Characterization of certain matrix classes involving generalized difference summability spaces

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Abstract. The main aim of this article is to generalize the notion of almost convergent, Cesàro summable and lacunary summable spaces by using a generalized difference operator defined associating a sequence of non-zero scalars and characterize some matrix classes involving these sequence spaces. In this article we also introduce the idea of difference infinite matrices. It is expected that these investigations will generalize several notions associated with thus constructed spaces as well as of matrix transformations.

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1 Introduction

Let w denote the space of all scalar sequences and any subspace of w is called a sequence space. Let ℓ_∞, c and c_0 be the spaces of *bounded*, *convergent* and *null* sequences $x = (x_k)$ with complex terms, respectively, normed by

$$\|x\|_\infty = \sup_k |x_k|$$

The notion of difference sequence spaces was introduced by Kizmaz [10]. The notion was further generalized by Et and Colak [4] by introducing the spaces $\ell_\infty(\Delta^s)$, $c(\Delta^s)$ and $c_0(\Delta^s)$. Another type of generalization of the difference sequence spaces is due to Tripathy and Esi [15], who studied the spaces $\ell_\infty(\Delta_r)$, $c(\Delta_r)$ and $c_0(\Delta_r)$. Tripathy, Esi and Tripathy [16] generalized the above notions and unified these as follows:

Let r, s be non-negative integers, then for Z a given sequence space we have

$$Z(\Delta_r^s) = \{x = (x_k) \in w : (\Delta_r^s x_k) \in Z\},$$

where $\Delta_r^s x = (\Delta_r^s x_k) = (\Delta_r^{s-1} x_k - \Delta_r^{s-1} x_{k+r})$ and $\Delta_r^0 x_k = x_k$ for all $k \in N$, which is equivalent to the following binomial representation:

$$\Delta_r^s x_k = \sum_{i=0}^{n} (-1)^i \binom{s}{i} x_{k+ri}$$

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Let r, s be non-negative integers and $v = (v_k)$ be a sequence of non-zero scalars. Then for Z, a given sequence space we define the following sequence spaces:

$$Z(\Delta^s_{(vr)}) = \{x = (x_k) \in w : (\Delta^s_{(vr)} x_k) \in Z\}, \text{ for } Z = \ell_\infty, \ c \text{ and } c_0$$

where$(\Delta^s_{(vr)} x_k) = (\Delta^{s-1}_{(vr)} x_k - \Delta^{s-1}_{(vr)} x_{k-r})$ and $\Delta^0_{(vr)} x_k = v_k x_k$ for all $k \in N$, which is equivalent to the following binomial representation:

$$\Delta^s_{(vr)} x_k = \sum_{i=0}^{n} (-1)^i \binom{s}{i} v_{k-ri} x_{k-ri}.$$

In this expansion it is important to note that we take $v_{k-ri} = 0$ and $x_{k-ri} = 0$ for non-positive values of $k - ri$.

In the next section we shall show that these spaces can be made BK-spaces under the norm

$$\|x\| = \sup_k |\Delta^s_{(vr)} x_k|$$

For s=1 and v_k=1 for all $k \in N$, we get the spaces $\ell_\infty(\Delta_r)$, $c(\Delta_r)$ and $c_0(\Delta_r)$. For r=1 and v_k=1 for all $k \in N$, we get the spaces $\ell_\infty(\Delta^s)$, $c(\Delta^s)$ and $c_0(\Delta^s)$. For $r = s = 1$ and v_k=1 for all $k \in N$, we get the spaces $\ell_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$.

Let E and F be two sequence spaces and $A = (a_{nk})$ be an infinite matrix of real or complex numbers a_{nk}, where $n, k \in N$.Then we say that A defines a *matrix mapping* from E into F, and denote it by writing $A : E \longrightarrow F$ if for every sequence $x = (x_k) \in E$ the sequence $Ax = \{(Ax)_n\}$, the A-transform of x, is in F, where

$$(Ax)_n = \sum_{k=1}^{\infty} a_{nk} x_k, (n \in N) \tag{1.1}$$

We denote be (E, F) the class of all matrices A such that $A : E \longrightarrow F$. Thus $A \in (E, F)$ if and only if the series on the right hand side of (1.1) converges for each $n \in N$ and every $x \in E$, and we have $Ax = \{(Ax)_n\}_{n \in N} \in F$ for all $x \in E$. A sequence x is said to be A-summable to l if Ax converges to l which is called the A-limit of x. Further we write (E, F, P) to the subset of (E, F) which preserves the limit or sum.

Interest in general matrix transformation theory was, to some extent, stimulated by special results in summability theory which were obtain by Cesàro, Borel and others, at the turn of the 20^{th} century. It was however the celebrated German mathematician O. Toeplitz who, in 1911, brought the methods of linear space theory to bear on problems connected with matrix transformations on sequence spaces. Toeplitz characterized all those infinite matrices $A = (a_{nk})$, $n, k \in N$, which map the space of convergent sequences into itself, leaving the limit of each convergent sequence invariant (See for instance Maddox [12]).

A linear functional L on ℓ_∞ is said to be a Banach limit (see Banach [1]) if it has the properties:

(*i*) $L(x) \geq 0$ if $x \geq 0$,

(*ii*) $L(e) = 1$, where $e = (1, 1, 1, ...)$,

(*iii*) $L(Dx) = L(x)$, where D is the shift operator defined by $(Dx_n) = (x_{n+1})$.

Let B be the set of all Banach limits on ℓ_∞. A sequence x is said to be almost convergent to a number l if $L(x) = l$ for all L in B. Let $\hat{c}$ denote the set of all almost convergent sequences. Lorentz [11] proved that

$$\hat{c} = \{x : \lim_{m\to\infty} \frac{1}{m+1} \sum_{i=0}^{m} x_{n+i} \text{ exists uniformly in n}\}.$$

By a lacunary sequence $\theta = (k_q); q = 1, 2, 3, ...,$ where $k_0 = 0$, we mean an increasing sequence of non-negative integers with $h_q = (k_q - k_{q-1}) \to \infty$ as $q \to \infty$. We denote $I_q = (k_{q-1}, k_q]$.

Let C_θ denote the space of all lacunary summable sequences. Then

$$C_\theta = \{x : \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} x_k \text{ exists}\}.$$

Let σ_1 denote the space of all Cesàro summable sequences. Then

$$\sigma_1 = \{x : \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} x_i \text{ exists}\}.$$

Let r, s be non negative integers and $v = (v_k)$ be a sequence of non-zero scalars. Then we define the following spaces:

$$\hat{c}(\Delta^s_{(vr)}) = \{x : \lim_{p\to\infty} \frac{1}{p} \sum_{i=0}^{p-1} \Delta^s_{(vr)} x_{n+i} \text{ exists uniformly in } n\},$$

$$\sigma_1(\Delta^s_{(vr)}) = \{x : \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \Delta^s_{(vr)} x_i \text{ exists}\},$$

$$C_\theta(\Delta^s_{(vr)}) = \{x : \lim_{q\to\infty} \frac{1}{h_q} \sum_{k\in I_q} \Delta^s_{(vr)} x_k \text{ exists}\}.$$

We call the spaces $\hat{c}(\Delta^s_{(vr)})$, $\sigma_1(\Delta^s_{(vr)})$ and $C_\theta(\Delta^s_{(vr)})$ as the spaces of $\Delta^s_{(vr)}$-almost convergent, $\Delta^s_{(vr)}$-Cesàro summable and $\Delta^s_{(vr)}$-lacunary summable sequences respectively. For $s = 0$ and $v_k = 1$ for all $k \in N$ we get the spaces of almost convergent([1], [11]), Cesàro summable and lacunary summable ([6], [8]) sequences respectively.

2 Main Results

In this section we show that the spaces $Z(\Delta^s_{(vr)})$, for $Z = \ell_\infty, c$ and c_0 are BK-spaces. We also characterize the matrix classes $(c, \hat{c}(\Delta^s_{(vr)}))$, $(c, \sigma_1(\Delta^s_{(vr)}))$, $(c, C_\theta(\Delta^s_{(vr)}))$, $(c, \hat{c}(\Delta^s_{(vr)}), P)$, $(c, \sigma_1(\Delta^s_{(vr)}), P)$ and $(c, C_\theta(\Delta^s_{(vr)}), P)$.

Theorem 2.1. *The spaces $Z(\Delta^s_{(vr)})$, for $Z = \ell_\infty, c$ and c_0 are BK-spaces under the norm defined by*

$$\|x\| = \sup_k |\Delta^s_{(vr)} x_k|$$

.

Proof. First we show that the spaces $Z(\Delta^s_{(vr)})$ are normed linear spaces normed by $\|.\|$.

For $x = \theta$, we have $\|x\| = 0$. Conversely, let $\|x\| = 0$. Then using definition of norm, we have

$$\sup_k |\Delta^s_{(vr)} x_k| = 0$$

It follows that

$$\Delta^s_{(vr)} x_k = 0 \text{ for all } k \geq 1.$$

Let $k = 1$, then $\Delta^s_{(vr)} x_1 = \sum_{i=0}^{s} (-1)^i \binom{s}{v} v_{1-ri} x_{1-ri} = 0$ and so $v_1 x_1 = 0$, by putting $v_{1-ri} = 0$ and $x_{1-ri} = 0$ for $i = 1, ..., s$. Hence $x_1 = 0$, since (λ_k) is a sequence of non-zero scalars. Similarly taking $k = 2, ..., rs$, we have $x_2 = . . . = x_{rs} = 0$. Next let $k = rs+1$, then $\Delta^s_{(vr)} x_{rs+1} = \sum_{i=0}^{s} (-1)^i \binom{s}{v} v_{1+rs-ri} x_{1+rs-ri} = 0$. Since $x_1 = x_2 = . . . = x_{rs} = 0$, we must have $v_{rs+1} x_{rs+1} = 0$ and thus $x_{rs+1} = 0$. Proceeding in this way we can conclude that $x_k = 0$ for all $k \geq 1$. Hence $x = \theta$.

Again it is easy to show that $\|x + y\| \leq \|x\| + \|y\|$ and for any scalar α, $\|\alpha x\| = |\alpha| \|x\|$.

Thus $Z(\Delta^s_{(vr)})$ is a normed linear space normed by $\|.\|$.

Now we show that $Z(\Delta^s_{(vr)})$ is a Banach space under the norm $\|.\|$.

Let (x^i) be a Cauchy sequence in $Z(\Delta^s_{(vr)})$, where $x^i = (x^i_k) = (x^i_1, x^i_2, ...)$ for each $i \geq 1$. Then for a given $\varepsilon > 0$, there exists a positive integer n_0 such that

$$\|x^i - x^j\| = \sup_k |\Delta^s_{(vr)}(x^i_k - x^j_k)| < \varepsilon \text{ for all } i, j \geq n_0$$

It follows that

$$|\Delta^s_{(vr)}(x^i_k - x^j_k)| < \varepsilon \text{ for all } i, j \geq n_0 \text{ and for all } k \geq 1.$$

This implies that $(\Delta^s_{(vr)} x^i_k)$ is a Cauchy sequence in C for all $k \geq 1$ and so it is convergent in C for all $k \geq 1$.

Let $\lim_{i\to\infty} \Delta^s_{(vr)} x^i_k = y_k$, say for each $k \geq 1$. Considering $k = 1, 2, ..., rs, ...$, we can easily conclude that $\lim_{i\to\infty} x^i_k = x_k$, say exists for each $k \geq 1$.

Now we can have

$$\lim_{j\to\infty} |\Delta^s_{(vr)}(x^i_k - x^j_k)| < \varepsilon \text{ for all } i \geq n_0 \text{ and } k \geq 1$$

Hence

$$\sup_k |\Delta^s_{(vr)}(x^i_k - x_k)| < \varepsilon \text{ for all } i \geq n_0.$$

This implies that $(x^i - x) \in Z(\Delta^s_{(vr)})$. Since $Z(\Delta^s_{(vr)})$ is a linear space, $x = x^i - (x^i - x) \in Z(\Delta^s_{(vr)})$.

Hence $Z(\Delta^s_{(vr)})$ are complete.

From the above proof we can easily conclude that $\|x^i - x\| \to 0$ implies $|x^i_k - x_k| \to 0$ as $i \to \infty$, for each $k \geq 1$.

Hence we can conclude that $Z(\Delta^s_{(vr)})$ are BK-spaces.

This completes the proof. □

Remark. One may find it interesting to see the above proof by taking particular values of r and s in the difference operator $\Delta^s_{(vr)}$. For example, let us take $r = 3$ and $s = 2$, then $\Delta^2_{(v3)} x_k = v_k x_k$ - $2v_{k-3}x_{k-3} + v_{k-6}x_{k-6}$. For $k = 2$, we have $\Delta^2_{(v3)} x_2 = v_2 x_2$ - $2v_{-1}x_{-1} + v_{-4}x_{-4} = v_2 x_2$. Also for $k = 7$, we have $\Delta^2_{(v3)} x_7 = v_7 x_7$ - $2v_4 x_4 + v_1 x_1$ etc.

Theorem 2.2. *$A \in (c, \hat{c}(\Delta^s_{(vr)}))$ if and only if*

(*i*) $\sup\{\sum_{k=1}^{\infty} \frac{1}{p} |\sum_{j=n}^{n+p-1} \Delta^s_{(vr)} a_{jk}| : p \in N\} < \infty, n = 1, 2, ...$

(*ii*) *there exists $\alpha_k, k = 1, 2, ...,$ such that*
$\lim_{p\to\infty} \frac{1}{p} \sum_{j=n}^{n+p-1} \Delta^s_{(vr)} a_{jk} = \alpha_k$, *uniformly in n and*

(*iii*) *there exists α such that*
$\lim_{p\to\infty} \frac{1}{p} \sum_{j=n}^{n+p-1} \sum_{k=1}^{\infty} \Delta^s_{(vr)} a_{jk} = \alpha$, *uniformly in n,*

where $\Delta^s_{(vr)} a_{jk} = \sum_{i=0}^{s} (-1)^i \binom{s}{i} v_k a_{j-ri,k}$ and we take $a_{j-ri,k} = 0$ for non-positive values of j-ri. (e.g., $\Delta^2_{(3)} v_1 a_{11} = v_1 a_{11} - 2v_1 a_{-2,1} + v_1 a_{-5,1} = a_{11}, \Delta^2_{(3)} v_1 a_{71} = v_1 a_{71} - 2v_1 a_{41} + v_1 a_{11}$ etc.)

Proof. Suppose $A \in (c, \hat{c}(\Delta^s_{(vr)}))$. Fix n and put $S_{pn}(x) = \frac{1}{p} \sum_{j=n}^{n+p-1} \Delta^s_{(vr)} (Ax)_j$, where

$$\Delta^s_{(vr)}(Ax)_j = \Delta^{s-1}_{(vr)}(Ax)_j - \Delta^{s-1}_{(vr)}(Ax)_{j-r} = \sum_{k=1}^{\infty} \Delta^s_{(vr)} a_{jk} x_k.$$

We write $B_j(x) = \Delta^s_{(vr)}(Ax)_j, j = 1, 2, ...$ Then we observe that (B_j) is a sequence of bounded linear operators on c and so $S_{pn}, p = 1, 2, ...$ is a bounded linear operator on c such that $\lim_{p\to\infty} S_{pn}(x)$ exists uniformly in n. Now (i) follows from an application of uniform boundedness principle. Let $e = (1, 1, ...)$ and $e_k = (0, 0, ..., 1, 0, ...), k = 1, 2, ...$ where 1 is in the k^{th} position. Since e and $e_k, k = 1, 2, ...$ are convergent sequences, $\lim_{p\to\infty} S_{pn}(e_k)$ and $\lim_{p\to\infty} S_{pn}(e)$ must exists uniformly in n. Hence (ii) and (iii) must hold.

For the converse part let (x_k) converges to l and the conditions (i), (ii) and (iii) hold. We write

$$(2.1)\quad \frac{1}{p} \sum_{k=1}^{\infty} \sum_{j=n}^{n+p-1} \Delta^s_{(vr)} a_{jk} x_k = \frac{1}{p} \sum_{k=1}^{\infty} \sum_{j=n}^{n+p-1} \Delta^s_{(vr)} a_{jk}(x_k - l) + \frac{1}{p} l \sum_{k=1}^{\infty} \sum_{j=n}^{n+p-1} \Delta^s_{(vr)} a_{jk}$$

Since (x_k) converges to l, using (i) and (ii) it follows that the first term on the right of (2.1) tends to $\sum_{k=1}^{\infty} \alpha_k (x_k - l)$ as $p \to \infty$. Again since (iii) holds, the second term on the right of (2.1) tends to $l\alpha$ as $p \to \infty$.

Therefore $\lim_{p\to\infty} \frac{1}{p} \sum_{k=1}^{\infty} \sum_{j=n}^{n+p-1} \Delta^s_{(vr)} a_{jk} x_k$ exists uniformly in n.

This completes the proof. □

Proof of the following two Theorems follow by applying similar arguments as applied to prove Theorem 2.2.

Theorem 2.3. *$A \in (c, \sigma_1(\Delta^s_{(vr)}))$ if and only if*

(*i*) $\sup\{\sum_{k=1}^{\infty} \frac{1}{p} |\sum_{j=1}^{p} \Delta^s_{(vr)} a_{jk}| : p \in N\} < \infty$,

(*ii*) *there exists* $\alpha_k, k = 1, 2, \ldots,$ *such that*
$\lim_{p\to\infty} \frac{1}{p} \sum_{j=1}^{p} \Delta^s_{(vr)} a_{jk} = \alpha_k$, *and*

(*iii*) *there exists* α *such that*
$\lim_{p\to\infty} \frac{1}{p} \sum_{j=1}^{p} \sum_{k=1}^{\infty} \Delta^s_{(vr)} a_{jk} = \alpha.$

Theorem 2.4. *$A \in (c, C_\theta(\Delta^s_{(vr)}))$ if and only if*

(*i*) $\sup\{\sum_{k=1}^{\infty} \frac{1}{h_q} |\sum_{j\in I_q} \Delta^s_{(vr)} a_{jk}| : q \in N\} < \infty$,

(*ii*) *there exists* $\alpha_k, k = 1, 2, \ldots,$ *such that*
$\lim_{q\to\infty} \frac{1}{h_q} \sum_{j\in I_q} \Delta^s_{(vr)} a_{jk} = \alpha_k$, *and*

(*iii*) *there exists* α *such that*
$\lim_{q\to\infty} \frac{1}{h_q} \sum_{j\in I_q} \sum_{k=1}^{\infty} \Delta^s_{(vr)} a_{jk} = \alpha.$

Theorem 2.5. *$A \in (c, \hat{c}(\Delta^s_{(vr)}), P)$ if and only if*

(*i*) $\sup\{\sum_{k=1}^{\infty} \frac{1}{p} |\sum_{j=n}^{n+p-1} \Delta^s_{(vr)} a_{jk}| : p \in N\} < \infty, n = 1, 2, \ldots$

(*ii*) $\lim_{p\to\infty} \frac{1}{p} \sum_{j=n}^{n+p-1} \Delta^s_{(vr)} a_{jk} = 0$, *uniformly in* n, $k = 1, 2, \ldots$

(*iii*) $\lim_{p\to\infty} \frac{1}{p} \sum_{j=n}^{n+p-1} \sum_{k=1}^{\infty} \Delta^s_{(vr)} a_{jk} = 1$ *uniformly in* n.

Proof. Suppose $A \in (c, \hat{c}(\Delta^s_{(vr)}), P)$. Then the condition (*i*) holds by Theorem 2.2. Also (*ii*) and (*iii*) hold since we must have $\hat{c}(\Delta^s_{(vr)}) - \lim Ae_k = 0, k = 1, 2, \ldots$ and $\hat{c}(\Delta^s_{(vr)}) - \lim Ae = 1$ respectively.

For converse part suppose the conditions hold and (x_k) converges to l. Then by Theorem 2.2, we have $A \in (c, \hat{c}(\Delta^s_{(vr)}))$ and using (2.1) we must have

$$\lim_{p\to\infty} \frac{1}{p} \sum_{k=1}^{\infty} \sum_{j=n}^{n+p-1} \Delta^s_{(vr)} a_{jk} x_k = l.$$

This completes the proof. □

Proof of the following two Theorems follow by applying similar arguments as applied to prove Theorem 2.5.

Theorem 2.6. *$A \in (c, \sigma_1(\Delta^s_{(vr)}), P)$ if and only if*

(i) $\sup\{\sum_{k=1}^{\infty} \frac{1}{p} |\sum_{j=1}^{p} \Delta^s_{(vr)} a_{jk}| : p \in N\} < \infty,$

(ii) $\lim_{p\to\infty} \frac{1}{p} \sum_{j=1}^{p} \Delta^s_{(vr)} a_{jk} = 0,\ k = 1, 2, ...$

(iii) $\lim_{p\to\infty} \frac{1}{p} \sum_{j=1}^{p} \sum_{k=1}^{\infty} \Delta^s_{(vr)} a_{jk} = 1.$

Theorem 2.7. *$A \in (c, C_\theta(\Delta^s_{(vr)}), P)$ if and only if*

(i) $\sup\{\sum_{k=1}^{\infty} \frac{1}{h_q} |\sum_{j\in I_q} \Delta^s_{(vr)} a_{jk}| : q \in N\} < \infty,$

(ii) $\lim_{q\to\infty} \frac{1}{h_q} \sum_{j\in I_q} \Delta^s_{(vr)} a_{jk} = 0,\ k = 1, 2, ...$

(iii) $\lim_{q\to\infty} \frac{1}{h_q} \sum_{j\in I_q} \sum_{k=1}^{\infty} \Delta^s_{(vr)} a_{jk} = 1.$

Remark. By taking $s = 0$, and $v_k = 1$ for all $k \in N$, in Theorem 2.2 and Theorem 2.5, we get the matrix classes $(c, \hat{c})$, $(c, \hat{c}, P)$ studied by King [9].

3 Conclusions

In the present paper we present a generalization of the notion of almost convergent, Cesàro summable and lacunary summable spaces by using the generalized difference operator $\Delta^s_{(vr)}$ and study the spaces of these difference sequences for BK-spaces. We also characterize the matrix classes involving these sequence spaces and the space c. Further we characterize the matrix classes which preserves the limit. One may find it interesting to see ([14], [2], [7]) for some results on matrices and functional analytic studies on spaces.

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