Availabilty and MTTF of a system with one warm standby component

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Abstract. This paper deals with the availability and MTTF of a series system with a warm standby component. The time-to-repair and the time-to-failure for each of the primary and warm standby component is assumed to be negative exponential distribution. We develop an explicit expression for the mean-time to failure (MTTF) and the steady-state availability A(∞) of the model.

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1 Introduction

Uncertainty is one of the important issues in management decisions. Two of the most useful uncertainty measures are system reliability and system availability. Also an essential requisite is to maintain a high or required level of reliability and availability. In this note, we study the behaviour of MTTF and availability for a series system with one (warm) standby components. Analytical solutions of the Markovian model for the machine repair problem with warm standbys were first developed by Sivazlian and Wang (1989). Wang and Kuo (2000) investigated the cost and probabilistic analysis of series systems with mixed standby components. The problem considered in this note provides a systematic methodology to develop the explicit expressions for the mean-time-to failure (MTTF) and the steady-state availability A(∞).

2 Description of the system

We consider the requirement of a 10 MW power plant. We assume that generators are available in units of 10 MW. Further that standby generators are allowed to fail before they are put into full operations and that the standby generators are continuously monitored by a fault detecting device in order to identify the failure of the system In addition, we assume that all switchover times are instantaneous and switching is perfect (e.g. never fails and never does any damage to the system). Primary component and standby components are repairable.

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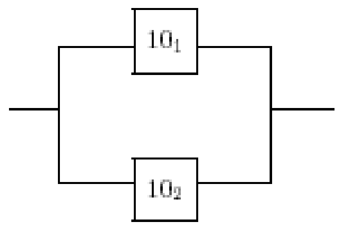

Figure 1: One 10 MW Components with One Standby components

Each of the primary components fails independently of the state of others and has an exponential time-to-failure distribution with parameter λ. Whenever one of these components fails, it is immediately replaced by a standby component, if available.

Also that the available standby components fails independently of the state of all the others and has an exponential time-to-failure distribution with parameter $\alpha(0< \alpha < \lambda)$.

Whenever a primary component or a standby component fails, it is immediately repaired in the order of their breakdowns, with a time-to-repair distribution with parameter μ.

Once a component is repaired, it is as good as new. Further, failure and repair times are independently distributed random variables. The configuration is a series system of one primary 10 MW component with one standby (warm standby) 10 MW component (see figure 1 below).

3 Problem solutions

3.1 Mean-time-to-failure (MTTF)

Let $P_n(t)$ be the probability that exactly n components are working at time t (t ≥ 0). If we let $P(t)$ denote the probability row vector at time t, then the initial conditions for this problem are

$$P(0) = [P_3(0), P_2(0), P_1(0), P_0(0)] = [1, 0, 0, 0]. \tag{3.1}$$

Omitting the arguments t in $P_n(t)$ so that $P_n(t)$=P_n, we obtain the following differential equations:

$$\begin{aligned}
\frac{dP_3}{dt} &= -(\lambda+\alpha)P_3 + \mu P_2 \\
\frac{dP_2}{dt} &= (\lambda+\alpha)P_3 + \mu P_1 - (\lambda+\alpha+\mu)P_2 \\
\frac{dP_1}{dt} &= (\lambda+\alpha)P_2 - (\lambda+\mu)P_1 + \mu P_0 \\
\frac{dP_0}{dt} &= (\lambda P_1 - \mu P_0)
\end{aligned}$$

This can be written in matrix form as

$$\dot{P} = QP$$

where

$$Q = \begin{bmatrix} -(\lambda+\alpha) & \mu & 0 & 0 \\ \lambda+\alpha & -(\lambda+\alpha+\mu) & \mu & 0 \\ 0 & \lambda+\alpha & -(\lambda+\mu) & \mu \\ 0 & 0 & \lambda & -\mu \end{bmatrix}$$

Without deriving the transient solutions, I propose the simplest procedure to develop the explicit expressions for the MTTF. To derive the MTTF, we take the transpose of matrix Q and delete the rows and columns for the absorbing state(s). The new matrix is called A. The expected times to reach an absorbing state is obtained from

$$E[T_{P(0)\to P(absorbing)}] = P(0)(-A^{-1}) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{3.2}$$

where

$$A = \begin{bmatrix} -(\lambda+\alpha) & \lambda+\alpha & 0 \\ \mu & -(\lambda+\alpha+\mu) & \lambda+\alpha \\ 0 & \mu & -(\lambda+\mu) \end{bmatrix}$$

The method is fruitful because of the following relations:

$$E[T_{P(0)\to P(absorbing)}] = P(0)\int_0^\infty e^{At}dt \tag{3.3}$$

and

$$\int_0^\infty e^{At}dt = -A^{-1}. \tag{3.4}$$

For this model, we obtain the following explicit expression for the MTTF as

$$E[T_{P(0)\to P(absorbing)}] = MTTF = \frac{3\lambda^2 + 4\lambda\alpha + 2\lambda\mu + (\alpha+\mu)^2 - \alpha\mu}{\lambda(\lambda+\alpha)^2}. \tag{3.5}$$

3.2 Availability

For the availability case, the initial conditions for this problem are the same as reliability case:

$$P(0) = [P_3(0), P_2(0), P_1(0), P_0(0)] = [1, 0, 0, 0]. \tag{3.6}$$

For the present model, the following differential equations written in matrix form can be obtained.

$$\begin{pmatrix} \dot{P}_3 \\ \dot{P}_2 \\ \dot{P}_1 \\ \dot{P}_0 \end{pmatrix} = \begin{pmatrix} -(\lambda+\alpha) & \mu & 0 & 0 \\ \lambda+\alpha & -(\lambda+\alpha+\mu) & \mu & 0 \\ 0 & \lambda+\alpha & -(\lambda+\mu) & \mu \\ 0 & 0 & \lambda & -\mu \end{pmatrix} \begin{pmatrix} P_3 \\ P_2 \\ P_1 \\ P_0 \end{pmatrix}$$

Let T_1 denote the time-to-failure of the system for this problem. To obtain the steady-state availability, we use the following method. In the steady-state, the derivatives of the state probabilities become zero. Hence this allows us to calculate the steady-state probabilities with

$$A(\infty) = 1 - P_0(\infty) \text{ and } QP(\infty) = 0 \tag{3.7}$$

or in matrix form

$$\begin{pmatrix} -(\lambda+\alpha) & \mu & 0 & 0 \\ \lambda+\alpha & -(\lambda+\alpha+\mu) & \mu & 0 \\ 0 & \lambda+\alpha & -(\lambda+\mu) & \mu \\ 0 & 0 & \lambda & -\mu \end{pmatrix} \begin{pmatrix} P_3(\infty) \\ P_2(\infty) \\ P_1(\infty) \\ P_0(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.8}$$

Using the relation

$$\sum_{i=0}^{3} P_i(\infty) = 1 \tag{3.9}$$

we substitute (9) in any one of the redundant rows in (8) to obtain

$$\begin{pmatrix} -(\lambda+\alpha) & \mu & 0 & 0 \\ \lambda+\alpha & -(\lambda+\alpha+\mu) & \mu & 0 \\ 0 & \lambda+\alpha & -(\lambda+\mu) & \mu \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} P_3(\infty) \\ P_2(\infty) \\ P_1(\infty) \\ P_0(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{3.10}$$

The solution of (10) gives the steady-state probabilities in the availability case. The explicit expression for $A(\infty)$ is given by

$$A(\infty) = \frac{\mu^3 + \mu(\lambda+\alpha)(\lambda+\alpha+\mu)}{A}, \tag{3.11}$$

where

$$A = \mu^3 - \mu(\lambda+\alpha)(\lambda+2\alpha) + (\lambda+\alpha)(\lambda+2\mu)(\lambda+\alpha+\mu) - \mu(\lambda+\alpha)(\lambda+\alpha-\mu).$$

4 Conclusion

The primary objective of this paper is to obtain explicit expression for the MTTF and steady-state availability using a very simple method without involving complicated analysis.

References

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