}(x)$. So the payoff function of the EFSF is given by:

$$f_2(x, y) = (yM)R_{1/2}(x), \tag{3.3}$$

where:

1. yM is the quantity of bonds expressed in money that the EFSF buys at time $1/2$;
2. $R_{1/2}(x)$ is the bonds yield at time $1/2$.

Assumption 2. We assume that the function $R_{1/2}$ is affine and defined by

$$R_{1/2}(x) = R_0 + mx = R_1(x, y) + ny.$$

The strategy x has impact on $R_{1/2}$ because at time 0 the Speculator has already operated in the market, changing the bonds yield.

The payoff function of the EFSF. Recalling the definitions of functions $R_{1/2}$ and f_2, we have

$$f_2(x, y) = My(R_0 + mx). \tag{3.4}$$

3.3 The payoff function of the State in the no tax game

Finally, after the payoff functions of the Speculator and of EFSF, we consider the payoff function of the State, our third player.

The payoff function, we are interested in, is defined, for any (x, y) by the quantity M of issued government bonds, multiplied by the yield R_0 (which the State would have had to pay without the intervention on the market of the Speculator and of the EFSF) minus the interests which the State should really pay (Mx by the yield $R_1(x, y)$, My by the yield $R_{1/2}(x)$ and $(1-x-y)R_1(x, y)$, interests which actually the

State pays in consequence of the strategies x of the Speculator and y of the EFSF). We have

$$\begin{aligned} f_3(x,y) &= MR_0 - MyR_{1/2}(x) - M(1-y)R_1(x,y) = \\ &= MR_0 - My(R_1(x,y)+ny) - MR_1(x,y) + MyR_1(x,y) = \\ &= MR_0 - Mny^2 - M(R_0 + mx - ny) = \\ &= M(ny-mx) - Mny^2, \end{aligned}$$

Note that the value $f_3(x,y)$ can be written in the meaningfull way:

$$My(R_0 - R_{1/2}(x)) + M(1-y)(R_0 - R_1(x,y)).$$

Assumption 3. *We assume, now, an intervention of the ECB to eliminate the term $-Mny^2$.*

Indeed, that intervention eliminates completely the bad influence of the Speculator on the yield payed by the State to EFSF on time 1/2:

- *the addition of the term $-Mny^2$ is the unique one that makes the function f_3 equal to $M(R_0 - R_1)$.*

Payoff function of the State. Finally, the payoff function of the State, with the further intervention of ECB, is defined by:

$$f_3(x,y) = M(R_0 - R_1(x,y)) = M(-mx+ny), \tag{3.5}$$

for every (x,y) in S. It is defined, for any (x,y) by the quantity M of issued government bonds, multiplied by the difference between the yield R_0 (which the State would pay without the intervention on the market of the Speculator and of the EFSF) and the yield $R_1(x,y)$ (which actually pays in consequence of the strategies x of the Speculator and y of the EFSF).

3.4 The payoff function of the no tax game

It is the function $f : S \to \mathbb{R}^3$ given, for every $(x,y) \in E \times F$, by:

$$f(x,y) = (Mx(mx-ny), My(R_0+mx), M(-mx+ny)) \tag{3.6}$$

Geometrical interpretation of the payoff function. We note that the function f is nothing more than a smooth parametric surface with compact support. The support of the surface f is (by definition) its image $f(S)$ and it is also, from a Game theory perspective, the payoff space of the three person game $(f,>)$. In the following figures we show the payoff space of f.

3.5 Payoff space of the no tax game

Let $g : S \to \mathbb{R}^2$ be the vector function from the bi-strategy space $E \times F$, having its components equal to the first two components of f. In [7], we have already studied the payoff space $g(E \times F)$ of our game $(g,>)$ (see Fig.6). For sake of illustration we chose $m = n = 1/2$, $M = 1$, $R_0 = 1/4$.

The results must now be interpreted according to the payoff function of the issuer State. Recalling the equation 3.5, we note that:

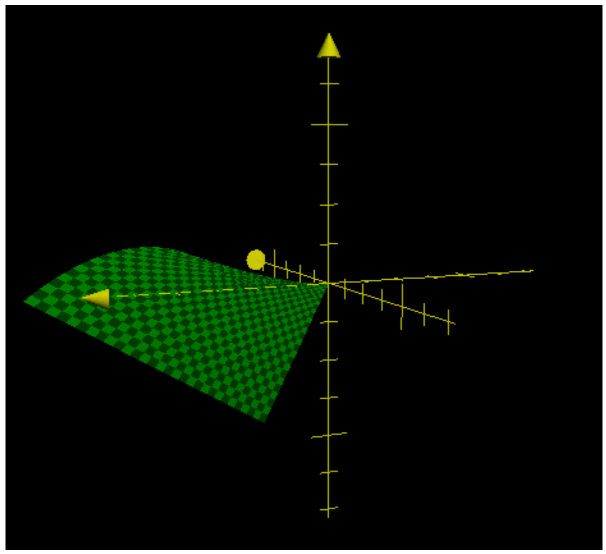


Figure 3: The payoff space of the game f

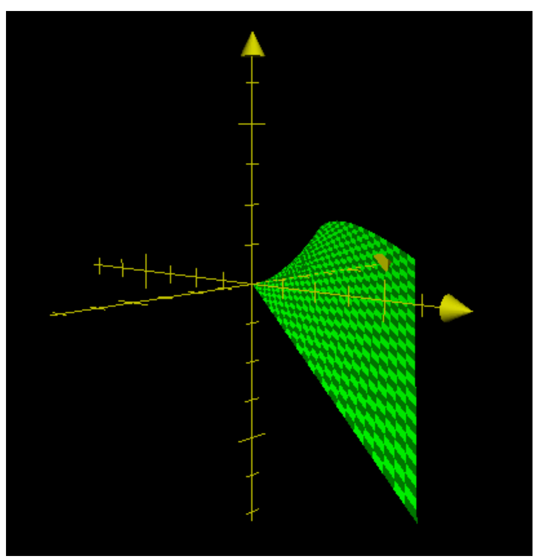


Figure 4: The payoff space of the game f

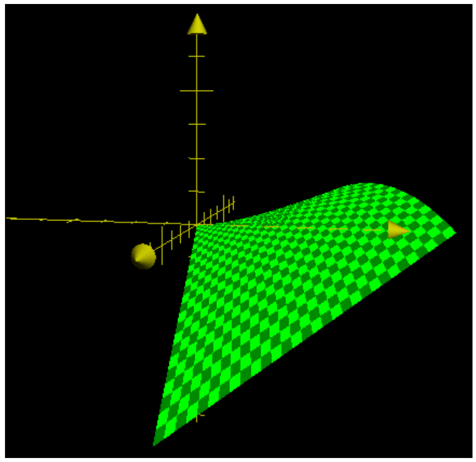


Figure 5: The payoff space of the game f

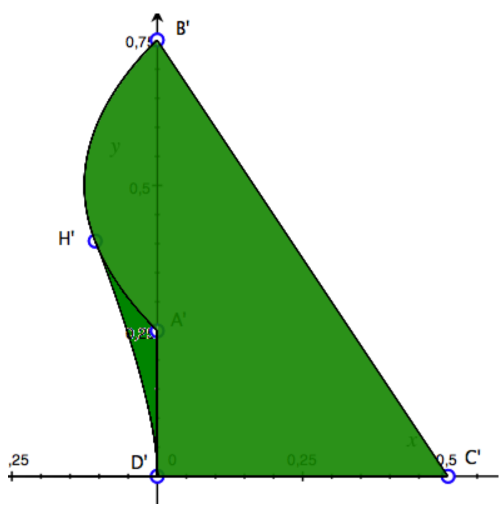


Figure 6: The payoff space of the no tax game $(g, >)$

- if the two players arrive on the point D of the bi-strategy space, the State obtains a zero profit ($f_3(0,0) = 0$). This solution is undesirable because it does not solve the problems of the State and not gives breath to its economy;
- if the two players arrive on the point B of the bi-strategy space, the State obtains a profit of
$$f_3(1,1) = M(-m+n).$$
This solution is not good for the State because the coefficient m has a negative effect on its profit and could eliminate the positive effect of n;
- if the two players arrive on the side $]B,C]$, the State obtains a profit of
$$f_3(1,y) = M(-m+ny).$$
This solution is very dangerous for the State because its profit is lower than the point B, until to arrive to minimum in C where the State have a negative profit $f_3(1,0) = -mM$;
- if the two players arrive in $[A,D[$, the State obtains a profit of
$$f_3(x,1) = M(-mx+n).$$
This solution is the best for the State because its profit increases until to arrive to maximum in $A = (0,1)$ where the State has a positive profit $f_3(0,1) = Mn$.

According to these considerations, it is morally, ethically and economically desirable that the Speculator and the EFSF arrive to the point $A = (0,1)$, so that the bonds yield R_1 goes down as more as it is possible and the State comes out of the crisis.

Remark. The points A' and B' of the payoff space could have a different collective gain about the three subjects of our game. In fact, if we arrive to the point A' the State in economic troubles has a profit equal to Mn, the Speculator gains 0, the EFSF gains MR_0 and so the total collective gain is $M(R_0+n)$. Instead if we arrive to point B' the State has a profit equal to $M(-m+n)$, the Speculator gains $M(m-n)$, the EFSF gains $M(R_0+m)$ and so the total collective gain is $M(R_0+m)$. We note that the payoff point A' and B' have the same collective gain only if $m = n$.

3.6 Study of the no tax game

As already studied in [7], without the introduction of the tax, the defensive cross and the Nash equilibrium lead most likely to the point B. But the point B is not a good point of arrival for the State in economic troubles, because the yield on its bonds remains at high levels and unchanged. At this point, the EFSF could consider splitting the gain $M(R_0+m)$ obtained in the most likely Nash equilibrium B with the issuer State, in order to cancel the negative effect $-mM$ of the strategy $x = 1$ by the Speculator. Thus, the EFSF would give mM to the State, taking for itself MR_0.

But this seemingly simple solution is not feasible for several reasons:

1. the EFSF has a policy that usually does not interfere with that one of European States, therefore this kind of action is difficult to accomplish;

2. the "payback" to the State by the EFSF could have very long timescales, and therefore the State could sink even deeper into economic crisis;
3. the amount cashed by the State cancels its loss (suffered because of the strategy of Speculator), but the yield on government bonds would remain high, and then this action is not a solution: it would simply postpone the problem over time without dealing with it. In fact, if in the future other financial institutions buy government bonds (also without speculative purposes), the State should pay them a yield which is remained at unsustainable levels, ending in bankruptcy.

For these reasons, it is important to find a method that allows the State to prevent speculations and not to be constantly "cured". Anyway, it is straightforward that a vaccine made only once, is better than a medicine taken continuously, medicine which in future will lose its effectiveness.

In these regards, a possible solution is the cooperation between the two players:

- the Speculator and the EFSF play the strategies $x = 0$ and $y = 1$, respectively, arriving to the payoff A' and dividing the collective gain by contract;
- at the same time, the yield on government bonds of the State decreases.

But the cooperative solution was not satisfactory. In fact, the cooperative solution is difficult to implement because the EFSF would have to achieve an agreement with the Speculator before that it plays a strategy $x > 0$, and it is almost impossible to know in advance the intentions of all the potential speculators in the bonds market. For this reason, it is necessary a preventive economic measure.

4 A new anti-speculative proposal

Many authors [26, 22, 25] have recognized the benefits of taxation on financial transactions and we follow exactly this lane of thought to model our game.

In [7] we propose, in order to avoid speculations of the first player about the current and future yield of the government bonds, to introduce - by regulatory authorities - a tax (see also [20]) that affects the gain obtained through speculative trades involving the government bonds.

We assume that the tax increased the reserves of the EFSF (see also [21]).

We obtained the payoff space in figure 7.

Moreover, we have noted the shift of the Nash equilibrium and defensive cross (which are again coincident) from strategic point B to the strategic point A. The point A is an optimal point for the State in economic troubles, because the yield of its government bonds decreases, and - at the same time - it is a quite good point for the Speculator and the EFSF because they are on the weak maximal boundary.

The new anti-speculative proposal. We propose that the tax is not cashed by EFSF, but directly by the State in economic troubles. In this way, even if the Speculator intervenes in government bonds market by a strategy $x \neq 0$, the State

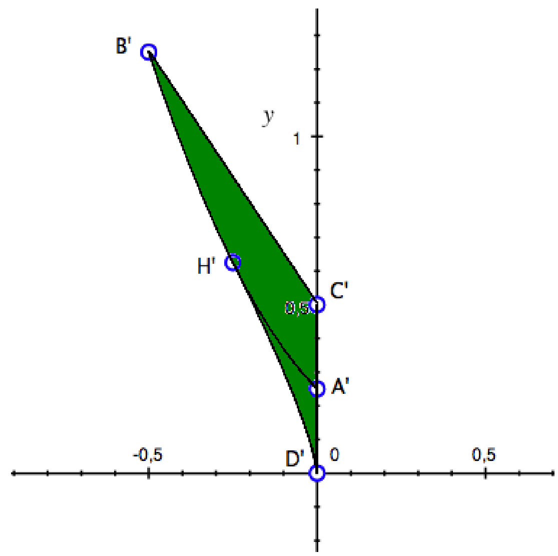


Figure 7: The payoff space of the game with tax cashed by the EFSF

cancels (at least in part) the upward effect caused by the strategy x on the government bonds yield.

5 The Game with the tax cashed by the State

The construction of our new tax game will proceed by a modification of the no tax game. We have to define a new vector function $f : S \to \mathbb{R}^3$.

5.1 The new payoff of the Speculator

We impose that the tax eliminates completely the possibility of speculative profits created by the Speculator itself, below we formally give this hypothesis:

Assumption 4. *We impose that the tax is equal to the incidence mx of the strategy x on the function yield R_1.*

By the introduction of the tax, recalling the Eq. 3.2, that is the definition of the payoff functionf_1 of the no tax game,

$$f_1(x, y) = xM(R_1(x, y) - R_0) = Mx(mx - ny),$$

the payoff function of the Speculator becomes:

$$f_1(x, y) = Mx(R_1(x, y) - T(x, y) - R_0),$$

for every (x, y). We have assumed that

$$T(x, y) = mx,$$

for every (x, y) in S. So, we obtain:

$$f_1(x, y) = -nMxy, \tag{5.1}$$

for any (x, y).

5.2 The new payoff function of the EFSF

We assume that the introduction of the tax has no effect on the payoff function of the EFSF. So its payoff function is equal to that in Eq. 3.4, that is

$$f_2(x, y) = M(R_0 + mx)y,$$

for every (x, y) in S.

5.3 The new payoff function of the State

In case of tax adoption, to obtain the payoff function of the State, we have to add the tax paid by the Speculator to the old payoff function of the State in the no tax case.

The tax paid by the Speculator is given by the amount of government bonds xM purchased by the Speculator, multiplied by the applied tax $T(x, y)$.

In formula, the new payoff function of the State with the tax is given by

$$f_3(x, y) = M(R_0 - R_1(x, y)) + MxT(x, y), \tag{5.2}$$

for every (x, y).

Now, recalling that

$$R_1(x, y) = R_0 + mx - ny$$

and assuming that $T(x, y) = mx$, we have

$$f_3(x, y) = M(-mx + ny) + Mmx^2. \tag{5.3}$$

5.4 The payoff function of the game with tax cashed by the State

The payoff function of the game with tax cashed by the State is, eventually, the vector function $f : S \to \mathbb{R}^3$ defined by

$$f(x, y) = (-nMxy, yM(R_0 + mx), M(-mx + ny) + Mmx^2),$$

for every (x, y) in the strategic square S.

Geometrical interpretation of the payoff function. We note that the function f is nothing more than a smooth parametric surface with compact support. The support of the surface f is (by definition) its image $f(S)$ and it is also, from a Game Theory perspective, the payoff space of the three person game $(f, >)$. In the following figures (8, 9 and 10) we show the payoff space of f.

Remark. The graphical representation of the payoff space of the game $(f, >)$ gives us an good qualitatively idea of the principal features of f, but this is not enough at our purposes. To perform an accurate analysis of the game and of its possible solutions, we examine accurately a section of the game f, namely, the section g determined by the two components of f.

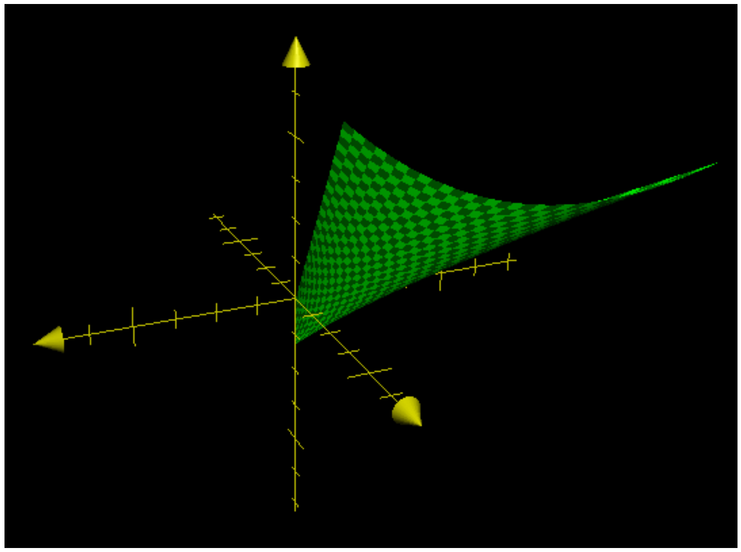


Figure 8: The payoff space of the game f

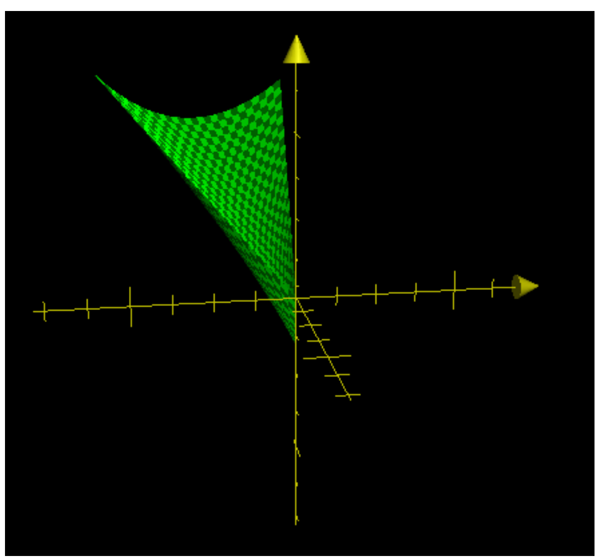


Figure 9: The payoff space of the game f

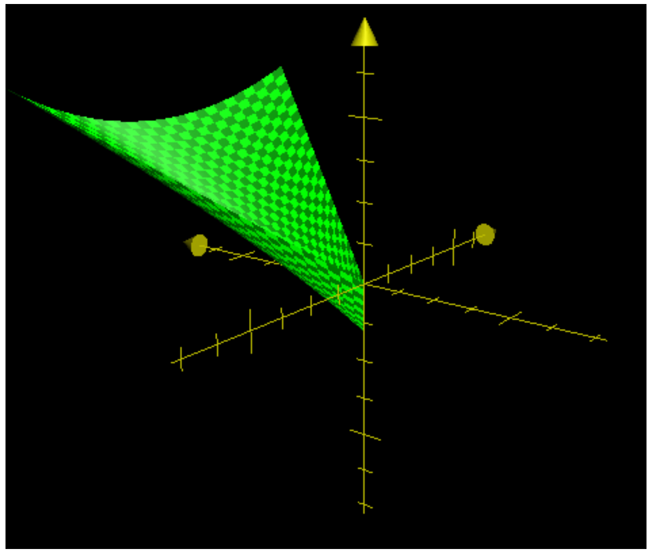


Figure 10: The payoff space of the game f

6 Complete study of the section game $g = (f_1, f_2)$

Let g be the vector function defined on the bi-strategy space $S = E \times F$ and having its components equal to the first two components of f, that is the function $g : S \to \mathbb{R}^2$ defined by

$$g(x, y) = (-nMxy, yM(R_0 + mx)),$$

for every (x, y) in the strategic square S. in this section we want to examine completely this section of f.

6.1 Critical space of the section g

As we are dealing with a non-linear game, we need to study its critical zone. In order to find the critical zone of the game g, we consider the Jacobian matrix of g and we set its determinant equal 0.

About the gradients of g_1 and g_2, we have

$$\nabla g_1(x, y) = (-nMy, -nMx) = -nM(y, x)$$

and

$$\nabla g_2(x, y) = (mMy, M(R_0 + mx)) = mM(y, x) + M(0, R_0).$$

The determinant of the Jacobian matrix of g is

$$\det J_g(x, y) = -M^2 ny(R_0 + mx) + M^2 mnxy.$$

Therefore, the critical space of the game $(g, >)$ is

$$Z_g = \{(x, y) \in S : -M^2 ny(R_0 + mx) + M^2 mnxy = 0\} = \{(x, y) \in S : y = 0\}.$$

Fig. 11 shows the (strategy) critical area of our game (the segment $[D, C]$).

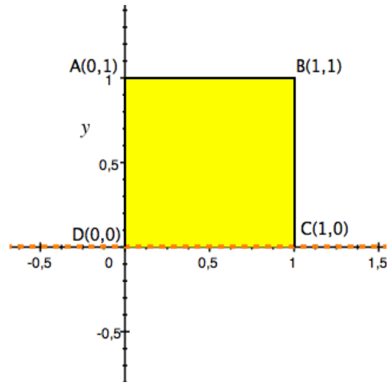


Figure 11: The critical zone of the game with tax cashed by the State

6.2 Payoff space of the game g

We transform, by the function g, all the sides of bi-strategy square $E \times F$ and the critical space Z_g of the game $(g, >)$. We so obtain the topological boundary of the payoff space $g(E \times F)$ of our game $(g, >)$ in Fig. 12.

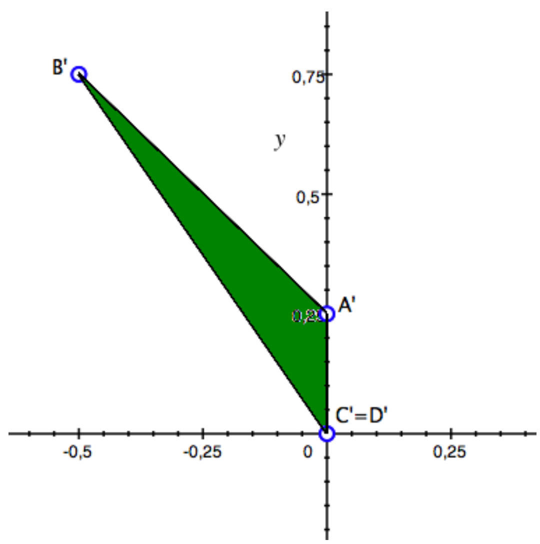


Figure 12: The payoff space of the game with tax cashed by the State (with $M = 1$ and $n = m = 1/2$)

6.3 Exam of f_3 on $\partial^*_g(S)$

Now, we exam the payoff function of the State on the maximal boundary of the game g. This examination is necessary, to our purpose, because, if the first two players decide to come to an agreement, then this agreement shall be a point on the Pareto boundary. The Pareto boundary of the game is the segment $[A, B]$ (since its image by g is the maximal boundary of the payoff space $g(S)$).

Recalling the Eq. 5.3, that is

$$f_3(x, y) = M(-mx + ny) + mMx^2,$$

we note that:

$$f_3(x, 1) = M(-mx + n) + mMx^2,$$

so that

$$f_3(., 1)'(x) = -mM + 2mMx > 0$$

if and only if $x > 1/2$. Since $f_3(A) = f_3(B) = Mn$, the most convenient points for the State, from the payoff point of view, are A and B.

Remark (The essential part of the game f). Observe, by the way, that the partial derivative D_2f_3 is the positive number Mn, so that the maximum of f_3 is attained on the segment $[A, B]$; similarly, the yield function R_1 attains its minimum on that segment (in the point A); so that the only part of the strategy space which is really significant for the three players is the Pareto boundary $[A, B]$.

Note, moreover, that the maximum collective gain is surely attained on this segment (this requires few calculations on the function $c := f_1 + f_2 + f_3$) and precisely it is attained:

(a) in A if $n > m$, and it is $M(R_0 + n)$;
(b) in B if $n < m$, and it is $M(R_0 + m)$;
(ab) on the entire segment $[A, B]$ if $m = n$, and it is $M(R_0 + n)$.

- if the two players arrive to the point A, the yield

  $$R_1(x, y) = R_0 + mx - ny,$$

  paid by the State for its government bonds, attains his minimum on $[A, B]$ (and also on the entire strategy space S):

  $$\min_S R_1 = \min_{[A,B]} R_1 = R_1(A) = R_0 - n,$$

  and thus the State improves his bond yield with respect to the initial condition R_0, contrasting also the crisis (note that the tax in this situation is the lowest possible, $T(A) = 0$);

- if the two players arrive in the point B, the yield paid by the State for its government bonds is

  $$\max_{[A,B]} R_1 = R_1(B) = R_0 + m - n,$$

  the bond yield is the worst possible on the Pareto boundary $[A, B]$. Nevertheless, the State cashes the Tax on the government bonds speculations (from the Speculator), tax which attains in this case his maximum value $T(B) = m$; and, as we already have seen, this tax determine the same maximum payoff for the state on $[A, B]$; this could encourage the State to emerge from the crisis, especially if $m \leq n$, as this condition is equivalent to $R_1(B) \leq R_0$;

- if the two players arrive on the side $]A, B[$, the yield paid by the State for its government bonds is less than R_0 less than at the point A', and the tax cashed cancels only partially the effect of the strategy x of the Speculator.

According to these considerations, is ethically and economically desirable that the Speculator and the EFSF arrive to the strategy points $A = (0, 1)$ or $B = (1, 1)$.

Remark. Comparing the payoff space of the no tax game (see Fig. 6) and that one of the game with the tax cashed by the State (see Fig. 12), we note that the latter seems smaller. At first glance, it seems that the tax has caused a loss of global wealth. But is not so: in fact the collective profit of the three players remains unchanged. The big difference is that the Speculator is effectively unable to make a profit, and the "loss" caused by the tax increases the gain of the State.

6.4 Nash equilibria of the game g

Recalling Eq. 5.1, that is

$$f_1(x, y) = -nMxy,$$

we have

$$\partial_1 f_1(x, y) = -Mny.$$

This derivative is positive if $y < 0$, and so:

$$B_1(y) = \begin{cases} \{0\} & \text{if} \quad y > 0 \\ E & \text{if} \quad y = 0 \end{cases}.$$

Recalling also the Eq. 3.4, that is

$$f_2(x, y) = M(R_0 + mx)y,$$

we have

$$\partial_2 f_2(x, y) = M(R_0 + mx).$$

This derivative is everywhere positive, and so:

$$B_2(x) = \{1\},$$

for every $x \in E$.

In Fig.13 we see, in red, the inverse graph of B_1, and, in blue, the graph of B_2.

The set of Nash equilibria, that is the intersection of the two best reply graphs, is

$$\mathrm{Eq(B_1, B_2) = (0, 1) = A.}$$

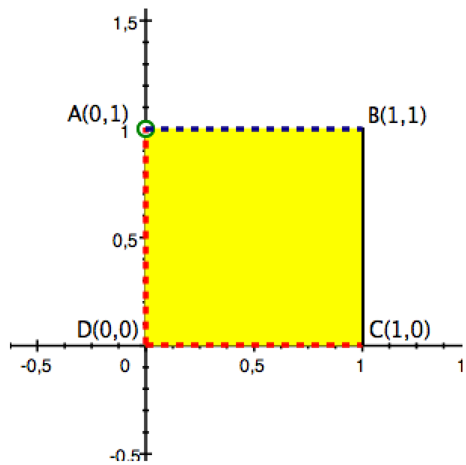


Figure 13: The Nash equilibria of the game with tax cashed by the State

6.4.1 Analysis of Nash equilibria.

The Nash payoff in the general game f is

$$f(0,1) = (0, MR_0, Mn).$$

The Nash equilibria is optimal for the two players, because it belongs to the proper maximal Pareto boundary. The selfishness, in this case, brings to the maximal boundary, even if the payoff solution advantages only second and third players.

The Nash equilibria of the game with tax is good for the State that issues the government bonds, because the yield (that the State has to pay) goes downward because of the strategy $y = 1$, while the strategy $x = 0$ does not affect upward. In a word, the State finances its public spending with a lower government bonds yield, and this allows to face economic crisis.

6.5 Defensive phase of the game g

When a players is not aware of the will of the other, or when it is, by its nature, cautious or risk averse, then it chooses strategies minimizing its own loss: the defensive strategies.

Conservative value of a player. It is defined as the supremum of its worst gain function $f_i^\sharp$. Therefore, the conservative value of the Speculator is

$$v_1^\sharp = \sup_{x \in E} f_1^\sharp(x),$$

where $f_1^\sharp$ is the worst gain function of the Speculator, and it is given by

$$f_1^\sharp(x) = \inf_{y \in F} f_1(x, y).$$

Recalling the Eq. 5.1, that is

$$f_1(x, y) = -nMxy,$$

we have:

$$f_1^\sharp(x) = \inf_{y \in F}(-nMxy).$$

Since the offensive correspondence of the EFSF is

$$O_2(x) = \begin{cases} \{1\} & \text{if} \quad x > 0 \\ F & \text{if} \quad x = 0 \end{cases},$$

we obtain:

$$f_1^\sharp(x) = \begin{cases} -nMx & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x = 0 \end{cases}.$$

In Fig. 14 we represent the function $f_1^\sharp$, when $M = 1$ and $n = 0.5$.

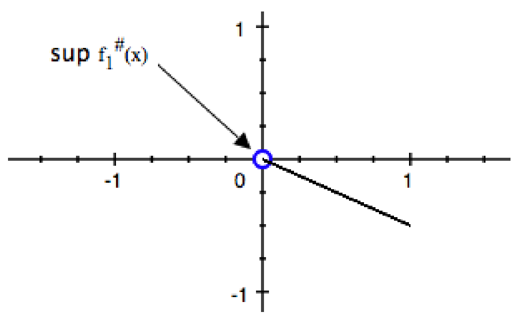


Figure 14: The worst gain function of the Speculator in the game with tax cashed by the State

So the defensive strategy of the Speculator is given by $x_\sharp = 0$, and the conservative value of the Speculator is

$$v_1^\sharp = \sup_{x \in E} \inf_{y \in F} xM(-ny) = 0. \tag{6.1}$$

On the other hand, the conservative value of the EFSF is given by

$$v_2^\sharp = \sup_{y \in F} f_2^\sharp,$$

where $f_2^\sharp$ is the worst gain function of the EFSF. It is given by

$$f_2^\sharp(y) = \inf_{x \in E} f_2(x, y).$$

Recalling the Eq. 3.4, that is

$$f_2(x, y) = M(R_0 + mx)y,$$

we have:

$$f_2^\sharp(y) = \inf_{x \in E} M(R_0 + mx)y.$$

Since the offensive correspondence of the Speculator is

$$O_1(y) = \begin{cases} \{0\} & \text{if} \quad y > 0 \\ E & \text{if} \quad y = 0 \end{cases},$$

we obtain:

$$f_2^\sharp(y) = \begin{cases} yMR_0 & \text{if} \quad y > 0 \\ 0 & \text{if} \quad y = 0 \end{cases}.$$

In Fig. 15 we show $f_2^\sharp$ with $M = 1, R_0 = 0.25$.

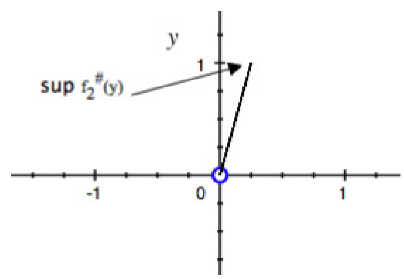


Figure 15: The worst gain function of EFSF in the game with tax cashed by the State

So the defensive (or conservative) strategy of the EFSF is given by $y_\sharp = 1$, and the conservative value of EFSF is

$$v_2^\sharp = \sup_{y \in F} \inf_{x \in E} M(R_0 + mx)y + Mmx^2 = MR_0. \tag{6.2}$$

Therefore, the conservative bi-value is

$$v_g^\sharp = (v_1^\sharp, v_2^\sharp) = (0, MR_0).$$

Choosing $M = 1$ and $R_0 = 0.25$ we have $(0, 1/4)$.

Conservative crosses. They are the bi-strategies of the type $(x_\sharp, y_\sharp)$, that is $A = (0, 1)$.

Analysis of conservative cross. The conservative cross A can be considered good because it is located on the proper maximal Pareto boundary, and it is also good for the State. In fact, recalling that

$$R_1(x, y) = R_0 + mx - ny,$$

the yield paid on government bonds passes from R_0 to $R_0 - n$, because of the strategy 0 of the Speculator and the strategy 1 of the EFSF. For completeness we note that the conservative value of the third player in the game f is

$$v_3^\sharp = \inf_{y \in F} \inf_{x \in E} (M(-mx + ny) + Mmx^2) = -(1/4)mM.$$

While the conservative the conservative value of the third player in the game f restricted to $[A, B]$ is

$$v_3^\sharp = \inf_{(x,y)\in[A,B]} (M(-mx + ny) + Mmx^2) = M(n - (1/4)m).$$

7 Cooperative solution of the game g

In the following, we assume that $n < m$, our sample payoff space changes.

Remark. The value n could be lower than the value m because the purchase of government bonds by the EFSF could be less accepted by the market players. In fact, the action of the EFSF could be seen as a behavior dictated (also, or even only) by political motivations, and not by economic reasons (like for example low risk and high profit about government bonds).

So, for the cooperative solution we assume that $n = 1/3$ and $m = 1/2$. In Fig. 16 we can see the new payoff space. We note that the point B' moves upward.

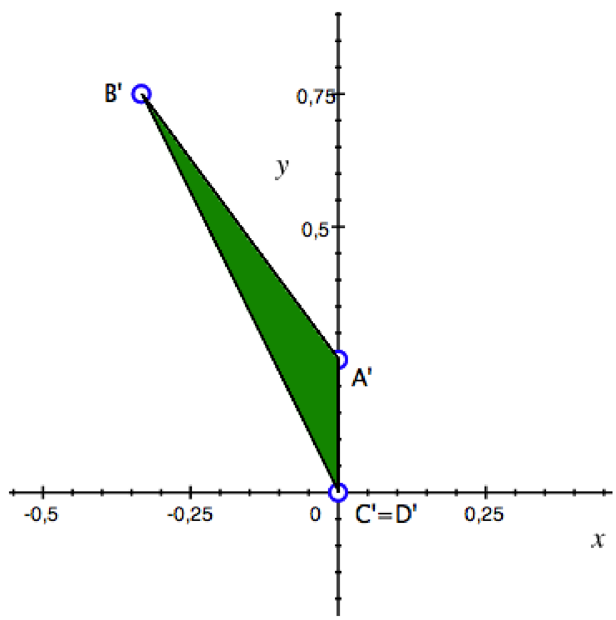


Figure 16: The payoff space of the game $(g, >)$ with $n < m$

7.1 Transferable utility solution

The Speculator and the EFSF play the strategies $x = 1$ and $y = 1$ in order to arrive at the point B', which is the point with maximum collective gain and which allows the State to gain the value 1/3 (in fact, the increase of the bond yield is totally balanced by the tax cashed). After that, the EFSF divides its gain 5/12 with the Speculator by contract.

Financial point of view. The Speculator plays the strategy $x = 1$, and the EFSF shares with the Speculator its gain $w = 5/12$, obtained arriving to B'. At the same time, the State in economic troubles gains.

For a possible quantitative division of max collective gain $w = 5/12$, between the EFSF and the Speculator, we apply a non-standard Kalai-Smorodinsky method. We solve the bargaining problem (Γ, β), where β is the infimum of the payoff space of g and Γ is the portion of the transferable utility Pareto boundary of the payoff space which has β as infimum, with respect to the Nash equilibrium A', as we explain.

We proceed finding the infimum β, which is

$$\beta := \inf \partial_* g(E \times F) = (-1/3, 0);$$

then we join it with the Nash equilibrium of the game $(g, >)$, which is given by $A' = (0, 1/4)$. We can see Fig. 18 in order to make us more aware of the situation.

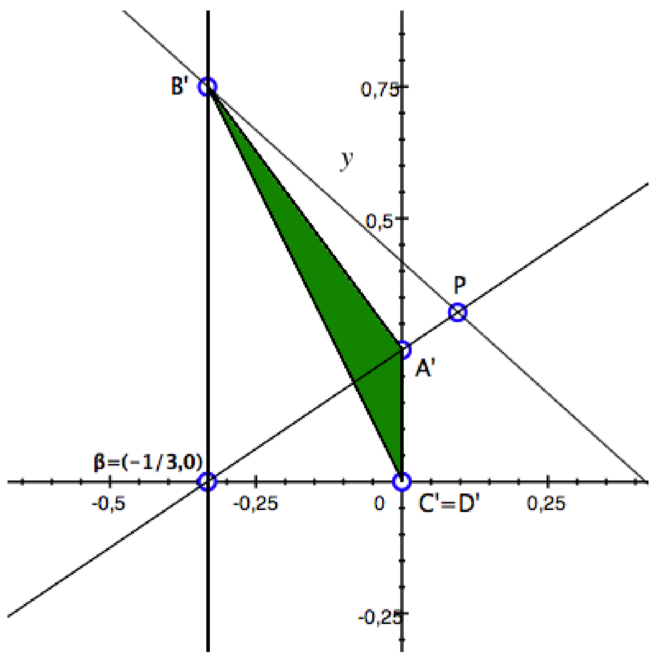


Figure 17: The cooperative solution of the game $(g, >)$ with $n < m$

The coordinates of the intersection of the point P, between the straight line of maximum collective gain (i.e. $X + Y = 5/12$) and the straight line joining the infimum of the maximal Pareto boundary with the Nash equilibrium (i.e. the line $Y = (3/4)X + 1/4$), give the desirable division of the collective gain $w = 5/12$ between the two players.

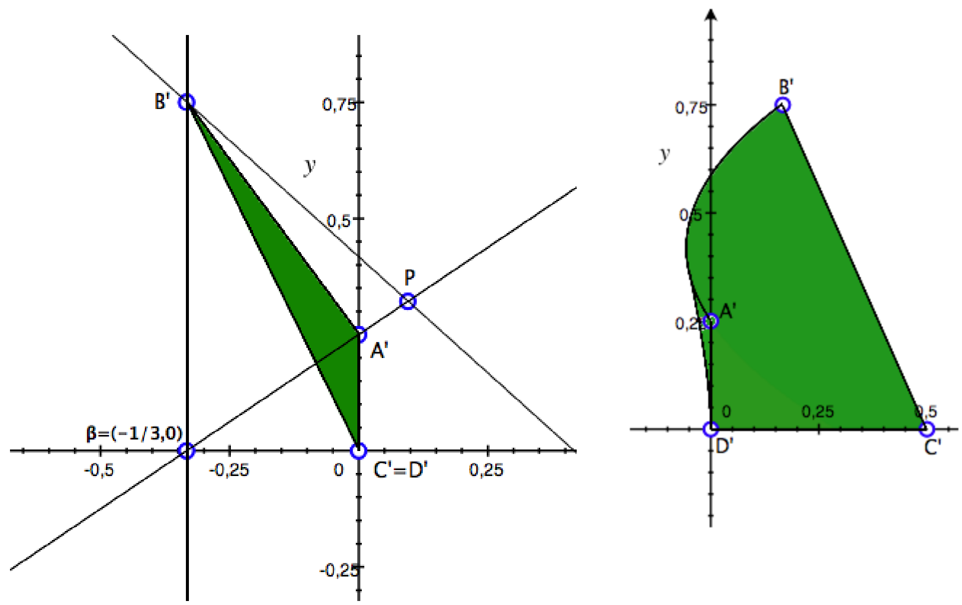
In order to find the coordinates of the point P is enough to put in a system of equations $X + Y = 5/12$ and $Y = (3/4)X + 1/4$. We have $X = 2/21$ and $Y = 9/28$.

Thus $P = (2/21, 9/28)$ suggests as solution that the Speculator receives $2/21$ by contract by the EFSF, while at the EFSF remains the gain $9/28$.

Remark. We note that

$$P = (2/21, 9/28) \gg A' = (0, 1/4) = v^\sharp = \mathrm{Eq}(B_1, B_2).$$

We note moreover, as represented in the last figure that: the point B' of the no-tax game gives to the speculator $1/6$, while in the tax game, the cooperative solution pays to the speculator $2/21$.



(a) Payoff space of tax game if $n < m$ ($n = 1/3, m = 1/2$) (b) Payoff space of the no-tax game if $n < m$ ($n = 1/3, m = 1/2$)

Figure 18: The cooperative solution of the game $(g, >)$ with $n < m$

8 Conclusions

In [7] we proposed the introduction of a Tobin tax on the government bonds cashed by the EFSF. In this paper we have made a further step.

Game with the tax cashed by EFSF. With the introduction of the tax cashed by the EFSF, we noted that

- Nash equilibrium and the defensive cross of the game move to the point A;
- the point A is an optimal point for the State in economic troubles, because the yield on its bonds is lower with respect to the initial condition, allowing the State to move the first step towards economic recovery;
- the point A' is also a quite good point for the Speculator and the EFSF, because they are on the weak maximal boundary;

- we have translated the politically more desirable solution in a solution convenient for all parties involved;

- the collective gain is not subject to social losses, such as in the no tax game, and we help to solve the problem of too high yield on government bonds;

- the introduction of the tax is a preventive deterrent for the presence of the speculators in the bonds market.

Game with the tax cashed by State. In this case, the results achieved with the introduction of the tax cashed by the EFSF remain valid and, moreover, we make a further step forward:

- the point A' becomes a point of the proper maximal boundary (in the game g between the Speculator and the EFSF), and not only a point of the weak maximal boundary;
- the point B' becomes an optimal point for the State too;
- the profit of the State is the same of that one in the point A': in fact, even if the Speculator for any reason decides to play the strategy 1, the tax cashed directly by the State balances the losses and the increasing in the yield on its government bonds and the State gains its maximum possible gain Mn.

Thanks to this result (and assuming a higher incidence on government bonds yield of the Speculator than these one by EFSF, that is $n < m$), we can propose also a mechanic (automatic) cooperative solution between the Speculator and the EFSF (the calculations was performed in the case $n = 1/3, m = 1/2$:

- they divide the gain $w = 5/12$ of the point B' by contract;
- the Speculator gains $2/21$, the BCE gains $9/28$ and at the same time the State in economic troubles gains $1/3$);
- in this way, all three economic subjects gain something;
- the Speculator and the EFSF gain more than in the Nash equilibrium and in the defensive cross, and the State can hope to go out of the government bond crisis.

Recapitulation. With our model we showed that:

1. the current normative framework (showed in a simplified fashion by our no tax game) allows the reduction of the effects of speculation, but it does not prevent them and does not solve them. For this reason, the current European economic measures could prove ineffective in the long term;
2. any prior agreement between EFSF and speculators, in order to cooperate and to avoid the negative effects of speculation, is Pareto-inefficient and it is practically impossible to implement because of asymmetric information problems;
3. the introduction of a government bonds tax that hits only the speculative profits would give the market a self-protection system against speculation (the speculators would not have particular interest to speculate);
4. the tax does not cause loss of Pareto-efficiency compared to the no tax game;

5. if the tax was collected directly by the State in economic troubles, even in the event of speculation the State could hope to get out of financial crisis, because the adverse effects of speculation on government bonds are compensated in whole or in part by the tax revenue;
6. for the above exposed reasons, it would be possible a cooperative solution that, in case of speculation, allows to obtain a profit for all parties (the State, the Speculator and the EFSF) without loss of collective profit.

In Fig. 17, we have shown a transferable utility solution of the tax game, when n is less than m. The solution we propose the solution $P < B'$ (see Fig. 18) - a Kalai-Smorodinky transferable utility solution, in which the speculator gains the *right flat rate payment* to participate in the game. We propose that this payment should be an automatic payment, that the EFSF has to pay to the speculator. At the point P, the speculator gains less than in the position B', as we have shown in Fig. 18, but nevertheless the speculator gains something. Note that without this *fair payment*, the speculator has no sensible reasons to participate in the game, since the payment scenarios given by the payoff space of the speculator in the tax game is negative or 0.

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