GENETIC APPROACH TO TEACHING GEOMETRY

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In this theoretical essay the genetic approach to teaching geometry is discussed. We offer the "genetic" techniques of geometry teaching connected with the genetic elaboration of important geometrical concepts including the analysis of the subject from historical, logical and epistemological, psychological and socio-cultural points of view, with revealing logical genealogies of concepts and theorems.

INTRODUCTION.

The aim of this paper is to offer some hints in order to contribute to methods of geometry teaching at modern stage. Since 1924 when N.Izvolky's "The didactics of geometry" was published, the genetic approach has been considered as appropriate for geometry teaching (Beskin, 1947, Bradis, 1949). However, there is no well-elaborated system of genetic teaching to geometry yet.

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PRINCIPLE OF GENETIC APPROACH

The framework of this article is genetic approach to teaching mathematics (Safuanov, 1999, 2005) which, in turn, integrates educational and philosophical ideas of G.W.Leibnitz (1880), F.A.W. Diesterweg (1962), H. Poincare (1990) a.o., psychological discoveries of Piagetian and Vygotskian schools as well as rich experience of practice of mathematical education.

The principle of genetic approach in teaching mathematics requires that the method of teaching a subject should be based, as far as possible, on natural ways and methods of knowledge inherent in the science. The teaching should follow ways of the development of knowledge. That is why we say: «genetic principle», «genetic method».

In history and modern state of genetic approach a significant variety of interpretations of the terms “genetic principle”, “genetic method”, “genetic approach to teaching mathematics” is observed... It is clear that today, as noted by Wittenberg (1968, p.127), nobody understands genetic approach as historical, and more appropriate is idea that genetic approach is connected to relevance, which here should be understood as conformity of a method of teaching (and learning) to the most expedient and natural ways of cognition inherent in the given subject (or topic). Wittenberg is certainly right also in that genetic approach is connected to epistemology, psychology and logic.
Analysing various interpretations of genetic approach to teaching mathematics in theory and history of mathematics education and taking into account today's experience of teaching undergraduate mathematics and latest achievements of psychology and methods of teaching mathematics, we can reveal the contents and features of genetic approach to teaching geometry.

We will call the teaching of mathematical discipline genetic if it follows natural ways of the origination and application of the mathematical theory. Genetic teaching gives the answer to a question: how the development of the contents of the mathematical theory can be explained?

Taking into account numerous descriptions of genetic approach in the literature on mathematics education, results of theories of cognition and also of the theory, practice and psychology of mathematics education, we can conclude that genetic teaching of mathematics should have the following properties:

Genetic teaching is based on previously acquired knowledge, experience and level of thinking of students;

For the study of new themes and concepts the problem situations and wide contexts (matching the experience of students) of non-mathematical or mathematical contents are used;

In teaching, various problems and naturally arising questions are widely used, which should be answered by students independently with minimal necessary effective help of the teacher;

Strict and abstract reasonings should be preceded by intuitive or heuristic considerations; construction of theories and concepts of a high level of abstraction can be properly carried out only after accumulation of sufficient (determined by thorough analysis) supply of examples, facts and statements at a lower level of abstraction;

The gradual enrichment of studied mathematical objects by interrelations with other objects, consideration of the studied objects and results from new angles, in new contexts should be carried out.

One of major aspects of genetic approach to teaching mathematics is psychological aspect. As indicated by E.Ch.Wittmann (1992, p. 278), genetic principle should use results of both genetic epistemology of J. Piaget and Soviet psychology based on the concept of activity. Synthesising not contradicting each other results of two theories concerning construction and development of concepts in the learning process, it is possible to take as a psychological basis of genetic approach to teaching mathematics the following principles of psychology of education:

1) Principle of problem-oriented teaching. S.L.Rubinshtein (1989, p. 369) wrote: «The thinking usually starts from a problem or question, from surprise or bewilderment, from a contradiction». It is similar to Piagetian phenomenon of the violation of balance between assimilation and accommodation. L.S.Vygotsky (1996, p. 168) indicated in 1926 that it is necessary to establish obstacles and difficulties in
teaching, at the same time providing students with ways and means for the solution of the tasks posed.

2) **Principle of continuity and visual representations**: introducing new contents, it is necessary to maximally use previously generated cognitive structures and visual representations of pupils, familiar contexts. This principle is connected to the Vygotsky's theory of development of scientific concepts (see, e.g., Vygotsky, 1996, p. 86 and 146), and also with his concept of «zone of proximal development».

3) **Principle of integrity and system approach**: the teaching should aim at the accumulation of integral systems of cognitive structures by the pupil (Itelson, 1972, p. 132). This principle also follows both from the activity approach (Vygotsky, 1996, p. 178-179 and 270; Davydov, 2000, p. 327-328, 400) and from the theory of operator structures of J.Piaget (1994, p. 89-91).

4) **Principle of «enrichment»**: «Accumulation and differentiation of experience of operating by an introduced concept, expansion of possible aspects of understanding of its contents (by inclusion of its various interpretations, increase of number of variables of different degree of essentiality, expanding interconceptual connections, use of alternative contexts of its analysis etc.)» (Kholodnaya, 1996, p. 332).

5) **Principle of «transformation»**: for revealing essential properties of an object, its essence, «genetically initial general relation» (Davydov, 2000), it is necessary to subject this object to mental transformations, to perform mental experiments, asking questions of the type: «What will happen with the object if? … ».

All of these principles of genetic teaching of mathematics may be applied in geometry teaching.

**GENETIC APPROACH TO TEACHING GEOMETRY IN SOVIET AND WESTERN MATHEMATICAL EDUCATION.**

Many years ago the original and deep understanding of the genetic approach (not reduced to the historical approach) to geometry teaching had been shown by N. A. Izvolsky (1924):

“In the usual course of teaching neither the text-book, nor the teacher do not make anything in order to answer (in some form) the question about the origin of the theorems. Only in rare instances we see exceptions: some teachers in this or that form pay their attention to the origin of the theorems; for the pupils of this teacher the geometry course accepts other character and ceases to be the mere set of the theorems. Moreover, sometimes some of the pupils, independently of both a text-book and the teacher, half-consciously come to the idea that a theorem has appeared not because of the wish of the author of a text-book or the teacher, but rather because it gives the answer to the problem that has naturally arisen during the previous work... Perhaps this idea of the development of the content of geometry does not reflect to a great extent the historical path of this development, but this view is the answer to the naturally arising question: how the development of the content of geometry could be explained? For the
teaching of geometry to have such view of the subject-matter is extremely valuable...” (p. 8).

Izvolsky expresses the essence of the genetic approach by the following sentence: “A view of geometry as a system of investigations aiming at finding answers to the consequently arising questions” (p. 9).

Such prominent mathematics educators of the post-war period as V.M. Bradis and N.M. Beskin also applied the genetic approach in methods of teaching geometry.

V.M. Bradis, considering principle of a genetic character of an account by a basic principle of teaching mathematician, wrote:

“The experience of teaching definitely shows that the quality of mastering of a mathematical subject matter will essentially win if each new concept, each new proposition is introduced so that its connection with things already familiar to the pupil is clear and the expediency of its study is visible. For pupil, most convincing justification of each new concept and proposition is a practical activity close, whenever possible, to their experience” (Bradis, 1949, p. 44-45).

N.M. Beskin (1947) wrote: “... It is necessary to show geometry to the pupils not in a complete, crystallised but in the process of development. The method, which we recommend, is called genetic. This method makes each pupil the active creator of geometry: we put before her/him a problem, the process of its solving gives rise to separate theorems and entire sections of geometry”.

One can find interesting examples of genetic approach in the article of T.J. Fletcher (1974) on geometry teaching: “The sequence of technique-followed-by-applications is being rejected for a teaching approach which is more subtle and certainly more difficult to carry out - a contextual approach which gives more recognition to the character and needs of human beings. This involves devising learning situations my which students generalise from the experience. Abstractions are too important to be told to the student, he must come to see them himself. In other words the pupil develops understanding not so much by following a logical exposition as by making for himself a sequence of conceptual reorientations. The problem of teaching is to set up learning situations from which the pupil acquires the experience which compels the reorientation...” (p. 23). And further: “…The guiding principle at this stage is for the student first to do something and then to consider how he did it; to ask what principles he was applying, and to ask how explicit recognition of the principles gives power to do more” (p. 27).

GENETIC APPROACH IN LEARNING GEOMETRICAL CONCEPTS AND THEOREMS.

We offer the following ways of developing problem situations (Safuanov, 2005, p. 262):
1) Based on historical analysis of the subject matter, the teacher reconstructs the development of the concept, shows the origin of the problem, puts forward the hypothesis, shows various ancient and modern solutions and assesses the results;

2) Based on the logical and epistemological analysis of the development of a mathematical idea, the teacher himself constructs a problem situation, and pupils solve that problem under the guidance and control of the teacher;

3) The teacher constructs a problem situation, but pupils themselves independently put forward hypotheses, find solutions and carefully check them;

4) Based on previously acquired knowledge and on the theme studied, pupils themselves state new problems, naturally arising questions and the ways of their solutions. In this case the teacher accomplishes the co-ordinating function.

We consider the genetic approach in two types of the theoretical learning: in learning concepts and in learning theorems and their proofs.

In learning concepts, one may apply the technique of the design of the system of the teaching of the concepts described in (Safuanov, 2005) which must be preceded by the analysis consisting of two stages: 1) genetic elaborating of a subject matter and 2) analysis of arrangement of a material and possibilities of using various ways of representation and effect on students. The genetic elaborating of a subject matter, in turn, consists of the analysis of the subject from four points of view: a) historical; b) logical; c) psychological; d) socio-cultural. In designing of the system of genetic teaching very important is to develop problem situations on the basis of historical and epistemological analysis of a theme.

As the history, epistemology and socio-cultural aspects of most geometric school material is well-studied in literature, the mathematics educators can easily construct systems of the teaching of geometric concepts similar to those for algebraic concepts described in (Safuanov, 2005).

For example, when learning the theme “Quadrangles”, the teacher may offer: "Choose superfluous quadrangles among those described on the sheet of paper (a square, a trapezoid, a rectangle, several parallelograms)". The superfluous figure is the trapezoid because each of other quadrangles has two pairs of parallel sides. Thus, the essential property of a parallelogram would be extracted. Further, pupils can reconstruct logical genealogies of such concepts as a rectangle, square etc.

In the next example, when learning geometrical transformations, say, symmetries, it would be appropriate to begin with the work with geometrical models representing geometrical figures. Manipulating them and finding their axes and centres of symmetry, pupils can easily can to the concept of symmetry. Similar activities had been proposed (in elementary school) by V.V.Davydov (1996) and his disciples.

When learning theorems, genetic approach demands the use of analytic proofs as described by Beskin (1947, p. 78).
“... Studying a theorem by genetic method we should not introduce the statement to pupils immediately.

We offer to pupils a specific problem the solution of which is the theorem.

... Prominent geometer discovered new theorem because he better knew this area than ordinary people did. We can understand this theorem, when it is already formulated, but we encounter difficulties attempting to reproduce the path by which the author has come to this theorem. In such case we should try only to facilitate, as far as possible, the understanding of that path. The genetic method can not be reduced to studying all the theorems by a completely uniform scheme.

Trying to come, whenever possible, to the theorems by a natural way, we not always can attain it.

The last observation concern not only complicated theorems, but also many rather simple theorems at the very beginning of geometry. The difficulty is sometimes explained by the fact that the theorem will be necessary in one of the further sections of geometry, and before the learning of that further section it is difficult to explain, why we have introduced the theorem” (pp. 67-70).

“...It is worthwhile, as far as possible, to raise before the pupils a veil, behind which the course of thought having brought for the first time to the discovery of new proofs is concealed.

...Using analytical method of a proof we first of all try to prove what is required immediately (by single logic step). If that fails, we find out the positions which do not suffice for a proof of this theorem, and try to prove those positions... and so on from unproved to known positions. The course of reasoning in an analytical proof is just inverse with respect to the corresponding synthetic proof.... Usually the proof contains both synthetic and analytical elements” (pp. 75-78).

Consider an example of studying a geometrical result at school.

Studying a formula for the area of a regular polygon, after the construction of a regular polygon, pupils come to the idea of necessity of the partition of a polygon into triangles. Properties of regular polygons imply the conjecture about the equality of the constructed triangles. The pupils check the conjecture, find the area of one of triangles and, executing appropriate operations, independently formulate a conclusion.

It is important not only to teach pupils how to prove, but also to shape in their minds the need for proofs and the aspiration to discover them independently. It is necessary to try to organise teaching so that the child would ask himself why an assertion is correct would try to get to the bottom of the reasons of its correctness.

Analytic activities and, in particular, analytic proofs, had been described in (Gusev and Safuanov, 2001) where also the example of analytic proof was presented.
CONCLUSION

In this paper we outlined some ideas and methods of teaching school geometry by genetic approach. We think that the further development of “genetic” techniques of geometry teaching may be connected with the genetic elaboration of important concepts including the analysis of the subject from historical, logical and epistemological, psychological and socio-cultural points of view, with revealing logical genealogies of concepts and theorems. Also, practical manipulations with geometrical objects using, in particular, dynamic geometry systems such as Cabri would be useful, too.

References


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