Controlling chaos in nuclear spin generator system using backstepping design

Ömür Umut

Abstract. In the present paper backstepping design is proposed to control nuclear spin generator system based on parameters identification. The observer is designed to identify the unknown parameter of NSG system. And on this basis, an efficient backstepping design is developed for controlling the uncertain NSG system to bounded points and tracking any desired trajectory. Numerical simulations are provided to show the effectiveness and feasibility of the proposed method.

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Key words: chaos control, backstepping design, nuclear spin generator system, uncertain system, parameter identification.

1 Introduction

Dynamic chaos is a very interesting nonlinear effect which has been intensively studied during the last four decades. The effect is very common, it has been detected in a large number of dynamic systems of various physical nature. However, this effect is usually undesirable in practice, and it resticts the operating range of many electronic and mechanic devices. Recently, controlling this kind of complex dynamical systems has attracted a great deal of attention within science and engineering.

Until now, many different techniques and methods have been proposed to achieve chaos control such as, OGY method [6], optimal control ([1], [10]), feedback control ([4], [9]), differential geometric method [3] and adaptive control ([2], [5]).

However, for many uncertain systems, the aforementioned methods may fail. An important problem in this field is how to achieve nonlinear control of complex dynamical systems with unknown parameters. This problem concerns the identification of the unknown parameters and the approach of controlling chaos. Recently, backstepping method has become one of the most important approaches for the design of nonlinear systems, ([11], [12]). In this paper, the observer is applied to the identification of the unknown parameters of nuclear spin generator system. Then an efficient backstepping design is developed for controlling nuclear spin generator system. The suggested tool enables synchronization of chaotic motion to a steady state as well as

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tracking any desired trajectory. Computer simulations also given for the purpose of illustration and verification.

2 System description

NSG is a high frequency oscillator which generates and controls the oscillations of the motion of a nuclear magnetization vector in a magnetic field. NSG was first carried out by Sherman in 1963 [8]. This system is described by

$$\begin{aligned} \dot{x} &= -\beta x + y \\ \dot{y} &= -x - \beta y(1 - \gamma z) \\ \dot{z} &= \beta(\alpha(1 - z) - \gamma y^2) \end{aligned} \tag{2.1}$$

where x, y and z are the components of the nuclear magnetization vector in the X, Y and Z directions and α, β and k are parameters where $\alpha\beta \geq 0$ and $\beta \geq 0$ are linear damping terms, the nonlinearity parameters βk are proportional to the amplifier gain in the voltage feedback. Physical considerations limit the parameter α to the range $0 < \alpha \leq 1$.

Sachdev and Sarathy studied in great detail the NSG system in [7]. They showed that it displays rich and typical bifurcation and chaotic phenomena for some values of the control parameters. For instance, when the parameters $\alpha = 0.15$, $\beta = 0.75$ and $\gamma = 10.5$ system (2.1) displays a chaotic attractor as shown in Figure 1.

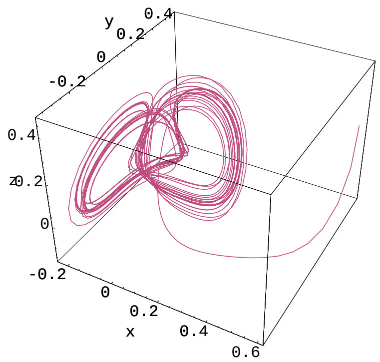

Figure 1: NSG chaotic attractor

Now, we design a control technique which can drive a strange attractor with unknown parameters to a stable state of nuclear spin generator system.

3 Identification of the unknown parameter

In this section, an observer will be designed to identify the unknown parameter α of system (2.1). Since parameter α is unknown, relevant dynamical information about the parameter α is not known. However we can attain the output vector (x, y, z). Noticing α is a constant, we assume that

$$\dot{\alpha} = 0 \tag{3.1}$$

Since the unknown parameter α can act as a status variable, the system (2.1) can be augmented by (3.1), i.e.,

$$\begin{aligned} \dot{x} &= -\beta x + y \\ \dot{y} &= -x - \beta y(1 - \gamma z) \\ \dot{z} &= \beta(\alpha(1 - z) - \gamma y^2) + u \\ \dot{\alpha} &= 0 \end{aligned} \tag{3.2}$$

In the following, we will design an observer to identify the unknown parameter α. From the third equation of system (2.1), we have

$$\alpha(z - 1) = -\gamma y^2 - \frac{1}{\beta}\dot{z} \tag{3.3}$$

Then we can design the following observer:

$$\dot{\bar{\alpha}} = -L(z)(z - 1)\bar{\alpha} + L(z)(-\frac{1}{\beta}\dot{z} - \gamma y^2) \tag{3.4}$$

where $L(z)$ is a gain function. Let

$$e = \alpha - \bar{\alpha} \tag{3.5}$$

then

$$\dot{e}(t) = \dot{\alpha} - \dot{\bar{\alpha}} = -L(z)(z - 1)e(t) \tag{3.6}$$

Thus, we can select an appropriate gain function $L(y)$ so that the system

$$\dot{e}(t) + L(z)(z - 1)e(t) = 0 \tag{3.7}$$

is exponentially asymptotically stable for all y. That is, $\bar{\alpha}(t)$ converges to $\alpha(t)$ with exponential rate as $t \to \infty$. We can choose

$$L(z) = k(z - 1), \quad (k > 0)$$

then we have

$$\dot{e}(t) = -k(z - 1)^2 e(t) \tag{3.8}$$

where the positive constant k determines the convergence rate. In fact, it is hard to observe $\dot{z}$, so the observer (3.3) is not applicable. We introduce an auxiliary variable

$$v = \bar{\alpha} + R(z)$$

where $R(z)$ is a design function that satisfies

$$L(z) = \beta \frac{dR(z)}{dz} \tag{3.9}$$

According to equations (3.4) and (3.9), we get

$$\dot{v} = \dot{\bar{\alpha}} + \dot{R}(z) = -L(z)(z-1)v + L(z)((z-1)R(z) - \gamma y^2) \tag{3.10}$$

and

$$\bar{\alpha} = v - R(z) \tag{3.11}$$

then (3.7) can be written as

$$\dot{e}(t) - \frac{1}{\beta}\frac{dR(z)}{dz}(z-1)e(t) = 0 \tag{3.12}$$

If we choose an appropriate design function $R(z)$ which can make $\bar{\alpha}(t)$ converges to $\alpha(t)$ at exponential rate as $t \to \infty$, the observers (3.10) and (3.11) can identify the unknown parameter α of system (3.2), where $L(z)$ and $R(z)$ are the gain and design functions respectively. Furthermore, we have

$$L(z) = \beta \frac{dR(z)}{dz}$$

Note that the observers (3.10) and (3.11) only rely on the third equation of system (3.2). That is, when the structures of the first and second equations of system (3.2) or the parameters β and γ are varied, the results of the identification are not changed. Therefore, the observers have strong robustness. Let

$$R(z) = \frac{k}{2\beta}(z-1)^2, \quad (k > 0)$$

then $L(z) = k(z-1)$, thus the observers become

$$\begin{aligned} \dot{v} &= -k(z-1)^2 v - k\gamma(z-1)y^2 + \frac{k^2}{2\beta}(z-1)^4 \\ \bar{\alpha} &= v + \frac{k}{2\beta}(z-1)^2 \end{aligned} \tag{3.13}$$

4 Controlling NSG system via backstepping design

In this section, we will use backstepping method to design a controller. In order to control the uncertain system we add a control input u to the third equation of system (2.1). Here we assume that the parameter α of the following control system has been identified, that is $\alpha = \bar{\alpha}$.

$$\begin{aligned} \dot{x} &= -\beta x + y \\ \dot{y} &= -x - \beta y(1 - \gamma z) \\ \dot{z} &= \beta(\alpha(1-z) - \gamma y^2) + u \end{aligned} \tag{4.1}$$

Then the objective is to find a control law u for stabilizing the state of system (4.1) at a bounded point.

Starting from the first equation, a stabilizing function $\alpha_1(x)$ has to be designed for the virtual control y in order to make the derivative of

$$V_1(x) = \frac{1}{2}x^2$$

i.e.,

$$\dot{V}_1 = -ax^2 + axy$$

be negative definite. Assume that $\alpha_1(x) = px$ and define an error variable $\bar{y} = y - \alpha_1(x)$. Then we obtain the $(x, \bar{y})-$subsystem

$$\begin{aligned} \dot{x} &= -(\beta - p)x + \bar{y} \\ \dot{\bar{y}} &= -(1+p^2)x - (\beta + p)\bar{y} + \beta\gamma z p x + \beta\gamma z \bar{y} \end{aligned} \tag{4.2}$$

Obviously, a candidate Lyapunov is

$$V_2(x, \bar{y}) = V_1(x) + \frac{1}{2}\bar{y}^2$$

Calculating its time derivative along system (4.2), we have

$$\dot{V}_2 = -(\beta - p)x^2 - \beta\bar{y}^2 + \bar{y}[(\bar{y} + px)(\beta\gamma z - p)]$$

we can choose $z = \alpha_2(x, \bar{y}) = \dfrac{p}{\beta\gamma}$ then

$$\dot{V}_2 = -(\beta - p)x^2 - \beta\bar{y}^2 < 0, \quad \text{if } \ 0 < p < \beta$$

Similarly, let $\bar{z} = z - \alpha_2(x, \bar{y})$, then we get the following system in the $(x, \bar{y}, \bar{z})$ coordinates:

$$\begin{aligned} \dot{x} &= -(\beta - p)x + \bar{y} \\ \dot{\bar{y}} &= -x - \beta\bar{y} + \beta\gamma(\bar{y} + px)\bar{z} \\ \dot{\bar{z}} &= \alpha(\beta - \frac{p}{\gamma}) - \alpha\beta\bar{z} - \beta\gamma(\bar{y} + px)^2 + u \end{aligned} \tag{4.3}$$

Repeating the previous steps, the derivative of

$$V_3(x, \bar{y}, \bar{z}) = V_2 + \frac{1}{2}\bar{z}^2$$

i.e.,

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2(x, \bar{y}) + \bar{z}\dot{\bar{z}} \\ &= -(\beta - p)x^2 - \beta\bar{y}^2 - \alpha\beta\bar{z}^2 + \bar{z}[\beta\gamma\bar{y}(\bar{y} + px) + \alpha(\beta - \frac{p}{\gamma}) - \beta\gamma(\bar{y} + px)^2 + u] \end{aligned} \tag{4.4}$$

becomes negative definite by choosing the input

$$u = \beta\gamma px(\bar{y} + px) - \bar{\alpha}(\beta - \frac{p}{\gamma}), \quad (\beta > p > 0) \tag{4.5}$$

In view of the equations

$$\bar{y} = y - \alpha_1(x), \quad \bar{z} = z - \alpha_2(x,\bar{y}), \quad \alpha_1(x) = px, \quad \text{and} \quad \alpha_2(x,\bar{y}) = \frac{p}{\beta\gamma} \tag{4.6}$$

we have $x \to 0$, $y \to 0$ and $z \to \dfrac{p}{\beta\gamma}$ as $t \to \infty$, i.e., $z(t)$ remains bounded. The control law u of system (4.1) with unknown parameter α is

$$\begin{aligned} u &= \beta\gamma pxy - \bar{\alpha}(\beta - \frac{p}{\gamma}) \\ \dot{v} &= -k(z-1)^2 v - k\gamma(z-1)y^2 + \frac{k^2}{2\beta}(z-1)^4 \\ \bar{\alpha} &= v + \frac{k}{2\beta}(z-1)^2 \end{aligned} \tag{4.7}$$

where $\beta - p > 0$ i.e., $\beta > p > 0$ and $k > 0$.
Therefore we have proved that system (4.3) has been stabilized at the point $(0,0,0)$ According to the equations in (4.6) the system (4.1) has been stabilized at the point $(0,0,\alpha_2)$.

In order to control NSG system to the point $(0,0,1)$, we add a control input w to the second equation of system (2.1). Thus the controlled system becomes:

$$\begin{aligned} \dot{x} &= -\beta x + y \\ \dot{y} &= -x - \beta y + \beta\gamma yz + w \\ \dot{z} &= \alpha\beta(1-z) - \beta\gamma y^2 \end{aligned} \tag{4.8}$$

For the virtual control y we design a stabilizing function $\alpha_1(x)$ to make the derivative of $V_1(x) = \frac{1}{2}x^2$, i.e.,

$$\dot{V}_1 = x\dot{x} = x(-\beta x + y) = -\beta x^2 + xy \tag{4.9}$$

be negative definite as $y = \alpha_1(x)$. We can choose $\alpha_1(x) = 0$ and define an error variable

$$\bar{y} = y - \alpha_1(x) \tag{4.10}$$

then we can obtain the following $(x,\bar{y})-$subsystem

$$\begin{aligned} \dot{x} &= -\beta x + \bar{y} \\ \dot{\bar{y}} &= -x - \beta\bar{y} + \beta\gamma z + w \end{aligned} \tag{4.11}$$

We can construct a Lyapunov function as follows:

$$V_2(x,\bar{y}) = V_1(x) + \frac{1}{2}\bar{y}^2$$

Calculating the time derivative of $V_2(x,\bar{y})$ along system (4.11), we have

$$\dot{V}_2 = \dot{V}_1 + \bar{y}\dot{\bar{y}} = -\beta x^2 - \beta\bar{y}^2 + \bar{y}(-x + \beta\gamma z + w) \tag{4.12}$$

in order to make $\dot{V}_2$ be negative definite, choose

$$w = x - \beta\gamma z \tag{4.13}$$

Therefore we have proved that in the $(x, \bar{y})$ coordinates the equilibrium $(0,0)$ of the subsystem (4.11) is asymptotically stable. According to (4.10), $\alpha_1(x) = 0$, $x \to 0$, $\bar{y} \to 0$ and from the third equation of system (2.1), we get that (x, y, z) in the controlled system (4.8) tends to $(0,0,1)$ as $t \to \infty$ when we choose the control input $w = x - \beta\gamma z$

4.1 Tracking any desired trajectory

In this section, we will find a control law w so that the scalar output $x(t)$ of NSG system can track any desired trajectory $r(t)$. Let $\bar{x}$ be the deviation between the output x and the desired trajectory $r(t)$, i.e., $\bar{x} = x - r(t)$. Define a function $U_1 = \frac{1}{2}\bar{x}^2$ and calculate its time derivative along the controlled system (4.8).

$$\dot{U}_1 = \bar{x}\dot{\bar{x}} = (x - r)(\dot{x} - \dot{r}) = (x - r)(-\beta x + y - \dot{r}) \tag{4.14}$$

becomes negative definite by choosing the virtual control y as

$$y = (\beta - 1)x + r + \dot{r} \tag{4.15}$$

Let $U_2 = U_1 + \frac{1}{2}\bar{y}^2$ where $\bar{y} = y - [(\beta - 1)x + r + \dot{r}]$ then

$$\begin{aligned} \dot{U}_2 = \dot{U}_1 + \frac{1}{2}\bar{y}^2 &= -(x - r)^2 + (y - ((\beta - 1)x + r + \dot{r}))(y - ((\beta - 1)\dot{x} + \dot{r} + \ddot{r})) \\ &= -(x - r)^2 + (y - (\beta - 1)x + r + \dot{r})((\beta^2 - \beta - 1)x - (2\beta - 1)y + \beta\gamma yz - \dot{r} - \ddot{r} + w) \end{aligned} \tag{4.16}$$

If we choose

$$w = \beta(2 - \beta)x + 2(\beta - 1)y - \beta\gamma yz + r + 2\dot{r} + \ddot{r} \tag{4.17}$$

then $\dot{U}_2$ is negative definite i.e.,

$$\dot{U}_2 = -(x - r)^2 - (y - ((\beta - 1)x + r + \dot{r}))$$

Hence we prove that the output $x(t)$ can track any given trajectory $r(t)$.
According to (3.13), the control law (4.17) with unknown parameter α is

$$\begin{aligned} w &= \beta(2 - \beta)x + 2(\beta - 1)y - \beta\gamma yz + r + 2\dot{r} + \ddot{r} \\ \dot{v} &= -k(z - 1)^2 v - k\gamma(z - 1)y^2 + \frac{k^2}{2\beta}(z - 1)^4 \\ \bar{\alpha} &= v + \frac{k}{2\beta}(z - 1)^2 \end{aligned} \tag{4.18}$$

For example, when $r(t) = 2\sin t$ the control law is

$$w = \beta(2 - \beta)x + 2(\beta - 1)y - \beta\gamma yz + 4\cos t \tag{4.19}$$

5 Numerical simulations

In this section, numerical simulations are given to verify the effectiveness of the observer and the control applicability of the proposed control laws. In all simulations, we assume $\beta = 0.75$, $\gamma = 10.5$, $k = 0.6$, initial conditions $x(0) = 0.6$, $y(0) = 0.5$, $z(0) = 0.2$. All solutions are obtained using Mathematica.

Figures 2(a)-(f) show the better control applicability of the control law (4.5), where $p = 0.3$ and the effectiveness of the observers (3.13) where $\alpha = 0.8$.

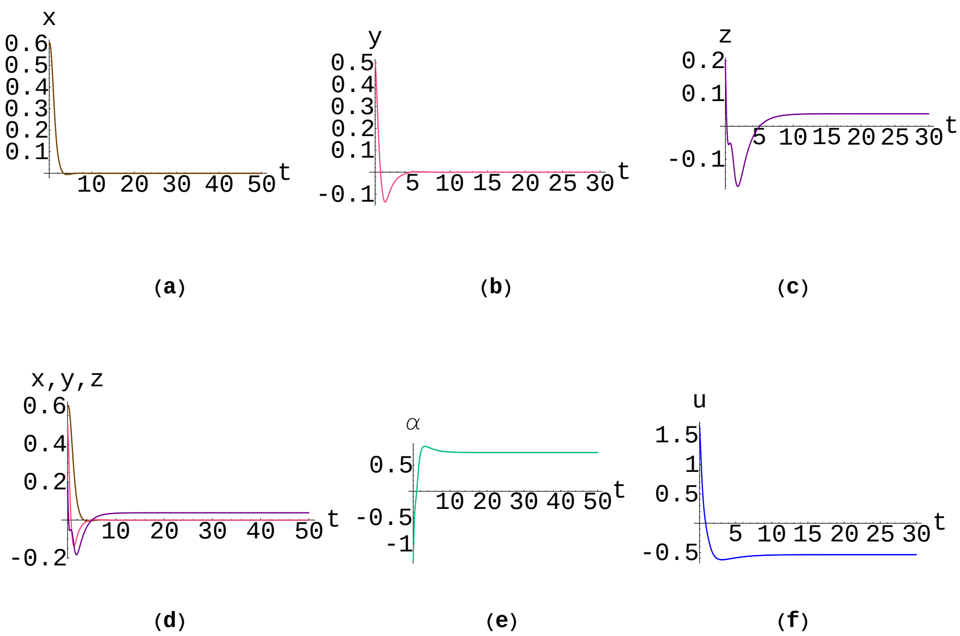

Figure 2: State of variants x, y, z of NSG system with control law u used ;(a) time variation of x, (b) time variation of y, (c) time variation of z, (d) time variations of x, y and z, (e) time variation of α and (f) control action u.

Figures 3(a)-(d) display the effectiveness of the control law (4.13), where $p = 0.3$ and the effectiveness of the observers (3.13) where $\alpha = 0.5$.

Figures 4(a)-(b) display the tracking results of the control law (4.17) when $r = 2 \sin t$

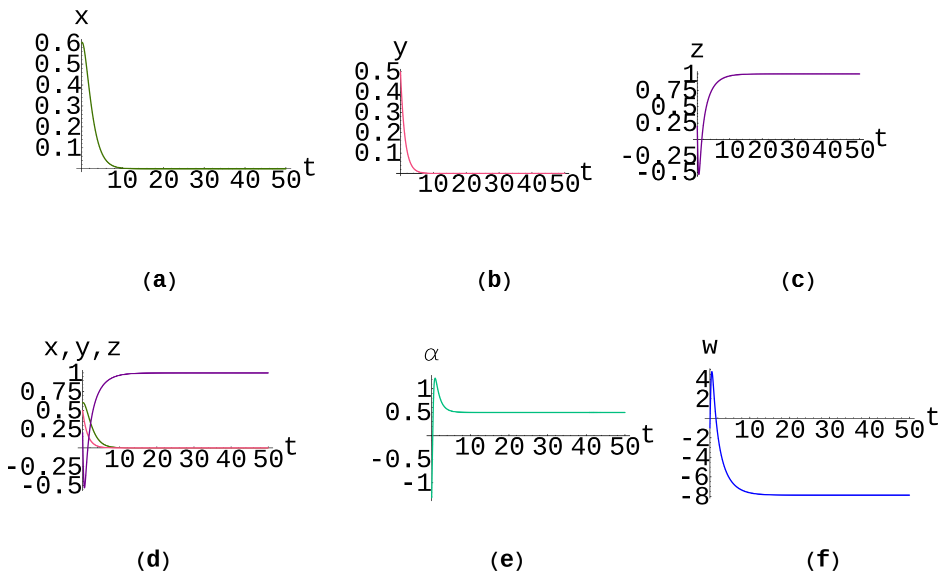

Figure 3: State of variants x, y, z of NSG system with control law w used; (a) time variation of x, (b) time variation of y, (c) time variation of z, (d) time variations of x, y and z, (e) time variations of α and (f) control action w.

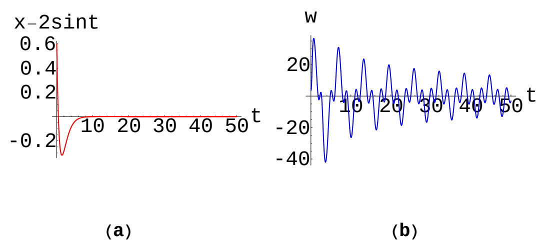

Figure 4: Track of $r(t) = 2\sin t$ with control law w; (a) Output of $x(t)$ of NSG system tracks $r(t) = 2\sin t$, (b) control action w.

6 Conclusions

In this paper, a Lyapunov based approach, called backstepping design, has been proposed for controlling nuclear spin generator system with unknown parameter. This effective control law can drive a strange attractor not only a steady state but also any desired trajectory. The effectiveness of the method is verified by numerical results.

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Authors' addresses:

Ömür Umut
Department of Mathematics, Faculty of Arts and Sciences,
Abant Izzet Baysal University, 14280 Bolu, Turkey.
E-mail: umut_o@ibu.edu.tr