CLASSROOM TEACHING EXPERIMENT: ELICITING CREATIVE MATHEMATICAL THINKING

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A classroom teaching experiment intended to elicit a high frequency of creative mathematical thinking is reported. It was designed to operationalise pedagogy enabling spontaneous creation of concepts through progressive increases in complexity of thought processes. A recent study of creative thinking in classrooms informed the design. Data was collected using a modification of the video/interview techniques from the Learners’ Perspective Study. Cameras were positioned to capture the activity of multiple student groups and their interim reports to the class. Four students were interviewed after each lesson. By providing insights into links between pedagogical moves, the quality of student thinking, and the creation of new knowledge, this study informs pedagogy intended to optimise student learning.

INTRODUCTION

Cobb and Steffe (1983, p. 83) defined a ‘teaching experiment’ as “a series of teaching episodes and individual interviews that covers an extended period of time”. This study extends the conception of a teaching experiment from researchers working with individuals or pairs outside, or within the classroom to a researcher as teacher (RT) working with the classroom teacher (T) and the whole class. The teaching experiment is part of a broader study of the role of optimism in collaborative problem solving. To gain insights into group ‘collaboration’ in class, access to collaborative activity was required. ‘Collaboration’, for the purposes of this study involves groups working together beyond their present conceptual level to explore questions they spontaneously set themselves as a result of identifying unfamiliar complexities and deciding to unravel them. The teaching experiment design was informed by a recent study of creative thinking in classrooms (Williams, 2005). The activities undertaken by these students who managed to manoeuvre their own ‘spaces to think’ in classrooms where this was not the explicit intention of the teacher provided insights into how the teacher could set up a classroom environment that increases opportunities for ‘creative thinking’ (‘spontaneous complex thinking’ called ‘complex thinking’ in this paper). Most of the theory upon which this paper relies is integrated into the design and analysis process. Complex thinking is analysed using Dreyfus, Hershkowitz, and Schwarz’s (2001) RBC model integrated with Krutetskii’s (1976) mental activities (Williams, 2005). The thinking of students becomes progressively more complex from ‘analysis’, ‘synthetic-analysis’, and ‘evaluative-analysis’ (Novel B) to ‘synthesising’ and ‘evaluating’ (Novel C). These terms are elaborated later.
SITES AND SUBJECTS

This study was undertaken in a Grade 5/6 class of 22 students in a government primary school in Australia. Students were from families that had been in Australia for more than one generation and families that had recently arrived in Australia. The students worked in six groups of three or four selected by the T and RT. The group of four students selected to illustrate frequent complex thinking were Patrick, Eliza, Gina, and Eriz (Group 1). They were selected because they worked well together and elaborated their thinking clearly in their interviews. The teacher (T) had worked previously with this researcher (RT), was aware of the pedagogical approach, had experimented with it, and considered that participation in this research could improve her expertise in this area. The task under study in this paper was the first task undertaken with the RT. It extended over three eighty-minute lessons.

TEACHING EXPERIMENT DESIGN

The six activities in the Space to Think found common to the creative development of novel ideas and concepts by five students in four classrooms in Williams’ (2005) study of ‘creative thinking’ were: a) exploring optimistically; b) identifying complexities within, beyond, or peripheral to the teachers’ task; c) manoeuvring cognitive autonomy; d) accessing mathematics through cognitive artefacts, or by focusing idiosyncratically on dynamic visual displays; e) spontaneously pursuing self-focused exploration; and f) asking questions to structure future exploration. Each of these six activities informed the pedagogy in the teaching experiment. Illustrations of pedagogical moves associated with each of these six activities are now described:

Enacting optimism was valued by the RT (Table 1, L1, 5:19): “we are always going to do- change … [our] mind whenever … [we] want to- because that’s how people learn- by having a try”. This was intended to encourage thinking about situations where students were not yet successful to find what they could change to improve the situation. Such activity is a characteristic of optimistic children.

Identifying Complexity: Task 1 (see Figure 1) provides many opportunities to explore mathematical complexities associated with it. Students can employ complex thinking through experimentation, and generation of specific examples (analysis, B), simultaneous analysis of the examples generated (synthetic-analysis, Novel B) for the purpose of making a judgment (evaluative-analysis, Novel B), and / or through finding patterns (analysis), and considering the relevance of patterns (evaluative-analysis or synthesis, Novel C if a logical mathematical argument given).

Initially, students were encouraged to use whatever language they were comfortable with: RT: “you don’t have to use maths words … use any words you like for a start” (Table 1, L1, 0:55). The small size of the blocks increased the likelihood that students...
would need to find language to describe boxes. Terms such as length, width, height, cube and cuboid were expected to emerge through the lessons. Because students could start working with specific examples, it was expected that all groups would have access to experimentation. By asking for a mathematical argument for why there were no more boxes (rectangular prisms) toward the end of Lesson 1, the mathematical structure associated with volumes of these boxes was expected to emerge. Reduction in the number of cubes per group (83, L1; 24, L2; 0, L3) and increase in the sizes of volume students considered (to a number beyond 24 in Lesson 2) was intended to shift students from counting to analysing the underlying structure.

**Task 1**

**Part 1:** Make boxes with 24 of these cubes. How many can you make? How do you know that you have got them all? Can you make a mathematical argument for how you know you have got them all? [Intention: elicit novel building-with and recognizing to support constructing]

**Part 2:** Late in Lesson 2, introduce a game for group competition. A ‘box’ with a volume of 36 little cubic blocks had been hidden in a big coloured container. Groups had 5 minutes to develop strategies. The aim is to be the first group to find the ‘box’ dimensions. Each group can ask a question that all class members could hear. RT and T will give Yes / No answers. Each group can state what they think the dimensions are when they are sufficiently sure. They cannot have a second turn at stating this until all groups have had a first turn. [Intention: to elicit consolidating and increased elegance to support constructing]

*Note:* Two terms were introduced at the start of the task.

*Box:* was elaborated by the students identifying the features of a large cuboid prior to the task. The RT drew attention to both cubic and non-cubic examples during this discussion.

*Volume:* was defined as the amount of 3D space taken up by the box and measured in cubic centimetre blocks for this task.

**Figure 1. Brief summary of the first task undertaken**

The purpose of the task was to develop an informal understanding of volumes of prisms, and to raise awareness of the meaning of factors and their relevance to this. It was anticipated that a students would become better at thinking mathematically if the RT drew attention to such thinking. E.g., RT: “Donald just made a mathematical argument for why he thinks there are twelve.” (L1, 5:19).

**Cognitive autonomy** was addressed in two ways. During group work, groups identified and pursued their own focus of exploration. Groups were composed so students who were likely to think at a similar pace were together. This criterion is not necessarily related to mathematical performance but rather related to the ability to think about new ideas. In an attempt to compose groups where students would work well together and think at similar paces, the RT used her prior experience in group composition to assist the T to form groups.
**Autonomous access** to mathematics was assisted by the minimal mathematical background required in the task, the use common language, and intermittent reports including visual displays in the form of concrete, written, and diagrammatic representations that often progressively changed as groups discussed their thinking. As the RT and T did not judge the correctness of the reports, groups made their own decisions about what might be relevant to their idiosyncratic explorations.

**Spontaneous Pursuit.** Spontaneity is crucial to creative activity (Williams, 2005). Steffe and Thompson (2000) called “‘spontaneous development’ development and learning not caused by the teacher” (Steffe & Thompson, 2000, p. 289). This teaching experiment was designed in the expectation that actions of the RT and T would influence but not cause creative thinking in this class.

We do not use spontaneous in the context of learning to indicate the absence of elements with which the student interacts. Rather we … refer to the non- causality of teaching actions… we regard learning as a spontaneous process in the student's frame of reference. (Steffe & Thompson, 2000, p. 291)

Williams (2004) operationalised spontaneity by subcategorising the social elements identified by Dreyfus, Hershkowitz, and Schwarz (2001) to identifying what can eliminate it. Spontaneity can be eliminated when there is lack of opportunity for a group to follow their own direction (External Control), explanations are provided by an external source (External Explanation), mathematical ideas are extended by an external source (External Elaboration), external sources dispute findings (External Query), external sources affirm the validity, correctness, or attainment of closure (External Agreement), and / or external sources focus attention on an aspect of student exploration and expect them to pursue it [External Attention/Control]. The RT and T did not provide mathematical input related to spontaneous explorations but did draw attention to aspects of findings and reports without judging the correctness of these aspects and without expecting students to explore them.

**Structuring Questions** asked by the RT to elicit complex thinking were based on questions students asked themselves and questions the RT had asked whilst a teacher (see Williams, 2005, see p. 384). Some of these questions are illustrated herein.

**RESEARCH DESIGN**

The research question is: Does this teaching experiment elicit a high frequency of complex mathematical thinking associated with developing new mathematical knowledge, and if so, does the pedagogy influence this process? To study this question, classroom pedagogy, student responses, and new understandings were captured through classroom video and video-stimulated interviews. The Learners’ Perspective Study methodology (Clarke, Keitel, & Shimizu, 2006) was modified to capture the private talk of three groups, the physical activity of the remaining groups, interim reporting sessions, and student reconstruction of their classroom thinking. Four cameras were used, group written work was collected, and post-lesson video-stimulated individual interviews were undertaken with four students after each
lesson. Students were selected from at least two groups each lesson. Selection was based on the positioning of video cameras, and the activity that occurred. In Group 1, Eliza was interviewed after Lesson 2, and Patrick and Eriz after Lesson 3. Gina was not interviewed during Task 1.

RESULTS AND ANALYSIS

Some of complexities groups discovered were: Would division help? (Group 2, Lesson 2). Could a cube be made with 24 blocks? (Group 3, Lesson 1). Would four always be a frequent dimension for boxes of any volume (as in boxes of volume 24)? (Group 4, Lesson 2). Why do some boxes need three numbers to represent them and other boxes need only two (Group 4, Lesson 2)? And how can we find how many cubic centimetres there are in a box when we do not have sufficient blocks to build it? (Group 1, Lesson 2). These diverse foci illustrate the potential for Task 1 to elicit spontaneous exploration.

Table 1 shows the common structure to the three lessons [Column 1] and the differences in foci between them [Columns 2, 3, 4]. It also shows parts of the lessons where complex thinking was elicited and illustrates this thinking. There were intervals in each lesson where complex mathematical thinking beyond that normally occurring in classrooms was identified (beyond analysis, Williams, 2005). Some of this thinking and the situations that influenced it are now elaborated. Student thinking tended to become more complex when the RT asked questions about patterns, reasons why patterns occurred, and whether there was a mathematical argument for why there were no more boxes.

Eliza, in her interview identified what had led to her understanding of boxes as layers of cubes. In Lesson 2, the group had only 24 cubes yet wanted to construct a 32 cubic cm box. The group made a box with six layers of four and then drew the last two layers of four on paper before counting all the cubic centimetres in groups of four. Eliza explained what had happened in her interview after Lesson 2:

“we had [sketching] six [lots of 4] stacked up like that … then we had … a drawing on a piece of paper … we needed that to pretend there was another bit of eight”

The quality of Eliza’s understanding that boxes contained layers, and her elegant use four layers of eight later was evident in her report in Lesson 2 [50:30] when she explained how the group counted the number of cubes in a 32 cubic centimetre box:

“start by making … some- four (pause) flat boxes (pause) out of eight (pause) one centimetre cubes (pause) … stack the four (pause) to make 32 (pause) … (pause) count them- there should be (pause) four (pause) in (pause) the height and (pause) eight in the length so you count- you use four times eight which is thir- so you have 32 in the box”.

Although Eliza used the term ‘length’ incorrectly in this instance, she had developed a language to explain what she is doing and by the end of Lesson 3 was using this term correctly. The hesitations in her communication could be due to her selection of new words, or to the fragility of her conceptual understanding or both.
### Table 1 Overview of lesson structure/ RT questions/ Group 1 student responses

<table>
<thead>
<tr>
<th>Lesson No./Interval</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td>Introduction</td>
<td>0:20:33 Task Part 1, Classroom Culture</td>
<td>1:12-9:42 RT: Introduced Part 2.</td>
<td>10:26-15:54 Think about 12, 27, 42 cubic cm boxes.</td>
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<tr>
<td>Group Work</td>
<td>23:21-31:30 Experimentation \textbf{\textit{(Analysis, B)}}. Recognized layers, simultaneously considered numeric and physical strategies, \textbf{\textit{(Synthetic-analysis, Novel B)}}; Changed orientation of box, Are they the same? \textbf{\textit{(Synthetic-analysis)}}</td>
<td>9:42-34:07 Group 1 Designed a way to count cubes 32 cubes when they had 24. Eliza: [Elegant method] Found second easier way to make same box. \textbf{\textit{(Evaluative-analysis, B)}} synthetic-analysis for the purpose of judgement] \textbf{\textit{(Evaluative-analysis, Novel B)}} synthetic-analysis with judgement]</td>
<td>15:54-35:30 Patrick began to focus on numbers and gave a tentative reasons for the numbers he found. \textbf{\textit{(Evaluative-analysis, Novel B)}} synthetic-analysis with judgement]</td>
</tr>
<tr>
<td>Focus of Reporting, How to Prime Reporter</td>
<td>31:30-33:34 RT: “[the reporter will] tell you what they're going to say... and you are going to [make it] match[es] what your group wants” RT “[don’t] comment on ... whether you agree or disagree ... when ... person’s talking”</td>
<td>34:07-35:15 RT: “listen carefully ... we don’t want you to repeat it- we want to be able to say ... ‘we agree with such and such a group on this [and maybe] but we do not agree on that and this is the reason why” G 1 developed method to find no. of cubes in a box when insufficient blocks to build. \textbf{\textit{(Synthetic-analysis, Novel B)}} towards Synthesis, Novel C structure recognised.</td>
<td>15:54-29:36 RT “Look ... at those numbers- what are they like? Why?”</td>
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<td>Priming Reporters</td>
<td>31:30-33:34 Patrick: “We made this one because we divided six into twenty four and we got four [moves hand in layers]” \textbf{\textit{(evaluative-analysis, Novel B)}}</td>
<td>29:36-35:13 G1 discussed one coming up a lot and the main number (12, 27, 42) coming up only once unless orientation of box changed. Initial thinking about why \textbf{\textit{(Evaluative-analysis)}}</td>
<td></td>
</tr>
<tr>
<td>Reporting</td>
<td>37:11-1:08:27 Patrick wondered whether the same cube in a different orientation counted.</td>
<td>35:15-1:16:21 G1: Showed understood box structure. \textbf{\textit{(Synthesis, Novel C)}} Consider last report (the term factors was used). Will think further in L3.</td>
<td>35:13-1:10:09 RT: “What mathematicians do is think about why ... are these the patterns that are working”?</td>
</tr>
<tr>
<td>Summarising</td>
<td>How will you know when you have them all? Give an argument.</td>
<td></td>
<td>Try to make a sentence for the role of factors in making boxes</td>
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4-262 PME31—2007
Having insufficient cubes to make a 32-cube box extended the thinking of Group 1. The layered structure was made transparent and the ability to find answers by multiplication slowly became apparent. This was simultaneous consideration of the numerical and physical representation (synthetic-analysis, Novel B). Synthetic-analysis occurs when two possibilities (e.g., solution pathways, or representations) are considered simultaneously. Eliza also demonstrated evaluative-analysis when she decided it would be easier to make four layers of eight than eight layers of four for the 32-cube box. Evaluative-analysis (more complex Novel B) is synthetic-analysis for the purpose of making a judgement. Eliza judged the relative elegance.

Eriz reported finding difficulty keeping up with the thinking of the other group members and identified the time when the group primed him to report in Lesson 3 as useful for consolidating his thinking. The depth of his understanding was demonstrated by his confident explanation to the class of the number of cubes in the box that was “two long, two wide, and six high”. He calculated the number in a layer (two by two), then multiplied by the number of layers. His use of this numerical form including all three dimensions (before its meaning was reported by others) showed he did create new understanding.

Patrick reported in his interview that incorrect reports of other groups had assisted his thinking. In Lesson 3, Group 2 made a box containing 24 cubic centimetres when they had meant to make one with 12. The Group 2 reporter stated: “the length was two- the width was two and the height was six”. Patrick in his interview stated:

“You know how they got it wrong- it made me think about (pause) how they could get it right (pause) um (pause) thinking that- it was 2 2 (pause) 2 2 6 (pause) and (pause). If it was 24- they got 24 and they have to get 12 what if they changed the 6 to 3 and that would just halve it and instead of 24 they would have 12.”

Although not stated explicitly, Patrick appeared to have halved the number of stacks in the height. He thought deeply about many ideas during Task 1 and was close to finding the role of factors in making these boxes. In his interview after L3 he stated:

“Mm … um … it was … when we were talking about the pattern it was … strange- four only came up once but when we were working with the 24 it came up a lot more. I think- it only come up in the 12s one … because 42 and 27 um four couldn’t fit into it.”

By simultaneously considering the findings for the boxes of different volumes, Patrick undertook synthetic-analysis (Novel B). By making a tentative judgement about what he found “because 42 and 27 um four couldn’t fit into it.” He had commenced evaluative-analysis (a more complex Novel B). By testing these ideas further using specific examples, he would continue to undertake evaluative-analysis as part of novel building—with (Novel B). If Patrick had begun to think about why these numbers mattered, he would have commenced synthesis as part of constructing (C). It is possible he was doing so but not verbalising this.
DISCUSSION AND CONCLUSIONS

In these three lessons, more instances of creative thinking were identified than in sixty single lessons in the Learners’ Perspective Study (Williams, 2005). This might be partly accounted for by changes in data collection processes leading to the capture of multiple groups rather than one student pair per lesson. The close links found between the lesson structure in the teaching experiment and the complex thinking elicited provide convincing evidence that the teaching experiment was successful. Complexity of thinking increased as students: experimented, developed language to think about and communicate ideas, considered the RT’s questions prior to reporting, developed mathematical arguments to support their ideas, clarified what they wanted to communicate during their reports (priming the reporter), and thought further about ideas presented by other groups. The new knowledge by various students included awareness of boxes as layers of cubes, ability to calculate the number of cubes using layers, and some understanding of factors as relevant to constructing boxes of given numbers of cubes. Further research is required to test the potential for this teaching experiment to elicit creative thinking in other contexts. An area for further study is the role of synthetic-analysis and evaluative-analysis in supporting constructing.

REFERENCES


