CAN YOU CONVINCE ME: LEARNING TO USE MATHEMATICAL ARGUMENTATION

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In the current mathematics education reform efforts, teachers are challenged to develop discourse communities in which the students learn to construct and evaluate mathematical arguments collectively. In this paper I examined the interactional strategies used by a teacher to constitute a classroom context in which the students participated in the discourse of collective argumentation. I report on the way the teacher used student explanations as the foundations for building justification and validation of reasoning.

INTRODUCTION

Over the past twenty-five years within both an international and New Zealand context the teaching and learning of mathematics has undergone substantial changes in the way it is conceptualised. Increased focus has been placed on student communication of mathematical reasoning including the development and presentation of mathematical arguments. Advocated in the New Zealand policy document is a requirement that teachers "provide opportunities for students to develop the skills of presentation and critical appraisal of mathematical argument or calculation" (Ministry of Education, 1992, p. 23). Similarly, the American policy document emphasises the need for teachers to create classroom environments in which their students learn to "construct mathematical arguments and respond to others' arguments" (National Council of Teachers of Mathematics, 2000, p. 18). These documents promote ambitious and challenging goals for change in the teaching and learning practices of many mathematics classrooms. They challenge traditional beliefs toward mathematics and its discourse as a non-contentious (Weingrad, 1998). They also challenge the media prevalent view of argumentation as oppositional behaviour considered to be interference to learning (Andriessen, 2006). Moreover, many teachers have not themselves experienced learning in such environments and nor is their role in them clear (Huferd-Ackles, Fuson, & Sherin, 2004). The research reported in this paper examines how one teacher engaged in a collaborative research project, purposely transformed the discourse practices used by her and that of her students. The focus of the paper is on the interactional strategies. I examine how those strategies were used to shift the discourse toward all students participating in collaborative argumentation.

For the purposes of this study collaborative argumentation is a form of mathematical dialogue in which all parties work together to critically explore and resolve issues which they all expect to reach agreement on ultimately (Andriessen, 2006). Considerable evidence is now available of the beneficial effects of students

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articulating their mathematical reasoning and inquiring and challenging the reasoning of others when engaged in productive collaborative argumentation (e.g., Manouchehri, & St John, 2006; Mercer, 2000; Wood, Williams, & McNeal, 2006). These many studies have provided evidence that when opportunities are made available to students to participate in rich forms of inquiry and argumentation, the quality of their own mathematical explanations and justification are enhanced. This is because argumentation is a powerful reasoning tool which allows the participants in the dialogue to refute, criticise, elaborate and justify mathematical concepts and facts and develop an understanding of the opposing perspectives as all participants work towards constructing collective consensus. Andriessen (2006) maintains that "when students collaborate in argumentation in the classroom, they are arguing to learn (p. 443).

Creating a classroom culture in which value is placed on collaborative inquiry and argumentation requires altering the students' perceptions and beliefs about what mathematics is and how it is used (Manouchehri, & Enderson, 1999) but also their attitudes and perceptions of argumentation. Andriessen (2006) describes how many individuals link argumentation to an aggressive and oppositional form in which the goal is not to work together but rather to score points—a form of arguing which has little to contribute to mathematics education. However, engaging in collaborative interaction and using inquiry and argumentation is not something many students can accomplish easily without specific adult intervention (Rojas-Drummond, Perez, Velez, Gomez, & Mendoza, 2003). Therefore, it is vital that teachers as the more expert members of the classroom community take an active role in orchestrating a social environment in which the students "listen to one another, respect one another and themselves, accept opposing views, and participate in a genuine give-and-take of ideas and perspectives" (Manouchehri, & Enderson, p. 6).

When students engage in 'arguing to learn' they are participating in activity grounded in the social and cultural practices of the classroom community and are learning to use a social language or speech genre which denotes a particular socially situated identity (Gee, 1992). However, without direct discussion of the structure of collaborative argumentation and its rules and norms some students may not be able to access the mathematical discourse and learning of the classroom. This paper explores the defining features of a classroom climate in which the teacher developed and extended student participation in mathematical argumentation.

The theoretical framework of this study is derived from a sociocultural perspective. From this perspective mathematical teaching and learning is inherently social and embedded in active participation in communicative reasoning processes (Lerman, 2001). In this environment, students successively gain increased levels of "legitimate peripheral participation" (Lave & Wenger, 1991, p. 53) as they access and use the discourse of inquiry and argumentation.

RESEARCH DESIGN

This research reports on one teacher case study from a study which involved four teachers in a one-year collaborative teaching experiment. The study was conducted at a New Zealand urban primary school where students came from predominantly low socio-economic home environments. Students were predominantly of Pacific Nations and New Zealand Maori ethnic groupings with many of whom spoke English as their second language.

Collaborative teaching experiment design (Cobb, 2000) was used in order to direct teacher and researcher attention on the social and analytical structuring (Williams & Baxter, 1996) of the mathematical discourse. In recognition of the two central characteristics of teaching experiment design research; the iterative cycles of analysis, and an improved process or product; a tentative communication and participation trajectory was used to map the progression of the discourse toward argumentation and to provide focus for the subsequent shifts in participation and communication. For example, after Ava (pseudonym for the teacher) had completed teaching a an early algebraic unit of work and before she returned to teaching a fractional number unit, the types of questions Ava and the students could use and the interactions anticipated to scaffold a further shift toward collaborative mathematical argumentation were considered and mapped out.

Data collection over one year included three semi-formal teacher interviews, classroom artefacts, field notes, twice weekly video captured observations of lessons, diary notes of informal discussions during and after lesson observations, written and recorded teacher reflective statements and teacher recorded reflective analysis of video excerpts. The on-going data collection and analysis maintained a focus on the developing mathematical discourse and argumentation. This supported the iterative cycles and revision of the communication and participation strategies. Data analysis occurred chronologically using a grounded approach in which codes, categories, patterns and themes were created. Through use of a constant comparative method which involved interplay between the data and theory, trustworthiness was verified and refuted.

RESULTS AND DISCUSSION

In the early stages of the study the participation and communication structure that Ava made available to students operated as a scaffold to begin to develop argumentation. As the study progressed the close relationship between a shift in the roles Ava and her students took and the changes enacted in the participation and communication structure is evident.

Creating a context for collaborative argumentation

Ava immediately worked with the students to establish a set of mutual expectations for behaviour as participants in a discourse community. She directly addressed the new ‘rules’ for talk, discussing with the students how they were required to work together to build a mathematical community. She emphasised that working together involved an increase in collaborative participation in mathematical dialogue, both as listeners and as talkers. She repositioned herself from the central position of 'mathematical authority' to that of 'participant in the dialogue'. She modelled the shift explicitly placing emphasis on words which placed her as a participant also.

> Ava: Can you show us with your red pen what would happen? We want to know.

In the first instance, Ava aimed to develop the students' skills to work collectively to build mathematical explanations. She stressed that all group members needed to engage in construction of mathematical explanations and be able to explain them to a wider audience. She outlined not only how these explanations needed to make sense for a listening audience but also how listeners needed to make sense of the explanations offered by others. To develop their skill in the examination and analysis of explanations she provided opportunities for small groups to construct, explain, and in turn question and clarify explanations step-by-step through her directives:

> Ava: They might say I think it is 59. That's cool but they have to back it up, explain how they came up with it. They have to say why. I want you before you even begin to go around in your group and actually talk about it. Someone in your group may ask you a question. For example, that's an interesting solution, why do you think that? Could you show us how you got it?

In this early stage, although mathematical argumentation was not a strong feature of how the students interacted Ava initiated discussion about the need for agreement and disagreement in the construction of reasoned explanations. For example, when a student stated that working as a group required agreement Ava responded:

> Ava: Yes you could be agreeing with what the person says…but are you always agreeing, do you think?

In accord with the trajectory, she carefully structured ways in which the students could approach disagreement and challenge. When the students worked together she pressed them toward considering the use of arguing productively:

> Ava: Arguing is not a bad word…sometimes I know that you people think to argue is…I am talking about arguing in a good way. Please feel free to say if you do not agree with what someone else has said. You can say that as long as you say it in an okay sort of way. If you don't agree then a suggestion could be that you might say I don't actually agree with you. Could you show that to me? Could you perhaps write it in numbers? Could you draw something to show that idea to me? That's fine because sometimes when you go over and you do that again you think…oh maybe that wasn't quite right and that's fine. That's okay.

Questioning, clarifying and beginning to challenge

The careful attention Ava gave to socially scaffolding the discourse led to the growth in student confidence to question and clarify sections of explanations when required. For example after a group had modelled an explanation using equipment and described their actions of repeatedly adding three sticks as 'squares times three' they are challenged:

Jo: Isn't that just plusing three sticks not timesing it? You are not timesing you're adding.

Pania: Well what she sort of means it is like it is going up.

Alan: Is that timesing going up?

Ava: When we talk about timesing what do we actually mean?

Jo: We mean multiplying not adding. Adding is plus [indicates a + with fingers] that sign.

Sandra: You mean when you add two more squares on, that is multiplying?

Ava intercedes and uses the reasoning under discussion to extend their thinking. Through use of the interactional strategy of revoicing (O'Connor & Michaels, 1993) Ava deepens their understandings of multiplication and enriches their language.

Ava: Rachel was saying she is adding three, adding another three, so that's three plus three plus three. So if you keep adding three all the time what is another way of doing it?

Alan: You can just times instead of adding. It won't take as long and it is more efficient.

Ava: Yes you are right. Did you all hear that? Alan said that you can just times it, multiply by three because that is the same as adding on three each time. What word do we use instead of timesing?

Alan: Multiplication, multiplying.

Pressing for multiple ways to justify and validate explanations

A need for active sense-making and a press to provide conceptual explanations provided the students with the foundations with which to build collaborative argumentation. Ava pressed the students toward constructing multiple explanatory means to justify and validate their reasoning. Problems were also used which required that the students develop multiple ways to convince others. Before they began constructing their explanations in small groups, Ava placed direct emphasis on a need for them to ask specific questions. With the students she listed the questions they could use to elicit more information about mathematical explanations. Then she introduced a second set of questions which related to their need to be convinced through mathematical argumentation. She recorded an initial set of questions and then regularly recorded additional questions which arose during the classroom dialogue—questions which asked why and led to justification and validation of reasoning. She also assisted them by asking that they prepare responses in their small groups to the types of questions they might be asked in the large group situation:

Ava: Think about the questions that you might be asked. Practise using some of those questions like why does that work or how can you know that is true. Try to see what happens when you say if I do that… then that will happen.

In this climate of intellectual autonomy Ava and her students began to regularly ask the question, 'can you convince us?' This press toward need for convincing through mathematical argumentation was accepted and modelled by the students. They recognised that this supported possibilities for confirmation or reconstruction of their reasoning. They would closely examine and rehearse each step in an explanation and if required for clarity or ease of sense-making rework, reformulate and re-present sections. Ava also instituted a further shift in how the students participated in communicating their mathematical reasoning introducing the concept of 'no hands up', particularly when there were many questions and challenges for an explanation. Her direct prompting for student interjection led to an interactive flow of conversation in which collaborative forms of argumentation were used to closely examine, analyse and validate the mathematical reasoning. When needed she intervened and facilitated slower exchanges, if she considered the mathematical concepts under consideration particularly challenging. For example, Ava participated in the following collective construction of an explanation, related to a problem which required naming a point which represented 5/100 on a numberline. She prompted for interjection but also intervened to maintain a focus on inquiry and challenge and ensure that the students reflectively considered and reconsidered their reasoning.

> Tipani [Draws a numberline, marks 0 then 9 and marks 1/10: Here is 0 and 1/10.
>
> Pania [Interjects]: Why are you doing those lines?
>
> Tipani [Records 5/100 in the middle]: Because each of those lines is representing one tenth, I mean ten tenths. I am thinking that this one is meant to be 5/100.
>
> Mahaki [Interjects]: Why?
>
> Tipani: Basically because of what you said Mahaki.
>
> Ava: Which was? Explain it in your own words and see if Mahaki agrees.
>
> Tipani: That if you times… ten by ten…well I am not actually that sure. I just think that it is five one hundredths. I don't think that it is five one thousandths.

Ava intervenes and attributes ownership back to the explainer but presses for further clarification.

> Ava: Well what do you think Mahaki, and you other people who heard what Mahaki explained? Let's take a look at these fractions and think about what Tipani and Mahaki were saying. What do you see when you look at these fractions…what other ways can they be represented apart from that?

Ava referred the argument back to the explainer. But within the context of collaborative argumentation another member appropriates and revoices what has been explained.

> Chanal [Looks at Mahaki who nods at him to speak, points at the 5/100 mark]: Mahaki said that one tenth can be ten percent because if you times one by ten you get ten and you times the ten by ten you get one hundred. So that will be one tenth is like ten percent. So in the middle that will be five percent there.

Ava revoices to ensure that all participants are able to access what is being argued. She then probes further, progressing the reasoning toward collective construction of rich conceptual knowledge using multiple levels of representations.

> Ava [Points at 5/10]: You accept that Mahaki? So what you are saying is that that means five parts out of a hundred and the one tenth there means ten out of a hundred. So what is this one?

Chanal: That is fifty percent.

Pania: So how?

Recognising that further arguments are required Ava facilitates an alternative view.

Ava: Yes you jump in here Chanal, if you can explain it in a different way.

Chanal [Points at the numberline showing one tenth divided into ten segments and points at the first segment on the numberline]: I know what. If you go back to there and just pretend you shrink that down to there. There's a hundred right? So that half way mark in brackets would be right there [Points at the position it would be in if you had a whole number line not just to one tenth and it represented 1/10] and that would be ten percent and if you halved that ten percent it would be five.

Pania: Five what?

Chanal [Records 5% and 5/100]: Five percent or five hundredth.

Ava: Are you all convinced? Or do you want to ask some more questions?

Mahaki: It is five hundredth because as Chanal said that thing there would be just like a little piece of this line…But the other way is to go the percent way. You get ten percent and then half that. That's the quickest way to explain it.

Ava and the students had maintained an extended flow of productive argumentation in which rich communal understandings of the equivalent relationship of rational numbers were constructed. Ava had positioned and repositioned the students to make visible their reasoning so that claims were collectively validated.

CONCLUSIONS

The teaching experiment was designed to successively press how the students participated in communicating their mathematical reasoning. The direct focus placed on collective construction and sense-making of explanatory reasoning acted as a scaffold to shift the discourse toward justification. Through these actions the social norms of sense-making were established in the community.

The introduction of notions of 'arguing' and disagreement were important to lay the foundations for further shifts toward argumentation. The adoption by Ava and her students of a metaphorical view of the need for 'convincing' provided motivation for the students to engage in the communal activity. Direct teacher actions built on the notion of convincing and supported the constitution of an environment in which the students participated in collective argumentation. The use of specific questions to frame inquiry and challenge, and the increased student autonomy in when and how to participate were important factors.

The findings of this study support Andriessen (2006) contention that young children can participate in collective argumentation when carefully scaffolded. Moreover, Wood and her colleagues (2006) identify differences in the cognitive demand and student participation in collective reasoning in classrooms where the use of discourse extends to justification and argumentation and this was evident in this study.

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