MISTAKE-HANDLING ACTIVITIES IN THE MATHEMATICS CLASSROOM: EFFECTS OF AN IN-SERVICE TEACHER TRAINING ON STUDENTS’ PERFORMANCE IN GEOMETRY

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In a quasi-experimental study with 619 students from 29 classrooms (grades 7/8) we investigated the effects of a teacher training on teachers’ mistake-handling activities and students’ learning of reasoning and proof in geometry. Teachers of the experimental group classrooms received a combined training in mistake-handling and teaching reasoning and proof, whereas the teachers of the control group classrooms only took part in a training on teaching reasoning and proof. Their students participated in a pre- and post-test. Moreover, they were asked to evaluate how the teachers handled their mistakes. Our findings show that the teacher training was successful: the teachers of the experimental group classrooms changed their mistake-handling behavior and, compared to the control group classrooms the students in the experimental group performed significantly better in the post-test.

INTRODUCTION

For many students and teachers mistakes are associated with negative feelings. Despite the fact that „mistakes are the best teachers“, according to a well-known everyday proverb, teachers and students hardly take advantage of mistakes in class. In this paper we present a study on an in-service teacher training regarding the role of mistakes for the learning process. The findings indicate that students’ mathematics achievement benefits from a change in their teachers’ behavior regarding mistakes.

THEORETICAL FRAMEWORK

Our study particularly takes into account the work of Oser and colleagues on the role of mistakes for learning processes (Oser & Spychiger, 2005). Accordingly, we postulate that mistakes are necessary to elaborate the individual idea about what is false and what is correct. According to the theory of negative expertise individuals accumulate two complementary types of knowledge: positive knowledge on correct facts and processes, and negative knowledge on incorrect facts and processes (Minsky, 1983). Learning by mistakes is regarded as the acquisition of negative knowledge. Detecting one’s own errors helps to revise faulty knowledge structures. Storing past errors and the cues that predict failure in memory may prevent individuals from repeating mistakes (Hesketh, 1997).

Mistake-handling in mathematics classroom – empirical results

In particular, in the 1970s and 1980s many research studies were conducted in mathematics education analyzing underlying patterns of students’ mistakes in different
mathematical domains (e.g., Radatz, 1979). This research followed a diagnostic perspective and aimed at the identification of reasons for typical students’ mistakes.

Beyond this diagnostic research approach, there are only few research studies on mistake-handling activities and on the question what might be a promising way to deal with mistakes in the mathematics lessons. Video based research in different countries indicates that students’ mistakes appear only rarely during classwork in mathematics lesson (cf. Heinze, 2005). Findings of a video study in Germany show that the average number of mistakes made publicly in a grade 8 geometry lesson dealing with reasoning and proof is less than five mistakes in 45 minutes. This is surprising since nearly all teachers in the study followed a discursive teaching style (teacher question, student answer, teacher feedback). According to questionnaire based studies in Switzerland and Germany mistake-handling activities of mathematics teachers are evaluated comparatively positive by their students (Oser & Spychiger, 2005; Heinze, 2005). In particular, students hardly fear making mistakes in “public” lesson phases. Hence, the conditions for integrating error management in the mathematics classroom are comparatively good. Nevertheless, it seems that teachers do not use errors as a chance to create learning opportunities for their students; instead they are following an implicit behavioristic style that avoids the occurrence and discussion of mistakes. Hence, it is not surprising that findings of Oser and Spychiger (2005) and Heinze (2005) show that Swiss and Germans students do not recognize the potential of their own mistakes in mathematics.

**Effectiveness of error management trainings**

Despite the fact that errors are regarded as important aspects for learning, research in mathematics education gives only few hints how to accomplish this task in the classroom. An exception is the teaching experiment of the Italian group Garuti, Boero and Chiappini (1999) for detecting and overcoming conceptual mistakes. They used the “voice and echoes game” as a special approach to deal with conceptual mistakes. An alternative way to use mistakes as learning opportunities is described in the research program of Borasi (1996). She conducted a series of case studies and developed the strategy of capitalizing on errors as springboards for inquiry. Her taxonomy describes three levels of abstraction in the mathematics discourse (performing a specific mathematics task, understanding technical mathematics content and understanding the nature of mathematics) and three stances of learning (remediation, discover and inquiry). For each of the possible combinations of levels and stances she gives a description how errors can be used productively in the specific situation. As Borasi (1996) summarizes the case studies and teaching experiments provide “anecdotal evidence” that learners can benefit from her approach.

There is hardly any quantitatively oriented empirical research about the effectiveness of error management trainings for mathematics classrooms. However, research in other disciplines indicates that error trainings are rather successful. Studies on the acquisition of word processing skills for example give evidence that a training in error management improves performance significantly better than a training based on error
avoidance (Nordstrom, Wendland, & Williams, 1998). During error management training, the learners commit errors, either themselves (active errors) or by watching someone else commit errors (vicarious errors), and receive feedback about their mistakes. In error avoidance training, however, the learner is prevented from experiencing errors; in a behavioristic manner the aim of training is to allow learners to practice skills correctly and focus on the positive. Similar findings concerning the positive role of mistakes for the learning process are reported by Joung, Hesketh, and Neal (2006) for a training program with fire fighters.

**Error management activities as part of the problem solving process**

In the mathematics classroom errors and particularly the individual error management may play different roles depending on the mathematical activities. If a learner wants to acquire a principle, a formula, or an algorithm, she or he has a clear learning goal: something has to be memorized and understood, such that one can apply this knowledge in specific tasks. Reasoning and proof differs. According to the model of Boero (1999) the proving process consists of different stages in which the exploration activities play an outstanding role. Exploring a given problem situation, investigating given assumptions, retrieving suitable facts from memory etc. should be based on heuristic strategies. Here we have a situation of a systematic trial-and-error approach, which requires a permanent evaluation and drawing consequences from mistaken working steps. Hence, the ability to manage errors can be considered as a particular aspect of metacognition and is a prerequisite for solving complex reasoning and proof tasks.

**RESEARCH QUESTIONS**

As outlined in the previous sections we consider mistakes as a necessary part of the learning process. Moreover, error management skills are of particular importance when applying heuristic strategies in the mathematical problem solving process. As empirical findings for Germany indicate teachers and students hardly use mistakes as learning opportunities. Error management trainings in other domains show positive effects for the learning process. In the present study we are interested whether a special in-service teacher training on mistake-handling in mathematics classroom has positive effects on students’ performance. Presently, we are not aware of empirical studies about the effectiveness of a teacher training in this field. Since we expect that error management abilities foster particularly problem solving competencies we focus particularly students’ performance in reasoning and proof in geometry.

In our study we address the question to what extent a teacher training about the role of mistakes for the learning process in mathematics has an effect on

1. students’ perception regarding their teachers’ mistake-handling activities in mathematics lessons and
2. students’ performance in geometric reasoning and proof.
DESIGN OF THE STUDY

The sample consists of 619 students (311 female and 308 male) from 29 grade 7 classrooms (about 13 years old students). At the end of grade 7 a pre-test on geometry (basic skills, reasoning and proof) and questionnaires on motivation regarding mathematics and the mistake-handling activities were administered. Based on the results in the pre-test and the motivation questionnaire the classes were assigned to an experimental (10 classes, N = 240) and to a control group (19 classes, N = 379).

At the beginning of grade 8 the teachers of the sample classes took part in an in-service teacher training. The training for the experimental and the control group were organized separately at the university and took two days for each group. The teachers of the experimental group received a training about mistakes and in teaching reasoning and proof, whereas the teachers of the control group had a training in reasoning and proof supplemented by aspects of the new German educational standards for mathematics. The training about mistakes included aspects of negative expertise, students’ learning by mistakes, and the productive use of mistakes in the mathematics classroom. The training about reasoning and proof encompassed a model of the proving process, teaching material regarding reasoning and proof and typical student problems in this field. After the first training day the teachers got as an exercise to analyze their own instruction with respect to certain criteria. Their observations were included in the second part of the training two weeks later.

Two months after the teacher training, the students of both groups took part in a post-test on reasoning and proof in geometry and they filled in the questionnaire on mistake-handling activities again. During the two months among others the regular teaching unit on reasoning and proof in geometry was conducted by the teachers.

The pre- and post-test on reasoning and proof in geometry are approved instruments from our own research (e.g. Reiss, Hellmich, & Reiss, 2002). They contain curriculum related items for example on properties of triangles and quadrangles and congruence theorems. Data concerning the students’ perspective on mistake-handling activities in the classroom were collected via an approved questionnaire adapted from Spychiger et al. (1998). Students had to rate statements on a four point Likert scale (see examples in table 1).

RESULTS

Students’ perception of mistake-handling activities

A principal component analysis led to four factors for the 27 items of the mistake-handling questionnaire explaining 51% of the variance (see table 1).

Though there are four common items loading on the two factors concerning the affective and cognitive aspects of the teacher behaviour in mistake situations (e.g., Our mathematics teacher is patient when a student has problems to understand.), we decided to distinguish between these two aspects of teacher behavior. By the screeplot criterion of the principal component analysis the four factor solution was better than
The three factor solution. However, one must have in mind that these two factors base partly on the same items. In Spychiger et al. (1998) a three factor solution was preferred with only one factor for the teacher behavior.

The results of the questionnaire indicate that students were not afraid of making mistakes and appreciated their teachers’ affective attitude. The individual use of mistakes and teacher behavior regarding cognitive aspects were rated moderately by the students (see table 2).

<table>
<thead>
<tr>
<th>Factor</th>
<th>Item example</th>
<th>Reliability (Cronbach’s α)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual use of mistakes for the learning process</td>
<td><em>In mathematics I explore my mistakes and try to understand them.</em></td>
<td>0.83 (9 items)</td>
</tr>
<tr>
<td>Affective aspects of the teacher behavior in mistake situations</td>
<td><em>Sometimes our math teacher gets an appalled face when a student makes a mistake.</em></td>
<td>0.88 (9 items)</td>
</tr>
<tr>
<td>Cognitive aspects of the teacher behavior in mistake situations</td>
<td><em>If I make a mistake in maths lessons my teacher handles the situation in such a way that I can benefit.</em></td>
<td>0.87 (7 items)</td>
</tr>
<tr>
<td>Fear of making mistakes in the mathematics lessons</td>
<td><em>I become scared when I make a mistake in mathematics classroom.</em></td>
<td>0.74 (6 items)</td>
</tr>
</tbody>
</table>

Table 1: Factors of the mistake-handling questionnaire.

<table>
<thead>
<tr>
<th>Mean (SD)</th>
<th>individual use of mistakes</th>
<th>positive teacher behavior affective</th>
<th>positive teacher behavior cognitive</th>
<th>fear of making mistakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-test</td>
<td>total 2.47 (0.56)</td>
<td>3.06 (0.71)</td>
<td>2.79 (0.71)</td>
<td>1.85 (0.61)</td>
</tr>
<tr>
<td></td>
<td>experimental 2.48 (0.55)</td>
<td>2.87** (0.77)</td>
<td>2.65** (0.72)</td>
<td>1.97** (0.63)</td>
</tr>
<tr>
<td></td>
<td>control 2.47 (0.56)</td>
<td>3.17** (0.64)</td>
<td>2.88** (0.69)</td>
<td>1.78** (0.59)</td>
</tr>
<tr>
<td>post-test</td>
<td>total 2.49 (0.57)</td>
<td>3.20 (0.63)</td>
<td>2.95 (0.69)</td>
<td>1.79 (0.59)</td>
</tr>
<tr>
<td></td>
<td>experimental 2.56* (0.57)</td>
<td>3.32** (0.53)</td>
<td>3.09** (0.59)</td>
<td>1.79 (0.56)</td>
</tr>
<tr>
<td></td>
<td>control 2.45* (0.57)</td>
<td>3.12** (0.68)</td>
<td>2.86** (0.73)</td>
<td>1.79 (0.61)</td>
</tr>
</tbody>
</table>

Likert scale: 1=strongly disagree, 2=disagree, 3=agree, 4=strongly agree

* p < 0.05  ** p < 0.01

Table 2: Mistake-handling in students’ perception – pre- and post-test results.

Though the experimental group and the control group were parallelized after the pre-test with respect to their achievement in geometry and their motivation towards mathematics, they significantly differed in their perception of mistake-handling.
situations. Students of the control group judged their teachers as more positive than the students of the experimental group. However, in the post-test the students of the experimental group rated their teachers significantly better in the mistake questionnaire than the students of the control group. In particular, there was hardly a change for the control group regarding students’ perception of mistake-handling situations (table 2). The development from pre- to post-test (as difference) differed significantly between experimental group and control group for the components “affective aspects of the teacher behavior in mistake situations” (t(617) = 8.325, p < 0.001, d = 0.67), “cognitive aspects of the teacher behavior in mistake situations” (t(617) = 7.049, p < 0.001, d = 0.57) and “fear of making mistakes” (t(617) = -3.942, p < 0.001, d = 0.32).

Though the control group teachers were rated better in the pre-test than the experimental group teachers, they were judged worse in the post-test. The results show that the teacher training was successful: the teacher behavior changed and became apparent to the students.

**Students’ performance in reasoning and proof**

Both geometry tests on reasoning and proof consist of items of three competency levels: (1) basic knowledge and calculations, (2) one-step argumentation and (3) argumentation with several steps (see Reiss, Hellmich, & Reiss, 2002, for details). As described previously experimental and control group were parallelized, i.e. there is no significant difference between students’ pre-test results, neither for the total test score nor for the results of the different competency levels (see table 3).

<table>
<thead>
<tr>
<th>Mean (SD) Score (percentages)</th>
<th>Items competency level 1</th>
<th>Items competency level 2</th>
<th>Items competency level 3</th>
<th>Test score</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-test</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>79.3 (19.5)</td>
<td>62.8 (36.4)</td>
<td>26.1 (24.1)</td>
<td>57.4 (18.9)</td>
</tr>
<tr>
<td>experimental</td>
<td>79.0 (19.3)</td>
<td>66.1 (37.1)</td>
<td>26.0 (24.2)</td>
<td>58.1 (18.8)</td>
</tr>
<tr>
<td>control</td>
<td>79.4 (19.6)</td>
<td>60.7 (35.8)</td>
<td>26.1 (24.0)</td>
<td>57.0 (18.9)</td>
</tr>
<tr>
<td>post-test</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>74.0 (23.0)</td>
<td>56.7 (23.4)</td>
<td>26.5 (22.9)</td>
<td>52.0 (17.6)</td>
</tr>
<tr>
<td>experimental</td>
<td>74.3 (22.9)</td>
<td>59.0* (21.1)</td>
<td>29.6** (21.9)</td>
<td>53.9* (17.0)</td>
</tr>
<tr>
<td>control</td>
<td>73.9 (23.1)</td>
<td>55.2* (24.7)</td>
<td>24.5** (23.3)</td>
<td>50.9* (17.8)</td>
</tr>
</tbody>
</table>

* p < 0.05  ** p < 0.01

**Table 3: Students performance in the geometry pre- and post-test.**

For the post-test after the treatment we observe significant differences between the two groups (experimental group M = 53.9%, control group M = 50.9%, t(617) = 2.08, p < 0.05, d = 0.17). The better improvement of the experimental group was mainly based on geometrical proof items on competency level 3 (control group: M = 24.5%, experimental group M = 29.6%, t(617) = 2.69, p < 0.01, d = 0.22). This means that
students of the experimental group achieved their better results particularly for the complex multi-step proof items.

DISCUSSION

In our study we trained in-service teachers regarding the role of mistakes in the teaching and learning process. Our findings indicate that this teacher training was successful from two points of view: On the one hand, the teacher of the experimental group changed their mistake-handling behavior in such a manner that it was recognized by the students. The effect sizes indicated moderate effects. On the other hand, the performance of the students in the experimental group improved significantly better in comparison to that of the control group. This improvement is mainly based on a better performance in solving geometrical proof items, i.e. items on a high competency level.

Analyzing the data in detail we can observe, that there is an improvement of the affective and cognitive teacher behavior in the perspective from the students. As described before these two factors base partly on common items, however, if we consider only the specific affective or cognitive related items we can observe the same tendency. Hence, it seems that the in-service teacher training has a positive effect for the teacher behavior which is noticed by the students.

In spite of this change in the teacher behavior we cannot observe a clear improvement in the self reported students’ behavior concerning their own mistakes. There is no significant difference in the pre-post-test development between students of the experimental and the control group. It seems that the effect of the teacher training is restricted to a modification of the teacher reaction in mistake situations. There is no clear evidence for a successful further step: the instruction of students how to use mistakes as individual learning opportunities.

Investigating the development of the achievement in geometric reasoning and proof we see that students of the experimental group outperformed their mates from the control group. The advantage of the experimental group particularly goes back to the complex proof items in the pre- and post-tests; however, we can observe only a small effect size (d = 0.22). Though we did not control the mathematics instruction in the 29 classes, we hypothesize that this effect is particularly influenced by the improved mistake-handling activities of the teachers. Since the teachers in the experimental group were more open-minded about students’ mistakes in mathematics classroom, a better improvement of students’ achievement for complex mathematics tasks is in line with the theoretical assumptions. Moreover, teachers from the experimental and the control group got the same teacher training on reasoning and proof and they taught mathematics on the basis of a detailed prescribed curriculum, i.e. there was a clear frame for their teaching.

The results of our intervention study give evidence that an in-service teacher training on mistake-handling activities has positive effects on the mathematics classroom. Nevertheless, further research studies are necessary to optimize the outcome of such
training sessions. In particular, one has to think about methods how to guide students to use their individual mistakes for improving their learning in mathematics. The creation and evaluation of specific learning material for this purpose may be one possible way.

References


