COMPARING KOREAN AND U.S. THIRD GRADE ELEMENTARY STUDENT CONCEPTUAL UNDERSTANDING OF BASIC MULTIPLICATION FACTS

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Elementary students who were currently third graders in Korea and the U.S.A. were asked to complete the Researcher-made Multiplication Questionnaire (RMQ) that contained four open-ended questions in order to examine their conceptual understanding of multiplication. The U.S. third graders provided better and more divergent definitions of multiplication than Korean students. However, Korean students created a multiplication word problem and represented it with a number sentence better than U.S. students did. Also, Korean students better identified the situations when multiplication was used in their real lives. But, Korean students lacked understanding of the different meanings of multiplication for all questions compared to the U.S. students’ understanding.

INTRODUCTION

An agenda for the reform of school mathematics in the field of education has been the topic of considerable research, resulting in a new teaching approach based on the current learning theory “Constructivism.” The teaching and learning of mathematics has moved worldwide from memorizing sets of established facts, skills, and algorithms to focusing on children’s active construction of meaning and mathematics as sense-making and meaningful learning.

In the U.S.A., this movement has been articulated with consistency by publications of the National Council of Teachers of Mathematics (NCTM) such as the Curriculum and Evaluation Standards for School Mathematics (1989) and the Principles and Standards for School Mathematics (2000). These publications call for reform in school mathematics by placing emphasis on the importance for all students in grades K-12 to study a common core of broadly useful mathematics through their active participation in the learning process in order to become intellectually autonomous learners.

Korean schools use a national curriculum. This mathematics curriculum has been developed and revised by a committee of educational leaders among classroom teachers in different grade levels, mathematics educators, and researchers from academic institutes under the authorization of the Ministry of Education and Human Resources Development (MEHRD). The current Korean mathematics curriculum, which is the 7th national curriculum, distinguished as “learner-centered,” was revised in 1998 (Lew, 2004) and has been implemented since 2000 (Paik, 2004, p. 12). This curriculum development reflected the current mathematics reform movement within...
the international context and was easily noticed in the history of Korean mathematics education (Paik, 2004).

International comparative studies in student academic achievement, such as the Trends in International Mathematics and Science Study (TIMSS, 1999 & 2003) and the Program for International Student Assessment (PISA, 2003), reported that Asian students achieved high scores in the subject of mathematics and outperformed their western counterparts. Particularly in 2003, Korean 8th graders ranked 2nd (M=589) while the U.S. students ranked 15th (M=504) in mathematics among 46 countries participating in TIMSS. After these findings, there is a growing research interest in Asian mathematics education within the U.S.A. Various research studies have been carried out to contrast the curriculum and instructional methods to examine differences between Asian countries and western countries (e.g., Chung, 2005; Hiebert & Stigler, 2000; Li, 2000; Watanabe, 2001; Yong, 2005). However, few research studies contributed to comparing students’ conceptual understanding of mathematics, which is one of the most important current issues in the teaching and learning of mathematics.

Multiplication has been much more difficult for children to perform than addition and subtraction in general (Oliver, 2005). Earlier studies have shown that young children can develop multiplication concepts in kindergarten or first grade (Carpenter, et al, 1993; Clark & Kamii, 1996). However, teaching multiplication facts is a basic part of the primary grade (K-3) mathematics curriculum. Students are introduced to multiplication concepts in second grade and are required to memorize their facts in third grade (Chung, 2005; Wallance & Gurganus, 2005). Some researchers (e.g., Behr et al., 1994; Bell et al., 1989; Confrey & Smith, 1995) claimed that once children reach the primary grades they are unable to solve problems involving multiplication or apply multiplicative number facts with meaning. When students reach grades 4-5, they experience difficulty in using multiplicative reasoning in a range of contexts and integrating their understanding of rational numbers with multiplication and division. This suggested that difficulties faced by older students can be attributed, at least in part, to the lack of development of conceptual understanding of multiplication in early primary grades (Mulligan & Watson, 1998). With the results of the international comparative studies and concerns about teaching and learning mathematical concepts, especially multiplication concepts in primary grades, researchers investigated how students from Korea and the U.S.A. differ in their conceptual understanding of basic multiplication facts.

**Purpose**

This study was established to compare the conceptual understanding of basic multiplication facts between Korean and U.S. 3rd grade elementary students. The specific objectives of this study were to investigate the following: 1) Define multiplication; 2) Represent their understanding of multiplication at the symbolic level (such as numbers and words, and number sentences); 3) Communicate their
understanding of basic multiplication facts with others; and 4) Relate their conceptual understanding of multiplication to their real life situations.

METHODOLOGY

Participants

Participants for this study included 129 male and 111 female, (total 240) third grade students of Korean (n=120) and American (n=120) heritage who were currently enrolled at two public schools in the suburban areas of the states of Indiana and Illinois in the U.S.A. and three schools in the suburban areas of Seoul and Pusan in Korea. The mean ages of the participants were 111.25 months (SD 3.48) for Korean schools and 106.35 months (SD 4.16) for the U.S. schools. Korean students were taught with the national elementary school mathematics curriculum which was developed by a mathematics curriculum committee under the authorization of the Ministry of Education and Human Resources Development (MEHRD, 2001). The U.S. students who participated in this study were taught with the mathematics curriculum, “Everyday Mathematics” published by the University of Chicago School Mathematics Project (2001).

Instrument

The Researcher-made Multiplication Questionnaire (RMQ), entitled “How Much Do I Know about Multiplication?” was used to examine the 3rd graders’ conceptual understanding of basic multiplication facts. It consisted of 4 open-ended questions. Students were asked to define multiplication, create a multiplication word problem and construct a number sentence for the problem, explain their understanding of a basic fact (e.g. 7 x 6) using their own words, and describe how they utilize multiplication skills in their real life situations.

The questionnaire was developed in English and reviewed by two professors. The first is a university professor with expertise in educational measurement and statistics and the second is a mathematics education professor. The questionnaire was translated into Korean by the researcher. This instrument was also used in a previous research study done by Kim, Anderson, and Chung in 2002. For the current study, the RMQ was reviewed and revised by two classroom teachers from each country prior to distribution to the participants.

Procedure and Data Analysis

The researcher in each country distributed a letter containing the information explaining the objectives of the study, a copy of the parent consent form, student consent form, and the questionnaire to the principals of three participating schools at the beginning of the fall 2006 semester. One school in the U.S.A. dropped out from the study. As a result, only five public schools participated in this study. Two schools were located in Seoul and one in the Pusan area in Korea. One school was located in Chicago, Illinois and one school was located in Granger, Indiana in the U.S.A.
The student questionnaires were collected by the researchers in early September from Korean schools and during the period of mid-November to early December from the U.S. schools.

Student responses on the questionnaires were categorized following the guidelines of the data coding system developed by the researchers and input on the computer using SPSS 14.0 software. The four multiplicative structures (additive/equal group, array/area, multiplicative comparisons, and combinations) identified by Greer (1992) were adopted to analyse student responses regarding multiplication stories. The coding system and input data were cross-examined by both researchers and reviewed by a Korean American sociology professor who is an expert in research measurement and statistics. Coded data were analysed using descriptive statistics. Frequencies, percentages, and cross-tabulation were used to analyse student responses to individual items on the RMQ. Cross-tabulation and the Chi-square statistics were employed to determine differences in the third grade students’ conceptual understanding of multiplication between the two groups.

RESULTS

When students were requested to provide a definition of multiplication using their own words, approximately three quarters of the U.S. students (n=88, 73.3%) provided a correct definition whereas only about one half of the Korean students (n=56, 46.7%) could do so. The Pearson Chi-Square statistics indicated that more U.S. students had significantly clearer definitions of multiplication than Korean students at p < .001 (see Table 1).

There are four distinctive models in multiplication. These are additive/equal groups, area/arrays, multiplicative comparisons, and combinations/Cartesian. The additive model tells how many groups or sets of equal size are being considered. The area/array model is a rectangular region defined as the units along its length and width and an arrangement of objects or pictures in rows by column. The multiplicative comparisons mean that there are two different sets that needed to be matched one-to-one to decide how much larger one is than the other. The combination/Cartesian model states that there are two factors representing the sizes of two different sets and the product indicates how many different pairs of things can be formed (Reys et al., 2004).

Students from both groups provided definitions in five different ways: 1) Additive/equal groups; 2) area/array; 3) multiplicative comparisons; 4) additive and array; and 5) additive and multiplicative comparisons. For the latter two ways, students explained the problem using two different models. Approximately forty-two percent (n=101) of two hundred forty students (48 Korean students, 53 U.S. students) defined multiplication as additive/equal groups (see Table 2). For the task to create a multiplication word problem, about seventy-one percent of Korean students (n=85) and about fifty-six percent of U.S. students (n=67) provided correct word problems and represented the problems in a number sentence. Pearson Chi-Square showed significant statistical difference between the two groups at p < 0.001 for the word
problem and at p < .01 for representing the problem by a number sentence (see Table 1). Approximately fifty-five percent of students (76 Korean students, 55 U.S. students) created additive/equal group models of multiplication problems (see Table 2).

Students were requested to explain to their younger siblings what 7 x 6 means in their own words. For this question, approximately seventy-three percent of Korean students (n=88) and seventy-six percent of U.S. students (n=91) explained the meaning of 7x6 correctly. There was no statistical significant difference between the two groups at p > .05 for explaining the meaning of the problem (see Table 1). Approximately twenty-eight percent of U.S. students (n=33) explained the problem with words and about twenty-six percent of students (n=31) used numbers and words in their explanation. Close to seventy-five percent of Korean students (n=85) described the problem in words and numbers/number sentences. The most frequently used approach for Korean students (n=81, 67.5%) was to describe the meaning of the problem with the additive/equal group multiplication model. For the U.S. students, about forty-one percent of the students (n=49) approached the problem using additive/equal groups and twenty-three percent of the students (n=28) explained the problem using the array/area model of multiplication (see Table 2).

Regarding the question of when students use multiplication skills in their real lives, nearly one half of the Korean students (n=56, 46.7%) and slightly more than one forth of the U.S. students (n=33, 27.5%) clearly identified a situation. Pearson Chi-Square statistics revealed that there was significant difference between the two groups at p < 0.01 for identifying situations, but no differences in terms of relating how to use the skills in their real lives (p>.05) (see Table 1). Thirty-five percent of Korean students (n=42) and twenty percent of U.S. students said they had used multiplication skills as the additive/equal group model in their real lives (see Table 2).

### Tables

<table>
<thead>
<tr>
<th></th>
<th>KOREA (n=120)</th>
<th>USA (n=120)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Correct</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Definition of multiplication***</td>
<td>46.7%</td>
<td>73.3%</td>
</tr>
<tr>
<td>Creating a word problem***</td>
<td>70.8%</td>
<td>55.8%</td>
</tr>
<tr>
<td>Connecting a word problem to a number sentence **</td>
<td>70.8%</td>
<td>54.6%</td>
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<tr>
<td>Explaining the meaning of the problem</td>
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<td>75.8%</td>
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<tr>
<td>Identifying the situations**</td>
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<td>27.5%</td>
</tr>
<tr>
<td>Explaining real life applications</td>
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</tr>
<tr>
<td><strong>Incorrect</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Resp.</td>
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</table>

**Table 1: Third graders’ conceptual understanding of basic multiplication facts (N=240)**

<table>
<thead>
<tr>
<th></th>
<th>Additive(A)</th>
<th>Array(R)</th>
<th>Multiplicative(M)</th>
<th>A &amp; R</th>
<th>A &amp; M</th>
<th>Incorrect</th>
<th>No Resp.</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
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</tr>
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<td>KOREA</td>
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<td>0%</td>
<td>0%</td>
<td>52.5%</td>
<td>0.8%</td>
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<td>4.2%</td>
<td>7.5%</td>
<td>0%</td>
<td>23.3%</td>
<td>0%</td>
</tr>
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</table>

* p<.05, ** p<.01, *** p<.001
DISCUSSION AND CONCLUSIONS

The analysed data of this study yielded results supporting previous international comparative studies. Korean third graders outperformed the U.S. students in creating multiplication word problems, constructing a number sentence to represent a word problem, and identifying real life situations in which they utilize multiplication skills. The Principles and Standards for School Mathematics (NCTM, 2000) advocates that a balance and connection between conceptual understanding and computational proficiency are required for students to develop fluency in mathematics. For example, when multiplication facts are taught for conceptual understanding and connected to other mathematics concepts and real-world meaning, students perform better on standardized tests and in more complex mathematics applications (Campbell & Robels, 1997). This study indicates that Korean students have good understanding of multiplication concepts.

One of the interesting findings in this study was that U.S. students gave better definitions of multiplication, yet they were not able to apply this knowledge to creating a multiplication word problem and a number sentence that corresponded with the problem. In the meantime, Korean students were better able to explain word problems and construct the number sentences. However, their ability to explain the meaning of the specific problem (7x6) was not significantly better than that of the U.S. students. Korean students better identified the situations in which they needed to use multiplication in their real lives, even though they were not more capable of explaining how multiplication skills were specifically used. This implies that Korean teachers focus more on having students practice constructing word problems and number sentences and understanding the word problems rather than letting students explore and connect relationships among symbolic representations and different concepts.

Another important finding in this study suggests that there is a major conceptual instructional challenge for classroom teachers of both groups. When teaching multiplication, teachers need to help children understand that multiplication has a variety of meanings, which are additive/equal group, array/area, multiplicative comparison, and combination/Cartesian. The models that were provided by third
graders in this study were not diverse. The greatest number of students in both countries used the additive/equal group model, which was defined by the students as repeated addition. Korean students, in particular, dominantly used the additive/equal group model. The array/area model was the second most frequently used model by the U.S. students to illustrate the multiplication problem. This indicates that the U.S. students possessed more various meanings of multiplication. Few students used multiplicative comparisons and no student provided multiplication meaning as the combination/Cartesian model.

Finally, one of the most significant findings in this study was that students from both countries did not clearly explain how multiplication skills were used in their real lives. More than three fifth of students (60.0% Korean students, 71.7% U.S. students) could not accurately address the question, which was to discuss how multiplication skills are used in everyday life by providing a specific example.

Wallace and Gurganus (2005) recommended that the most effective sequence of instruction to help children acquire the concepts of multiplication facts is to introduce the concepts through problem situations and link new concepts to prior knowledge. With this strategy, students also should be allowed to have concrete experiences and semi-concrete representations before purely symbolic notations, explicit instruction of rules, and mixed practice are introduced.

**References**


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