

Some elements of Dynamic Geometry and optimization in Physics

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Abstract. The strong connection of the Physics and Geometry is well-known, beginning with the roots in the geometrical similitude of the important method of the Physics similarity, up to the recent development of the theory of physical fractal structures following the elaboration by the geometers Benoit Mandelbrot et al. of the foundations of Fractals theory.

This work aims to point out the presence of some elements of Dynamic Geometry and Optimization in themselves foundations of the modern Physics, as well as to emphasize their presence in frame of some new formalisms of Physics, in the Complexity theory, particularly.

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1 Introduction

As it is well-known, the United Nations (UN) General Assembly has adopted the resolution A/58/L.62, declaring 2005 as the International Year of Physics, and invited the United Nations Educational, Scientific and Cultural Organization (UNESCO) to organize activities celebrating this Year (see also the web page: <http://www.un.org/Depts/dhl/resguide/r58.htm>). In frame of the UN press release GA/10243 it was shown: “... In 1905, Albert Einstein had published several scientific articles that profoundly influenced understanding of the Universe. He had introduced utterly revolutionary ideas on fundamental questions, including the existence of atoms, the nature of light and the concepts of SPACE, energy and matter. The aim of the International Year went beyond the mere celebration of one of the greatest minds in Physics in the twentieth century. The Year would provide an opportunity for the largest possible audiences to acknowledge the progress and importance of the great field of science“.

The most important scientific articles published by Albert Einstein in 1905 remain those referring to the Special Relativity Theory, which led to the discovery of the nuclear reactions, to the design and building of the elementary particles accelerators, of the nuclear reactors, etc [unfortunately also to the obtainment (as a undesired consequence, by A. Einstein and the largest part of scientists) of the nuclear weapons].

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We have to underline here that the most important results obtained in following (1907-1920) by Albert Einstein refer to the General Relativity Theory, in fact an absolutely outstanding Gravitation Theory, in strong connection with the modern (physical) meaning of Geometry.

2 Elements of Dynamic Geometry in foundations of modern Physics

Presently, the definitions of the space and time in domains without matter and intense gravitational fields start from the propagation equation of electromagnetic pulses in free spaces [1]:

$$(2.1) \quad \nabla_4^2 \bar{E} = 0, \text{ where } : \nabla_4^T = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \frac{\partial}{\partial(ict)} \right)$$

is the transposed (row) 4-dimensional “nabla” operator.

As it is well-known, the light beams passing in proximity of some stars (of Sun, particularly) of mass M and radius R are deflected. Assuming that the inertial mass is equal to the gravitational one (for photons, inclusively), the expression of the deviation angle corresponding to the Newton’s gravitation theory is [2]:

$$(2.2) \quad tg \frac{\delta_N}{2} = \frac{kM}{c^2 R}$$

In the classical physics, such deviations of the light beams are met for the light propagation in dielectric (optical) media, when the light beam trajectory between 2 arbitrary points A and B is given by the Fermat’s principle:

$$(2.3) \quad \delta \int_A^B n \cdot dl = 0 \quad ,$$

where the symbol δ stands for the variation around the light beam real (physical) trajectory, n is the optical refraction index, and dl is the differential element of length of the considered trajectory.

The integral expression [equivalent to equation (2.3) of Fermat’s principle] states that the electromagnetic signal (pulse) propagation (of the light beam, particularly) accomplishes along that trajectory for which the optical path, defined by means of relation: $\Delta_{AB} = \int_A^B n \cdot dl$ has the minimal value (the most frequent case), the maximal value (possible situation, but rarely met), or a constant value, independent on trajectory (a situation valid for 2 conjugated points A, B relative to an optical lens). It results that – usually in optics – the electromagnetic (of the light beam, particularly) propagation produces along the space geodesics for which the classical geometry path is replaced by the so-called optical path:

$$(2.4) \quad d\Delta = n \cdot dl$$

We have to underline also that in expression (2.4) the optical refraction index n multiplies only the component of geometrical path along the tangent to the light beam trajectory.

Because the intense gravitational fields act also on the duration of the electromagnetic signals propagation, the modern gravity theories adopt the ideas concerning: a) the physical space curvature [analogous to equation (2.4)] by means of a “metrical” tensor g_{ij} of the coefficients of the space-time interval expression:

$$(2.5) \quad ds^2 = \sum_{i,j=1,4} g_{ij} dx_i dx_j = d\bar{r}_4^T \cdot \bar{g} \cdot d\bar{r}_4$$

(where: $x_1 = x$, $x_2 = y$, $x_3 = z$, $x_4 = ict$), b) the geodesics character $\left(\int_A^B ds = \min\right)$

of the trajectory of electromagnetic signals.

Starting from the expression of the d’alembertian operator corresponding to the metric tensor \bar{g} :

$$(2.6) \quad \nabla_{\bar{g}}^2 = \frac{1}{\sqrt{\det \bar{g}}} \nabla_4^T \left(\sqrt{\det \bar{g}} \cdot \bar{g}^{-1} \cdot \nabla_4 \right)$$

and taking into account that the expressions of the space-time interval (2.5), and of the d’alembertian operator corresponding to the metric tensor \bar{g} (2.6), respectively, are invariant (see [3]) to the accomplishment of a unitary transform ($\bar{U}^{-1} = \bar{U}^T$, therefore: $\det \bar{U} = \pm 1$):

$$(2.7) \quad d\bar{r}_4^i = \bar{U} \cdot d\bar{r}_4 \quad ,$$

it results that the space coordinates and the time are defined in the Einstein’s gravitation theory starting from the equation of electromagnetic signals propagation (of the light, particularly):

$$(2.8) \quad \nabla_{\bar{g}}^2 \bar{E} = 0 \quad .$$

As a particular example, we will mention that in the proximity of a spherical gravitation source of mass M and radius R , the space-time interval can be expressed by means of Schwarzschild’s relation:

$$(2.9) \quad ds^2 = \frac{dr^2}{1 - \frac{2kM}{c^2 r}} + r^2 d\theta^2 + r^2 \sin^2 \theta \cdot d\phi^2 - c^2 \left(1 - \frac{2kM}{c^2 r} \right) \cdot dt^2$$

where r , θ , ϕ are the usual spherical coordinates, t is the time, and k is the Newton’s constant of the universal gravitation.

In the general case, the metric tensor \bar{g} will be time-dependent, which indicates that even the modern Physics foundations are related to some problems of Dynamic Geometry.

Finally, we have to underline the absolutely outstanding character of the gravitational interaction, relative to the other 3 known (and recognized) fundamental physical interactions (see Diagram 1), which indicates the real possibilities of some future improvements of the present (modern) definitions of the space and time (see also [4], [5]).

truth, for increasing accuracy measurements, all these relations will be denied, the basic decision in the statistical studies of the experimental results being so the *rejection of the compatibility of some relations (or theoretical models) relative to the studied experimental results*. As to any statistical hypothesis, it is associated to the hypothesis of compatibility rejection a certain error risk, that has to be always known, but which is . . . rarely studied!

In order to advance in this direction, we will remark that – as it is well-known: a) to each set of experimental results concerning N different parameters corresponding to the same state of the studied system (let x_{1mp}, \dots, x_{Nmp} - the most probable values of these parameters) it is associated a confidence domain, that has – in the most frequent case of a normal distribution – the shape of a N -dimensional ellipsoid:

$$(3.1) \quad \varepsilon^T \Gamma^{-1} \varepsilon = f_N(N_i)$$

where ε is the “column” vector of errors ($\varepsilon_i = x_i - x_{i,cmp}$), ε^T is its transposed (“row”) vector, Γ is the matrix of co-variances (each element of Γ being equal to the statistical average of the product of the corresponding errors: $\Gamma_{ij} = \langle \varepsilon_i \varepsilon_j \rangle$), and $f_N(N_i)$ is a certain function on the confidence level N_i corresponding to the considered confidence domain, b) in the frequent case of the study of a pair of physical parameters (X and Y), the confidence domain associated to the normal 2-dimensional distribution will correspond to the internal part of the ellipse:

$$(3.2) \quad \left(\frac{x_k - x_{k,cmp}}{s(x_k)} \right)^2 + \left(\frac{y_k - y_{k,cmp}}{s(y_k)} \right)^2 - 2r_k \left(\frac{x_k - x_{k,cmp}}{s(x_k)} \right) \left(\frac{y_k - y_{k,cmp}}{s(y_k)} \right) = f_2(N_{ik})$$

where $s(x_k)$ and $s(y_k)$ are the square mean deviations corresponding to the values of parameters X and Y for the state k , and r_k is the correlation coefficient of these parameters values for the studied state

$$(3.3) \quad r_k = \frac{\Gamma(x_k, y_k)}{s(x_k) \cdot s(y_k)} = \frac{\langle (x_k - x_{k,cmp})(y_k - y_{k,cmp}) \rangle}{s(x_k) \cdot s(y_k)},$$

and

$$(3.4) \quad f_2(N_{ik}) = -2(1 - r_k^2) \cdot \ln(1 - N_{ik}) \quad .$$

One finds so, that *the fundamental problem of the definition of physical states is reduced to an optimization problem corresponding to the minimum value of the sums of weighted squares (3.1) [in the general case], or (3.2) [in the case of only 2 parameters]*.

Usually, the correlation coefficient r_k is considered as the main criterion in order to appreciate the compatibility of some relations $y = f(x)$ relative to certain sets of experimental data. In fact, this coefficient “measures” only the neighborhood degree of the centers of the confidence domains relative to the studied regression line (or curve, generally); for instance, even that $|r_a| > |r_b|$, the ensemble of experimental values from Fig.1a is not compatible to relation $y = f(x)$, while the ensemble from Fig.1b is compatible with this relation, because the corresponding confidence domains

are crossed by the regression line (function) $y = f(x)$. Of course, the solution of such problems, very important for the experimental data processing can be accomplished only using the computers. Particularly, some too small values (e.g., less than 0.01) of $q_k = 1 - N_{ik}$ (obtained from relations (3.2) or (3.4) for $x_k = x_{tk}$, $y_k = y_{tk}$, where x_{tk} , y_{tk} are the coordinates of the tangency point of the confidence ellipse tangent to the regression line (function) $y = f(x)$ (see the broken line ellipse from Fig.1b), can justify the hypothesis of incompatibility of the studied relation $y = f(x)$ relative to the considered ensembles of experimental results [6].

As the error risk $q_k = 1 - N_{ik}$ at the rejection of the compatibility of the experimental results corresponding to the physical state k relative to the studied theoretical relation $y = f(x)$ is less or it is larger than a certain threshold (chosen usually between 0.001 and 0,2), the respective compatibility is rejected, or it has to be accepted, respectively [7] - [10]. One finds so that – not only the definition of physical states – but even the decision about the compatibility/incompatibility of some physical theoretical models relative to the experimental data reduces to some problems of optimization and (implicitly, dynamic) geometry.

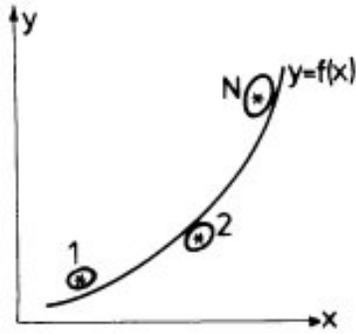


Fig. 1a

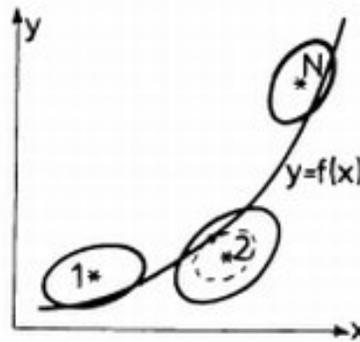


Fig. 1b

4 Dynamic Geometry in fluids flows descriptions and fractal structures

We have to answer in following to the question: is it involved also the Geometry in the general theory of Complexity? As it is well-known, the Complexity theory involves many fields of Mathematics, Physics, Chemistry, Biology, Electrical Engineering, Computer Science, Economics, Social Sciences, Cognition theory, etc. (see e.g. [11]). The most important general features of Complexity refer to the: a) (auto-catalytic) growth and accommodation, b) features of strong or weak chaos, c) power laws (and fractals, particularly), d) limit laws (and the auto-organizing criticality states, particularly), etc. Could we identify such features in frame of problems described by (dynamic) geometry?

In the particular case of fluids flows, the trajectories of fluid particles form (express) a certain (dynamic, for non-steady flows) geometry. For low speed (laminar) flows, the trajectories could be parallel, but for quicker flows appear some vortices (curved trajectories), at beginning of rather large dimensions. In order to explain the

rather strange similitude indices (analogous in Physics to the power exponents from the geometrical similitude relations) intervening in the description of turbulent flows parameters, Kolmogorov [12] proposed a hierarchical structure of vortices, the energy being injected firstly in the largest vortices and transferred in a cascade from the larger to the smaller vortices, up to the smallest ones, where the energy is dissipated. This hypothesis was strengthened by the contributions of Mandelbrot [13]. Taking into account that (for samples of different sizes) it is difficult to have exactly equal values of all uniqueness parameters (excepting the sample size), in order to fulfill the classical relation: $P = C \cdot L^D$ (with a non-rational similitude index D , called *the fractal dimension*) of the fractal theory [14], it results that the physical applications of the fractal theory correspond to *the prevalence (dominance) of the size (length) uniqueness parameter*. If for different size domains, the values of D belong to a set (discrete or continuous) of real numbers, the corresponding physical structure is called *multi (poly)fractal*.

In the last years, there were published several identifications of multi (poly)fractal structures, as those corresponding to: (i) the fracture surfaces of metals [15], (ii) fracture surfaces of concrete specimen [16], (iii) several parameters of disordered and porous media, aggregates, polymers and membranes [17], (iv) electrode surfaces (of fractal dimension $D_s \approx 2.6$) of some super-capacitors [18], [19], etc.

5 Dynamic Geometry in descriptions of growth / accommodation processes

A typical plot of the growth/accommodation processes is that presented by fig.2, p. 807 [20] for the relationship between the force generated by the skeletal muscle contraction and the myoplasmic Ca^{2+} concentration. For an arbitrary parameter p of the growing system, the growth/ accommodation plot is that from Fig.1 (below), where p_0 and p_∞ are the initial and final values, respectively, of the considered parameter.

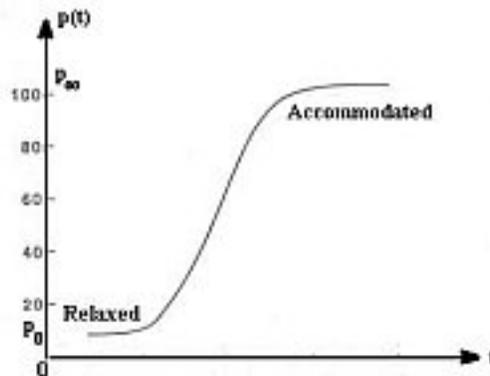


Fig. 2. The typical dependence of physical parameters for growth/accommodation processes

A possible quantitative model of the growth/accommodation phenomena is given by the differential equation:

$$(5.1) \quad \frac{dp}{dt} = a \cdot p(p - p_\infty), \text{ where } : a \cdot p_\infty = \frac{1}{\tau}$$

with the solution:

$$(5.2) \quad p(t) = \frac{p_\infty}{1 + \frac{p_\infty - p_o}{p_o} \exp\left(-\frac{t}{\tau}\right)}$$

For $p \ll p_\infty$, the equation (5.1) becomes: $\frac{dp}{dt} \cong \frac{p}{\tau}$, with the solution:

$$(5.3) \quad p(t) = p_o \exp \frac{t}{\tau} .$$

Because the dynamics of a quantity is said to be auto-catalytic if the time variations of that quantity are proportional (via stochastic factors) to its current value [11], it results that the validity domain of equations (5.3) represents the *auto-catalytic growth* domain of the growth/ accommodation processes.

Conversely, if: $p_\infty - p \ll p_\infty$, then the equation (1) becomes: $\frac{dp}{dt} = -\frac{p - p_\infty}{\tau}$, with the solution:

$$(5.4) \quad p(t) = p_\infty \left[1 - \frac{p_\infty - p_o}{p_o} \exp\left(-\frac{t}{\tau}\right) \right],$$

which corresponds to the “relaxation” domain of the growth/accommodation processes.

Because the characteristic growth/accommodation or relaxation times corresponding to different physical (as those corresponding to the strain τ_σ and stress τ_ε relaxation, resp. (23a, b) or biophysical (e.g. as those corresponding to the head and lungs growth, resp. of a child) parameters are different, it results that during the auto-catalytic growth/accommodation – according to relations (3) – we have: $\ln \frac{p_1}{p_{o1}} = \frac{t}{\tau_1} = \frac{\tau_2}{\tau_1} \ln \frac{p_2}{p_{o2}}$, and:

$$(5.5) \quad \frac{p_1}{p_{o1}} = \left(\frac{p_2}{p_{o2}} \right)^\nu, \text{ with } : \nu = \frac{\tau_2}{\tau_1} \in R$$

i.e. *the auto-catalytic growth leads to some specific (fractal) power laws between the different parameters p_1, p_2 of the growing system.*

Conclusions

The accomplished analysis points out the total interdependence of Physics, Geometry and Optimization theory, starting with the definitions of the main notions of Physics and Geometry, as: a) geodesics, b) space coordinates, c) true values of physical parameters, etc., as well as the strong dependence of these notions on the features of the specific existing physical interactions.

One finds also the common features of Geometry and Physics, related to the modern theory of complexity, as those referring to the: (i) power laws and fractals, particularly, (ii) growth theory and equations, (iii) symmetries in the different specific physical spaces, etc.

Because all these features are related (via physical interactions) also to time evolutions, one finds that the Geometry problems met in Physics are essentially Dynamic Geometry problems.

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