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Erdős, Paul; Sárkőzy, A.; Sós, T.
On sum sets of Sidon sets. I. (In English)
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For a finite or infinite set $A \subseteq \mathbb{N} = \{1, 2, \dots\}$ let $A(n) = |A \cap [1, n]|$ and $2A = \{a + a' \mid a, a' \in A\}$. A is called a Sidon set if all sums $a + a'$ in $2A$, $a \le a'$ are distinct.

Sum sets $2A$ of Sidon sets A cannot consist of "few" generalized arithmetic progressions of the same difference. To be more precise let $B_d = \{a \in 2A \mid a - d \notin 2A\}$ for $d \in \mathbb{N}$. There are absolute constants $c_1, c_2 > 0$ such that for all $d \in \mathbb{N}$ we have $|B_d| > c_1|A|^2$ if A is a finite Sidon set and (∗) $\limsup_{N\to+\infty} B_d(N)(A(N))^{-2} > c_2$ if A is an infinite Sidon set. For the proof in the case of infinite A the generating function $f(z) = \sum_{a\in A} z^a$, where $z = e^{-1/N}e^{2\pi i\alpha}$ for large $N \in \mathbb{N}$ and real α is considered. Assuming the contrary of the proposition, ingenious estimates of $I := \int_0^1 |(1 - z^d)f^2(z)|^2 d\alpha$ lead to contradicting lower and upper bounds for I. By example it is shown that $(A(N))^{-2}$ in (∗) cannot be replaced by $(A(N))^{-2} \log^{-1} N$.

While these results in the case $d = 1$ deal with blocks of consecutive elements in $2A$ for Sidon sets A, the next theorems give information about gaps between consecutive elements of $2A$. Let $2A = \{s_1, s_2, \dots\}$, $s_1 < s_2 < \dots$. For $n \in \mathbb{N}$, $n > n_0$ there exists a Sidon set $A \subseteq \{1, 2, \dots, n\}$ such that $s_{i+1} - s_i < 3\sqrt{n}$ for all $s_{i+1} \in 2A \setminus \{s_1\}$. The prime number theorem is used for constructing such sets A. For infinite Sidon sets the probabilistic method of Erdős and Rényi is adapted to prove the following result: For $\varepsilon > 0$ there is a Sidon set A such that

$$s_{i+1} - s_i < \sqrt{s_i}(\log s_i)^{(3/2)+\varepsilon}$$

for all $i > i_0(\varepsilon)$ and $s_i \in 2A$. Also given are lower estimates for $s_{i+1} - s_i$. A catalog of unsolved problems concerning Sidon sets and $B_2[g]$ sets closes this part I.

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