CONSTRUCTING PEDAGOGICAL REPRESENTATIONS TO TEACH LINEAR RELATIONS IN CHINESE AND U.S. CLASSROOMS

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This study investigates Chinese and U.S. teachers’ constructions of pedagogical representations by analyzing the video-taped lessons from the Learner’s Perspective Study, involving 10 Chinese and 10 U.S. consecutive lessons on the topic of linear equations or linear relations. This study allows not only for the examination of what pedagogical representations Chinese and U.S. teachers construct, but also for the examination of the changes and progressions of constructed representations in these Chinese and U.S. lessons. This study is significant because it contributes to our understanding of the cultural differences involving U.S. and Chinese students' mathematical thinking and has practical implications for constructing pedagogical representations to maximize students’ learning.

THEORETICAL BASIS OF THE STUDY

Cross-national studies provide unique opportunities for understanding how classroom teaching affects students’ mathematical thinking, and then such studies provide diagnostic and decision-making information about how we can improve students' learning (Bradburn & Gilford, 1990; Cai, 2001; Ma, 1999; Stigler & Hiebert, 1999). Previous studies have revealed remarkable differences between U.S. and Chinese students' mathematical thinking and reasoning (Cai, 2000). Yet, we are just beginning to uncover how teaching in the two cultures may contribute to the cross-national performance differences. Because the use of mathematics representations is an important instructional feature that exerts great influence on students’ mathematical thinking and reasoning, studies comparing the use of mathematics representations in U.S. and Chinese classrooms can provide us with insights into how teaching in different cultures may affect students’ learning and mathematical thinking.

Pedagogical representations refer to the representations that teachers and students use in their classroom as expressions of mathematical knowledge. They help explain concepts, relationships, connections, or problem-solving processes. In mathematics instruction, some representations might be more effective than others as expressions of knowledge and thinking tools to explain problem-solving processes (Cobb et al., 1992; Leinhardt, 2001). Choosing pedagogically sound representations is an important decision to make when a teacher selects instructional strategies for the mathematics classroom. To select a desirable pedagogical representation, a teacher should integrate at least two perspectives for consideration: the nature of the mathematical content being taught and the minds of students learning the content (Ball, 1993). First, the representation should highlight the features of the mathematics content the teacher
wants to teach. Second, the representation should provide students with a familiar and accessible context in which they can extend and develop their capacity to reason and understand the idea. In mathematics classroom practice, Perkins and Unger (1994) also found that a powerful and effective representation often bears these two features. On the one hand, they argued, its extraneous clutter is often “stripped” in order to highlight the critical mathematical characteristics. On the other hand, it is also “concrete” to learners. Although it is not clear yet if there is a universally “good pedagogical representation” in terms of its strippedness and concreteness in teaching a mathematics idea to students in different cultural contexts, it is generally agreed that the teachers’ selection of desirable pedagogical representations of specific mathematics knowledge reflects the teachers’ conceptions, knowledge of mathematics, and their beliefs about learning and teaching (NCTM, 2001). Put another way, the pedagogical representations that teachers develop is related not only to the theory and research about student understanding, but also to teachers’ beliefs about the functions of particular representations in students’ learning and understanding (Greeno, 1987). Pedagogical representations are effective in classroom instruction only if they are either known by students or easily knowable (Leinhardt, 2001).

Recently, an attempt has been made to compare Chinese and U.S. teachers’ conceptions and constructions of pedagogical representations in mathematics instruction (Cai, 2005; Cai & Wang, 2006). For example, Cai (2005) examined U.S. and Chinese teachers’ construction, knowledge, and evaluation of representations to teach the concept of arithmetic average and found that the Chinese teachers and U.S. teachers in the study used representations differently. For example, while the Chinese teachers used concrete representations exclusively to mediate students’ understanding of the concept of average, the U.S. teachers tended to use concrete representations not only to foster students’ understanding of the concept but also to generate data. Cai and Wang (2006) further examined U.S. and Chinese teachers’ construction, knowledge, and evaluation of representations to teach the concept of ratio and found the generalities of U.S. and Chinese teachers’ construction of pedagogical representation across the content areas.

In this paper, we examined how U.S and Chinese teachers construct representations to teach linear relations over a sequence of video-taped lessons. This allows not only for the examination of what pedagogical representations Chinese and U.S. teachers construct, but also for the examination of the changes and progressions of constructed representations in these Chinese and U.S. lessons.

**METHODOLOGY**

**Data resources**

The data for this study came from the Learner’s Perspective Study (LPS for short), which examines the patterns of participation in competently taught seventh or eighth grade mathematics classrooms in thirteen countries in a more integrated and comprehensive fashion than has been attempted in previous international studies.
(Clarke et al., 2006). In this study, we selected one Chinese school data set and one U.S. school data set. The main topics taught over 10 consecutive lessons in the Chinese and U.S. classroom are shown in Table 1.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Chinese lessons</th>
<th>U.S. Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Concept of linear equations with two unknowns and its solution (CH1)</td>
<td>Concepts of linear and non-linear relations (US1, US2)</td>
</tr>
<tr>
<td></td>
<td>Concepts of coordinate plane and the coordinates; graph of linear equations with two unknowns (CH2-CH4)</td>
<td>General features of linear relations and multiple representations for linear and non-linear relations (US3, US4)</td>
</tr>
<tr>
<td>2</td>
<td>Concepts of system of linear equations with two unknowns and their solutions (CH5)</td>
<td>Comparison and contrast of different representations (US5-US8)</td>
</tr>
<tr>
<td>3</td>
<td>Methods of solving linear equation with two unknowns such as the elimination method and the graph method(CH6-CH10)</td>
<td>Comparison and contrast of different representations; application of linear and non-linear relations (US9, US10)</td>
</tr>
</tbody>
</table>

Table 1. Knowledge construction in Chinese and U.S. lessons

The Chinese school was from Shanghai, which is a key school at the district level, and the U.S. school was in top 20 percentile of schools in the state of California. The Chinese teacher, Mr. Tang, has a bachelor degree in mathematics from Teacher’ Education Institute and has 24 years of teaching experience. There were 55 seventh-grade students in the classroom; the textbook used was the only unified official textbook in Shanghai. The duration of each lesson was approximately 45 minutes. The U.S. teacher, Ms. Nancy has a bachelor degree in mathematics with some teacher education training. She has more than 15 years of teaching experience. There were 37 grade eight students in the classroom. The textbook used was the Integrated Mathematics (Algebra) published by McDougal Little Inc. The duration of each lesson was approximately 50-minutes.

**Data analysis**

The data was analysed in two dimensions. First, we analyzed how knowledge was constructed during ten Chinese and ten U.S. consecutive video-taped lessons. Then we focused on the first four Chinese lessons (CH1- CH4) and the middle six U.S. lessons (US3 - US8), which included extensive coverage of linear relations, for further examination of the instructional tasks and the pedagogical representations involved in these Chinese and U.S. lessons. The instructional tasks, or mathematical tasks, can be defined broadly as projects, questions, problems, constructions, applications, and exercises in which students engage. The instructional tasks provide an intellectual environment for students’ learning and development of mathematical thinking. Pedagogical representations of mathematics concepts were put into four categories: symbolic representation, numeric representation, tabular representation, graphic representation and verbal/literal representation. With respect to the code of
pedagogical representations, one researcher developed a coding system by using special video data analysis software, Studio-code through carefully watching the video-taped lessons. Then the first author did a careful check. If the first author did not agree the code in certain episodes, then, a discussion with the research assistant was conducted until an agreement was achieved.

RESULT

Knowledge construction

Table 1 shows the sequence of how the topic was presented in Chinese and U.S. lessons. The Chinese teacher started with an introduction of the concept of linear equations and its solutions. He introduced the concept of rectangle coordinate planes to graph linear equations and then explained the concept of system of linear equations with two unknowns and its solution. After that, several methods to solve a system of linear equations with two unknowns were introduced and consolidated. It should be indicated that the Chinese teacher emphasizes the procedures for solving linear equations more than the concept involved. The U.S teacher started by introducing the concept of linear and non-linear relations in general, and then the teacher discussed extensively the features of linear relations and focused on transformation of multiple representations of linear and non-linear relations through group activities. Finally she applied the knowledge to solve word problems. The U.S. teacher intended to develop the concepts (linear and non-linear relations) and foster understanding of the features of linear and non-linear relations through multiple representations and students’ group work. However, it is clear that the various activities that she used were to help students recognize the different representations, instead of using the representations to actually foster understanding of linear equations.

Comparing the Chinese and U.S. lessons, there are a number of differences in terms of lesson structures. Chinese lessons were dominated with whole class instruction, while group activity dominated the U.S. lessons. In the U.S. classroom, the students were divided into several groups, and the lessons were delivered through group activities. In the Chinese classroom, the lessons were delivered through whole classroom teaching, although there was frequent peer discussion. Each U.S. lesson included “warm-ups,” which were related to the new topic to be learned in the lesson, but not related to the topics in the previous lessons. In the Chinese lessons, all lessons started with a review of knowledge learned in the previous lessons. This suggests that there were better connections between the Chinese lessons than between the U.S. lessons. In the U.S. lessons, the teacher usually did not present a summary for each lesson, while the Chinese teacher regularly summarized the key points of each lesson.

In addition, the Chinese teacher emphasized on the procedures for solving linear equations. That was not the case in the U.S. lessons. The U.S. teacher put heavy emphasis on multiple representations, and transforming among different representations is the goal in several of her lessons. That was not the case in the Chinese lessons. We will examine the representations further in the next section.
Representation construction

In this section, we examine representation constructions by comparing instructional tasks in six U.S. lessons (from US3 to US8), totalling around 300 minutes, with four Chinese lessons (from CH1 to CH4), totalling around 180 minutes. These lessons were chosen because of their extensive coverage of linear relations. The six U.S. lessons include 10 instructional tasks, and the four Chinese lessons include 23 instructional tasks. To examine the kinds of representations used, we looked at the total duration for solving each task as a whole. The proportion of different representations for each task in the U.S. classroom is depicted in Figure 1.

![Figure 1. Distribution of representations in U.S. lessons](image)

The above figure shows that there is one task L3T1 for which five presentations (verbal, tabular, numerical, symbolic and graphic) were used. There are four tasks (L3T2, L6T3, L7T1 and L8T3) for which four representations were used. There are three tasks for which two representations were used, while there are two tasks for which one representation was used. The order of popularity of using representations is as follows: verbal (100%), symbolic (80%), graphic (50%), tabular (50%), and numerical (10%). Similarly, we can show the proportion of different representations for each task in the Chinese classroom in Figure 2.

![Figure 2. Distribution of representations in Chinese classroom](image)
Obviously, the Chinese figures have a relatively “simpler” appearance. In one task (L4T2), there are four representations. There are three tasks (L1T1, L3T6 and L4T6) for which three representations were used. There are fourteen tasks for which two representations were used, and there are five tasks for which only one representation was used. The order of popularity of using representations is verbal (70%), numerical (56%), symbolic (30%), graphic (26%), and tabular (4%).

When comparing the construction of representation in the U.S and Chinese classrooms, it was found that the U.S. teacher preferred using multiple representations simultaneously (in 50% of the cases, more than three presentations were used), while the Chinese teacher preferred using one or two representations (in 83% of the cases, only one or two representations were used). In addition, verbal and numerical presentations were most commonly used and tabular representations were least commonly used in the Chinese classroom; however, the verbal, symbolic, graphic, and tabular representations were most commonly used and the numerical representations were the least used in the U.S. classroom.

Ways of constructing representations

Overall, we can present the development of representations of linear equations in the Chinese and U.S classrooms in the following diagram (Figure 3).

![Diagram of representation development](image)

Figure 3. Representation development of linear relation in Chinese and U.S. lessons

The U.S teacher developed multiple representations simultaneously over sequential lessons through different activities, such as visual sorting, manipulative drawing and matching, and comparative and contrasting exploration. The teacher tried to deepen the students’ understanding and integration of different representations through progressive activities. This diagram shows that the Chinese teacher tried to develop the concept of linear equations (symbolic representations) and graphs of linear equations (graphic representations) through solving problems (verbal daily problem and
symbolic linear equations) by making use of tabular and numerical representations. Thus, students may have a deep understanding of linear equations and its figures and also understand the ways of drawing a figure of linear equation by plotting two points for the linear equation. However, they may not realize that numerical and tabular representations are of the same importance as other representations.

CONCLUSION
With regard to the construction of representations when implementing instruction tasks, it was found that the U.S. teacher preferred using multiple representations simultaneously, while the Chinese teacher preferred using only one or two representations. In addition, verbal, symbolic, graphic and tabular presentations were most popularly used by the U.S. teacher while for the Chinese teacher verbal, numeric, symbolic and graphic representations were more popular. Numerical representation was least frequently used by the U.S. teacher, while tabular representation was least frequently used by the Chinese teacher. In addition, the U.S teacher seemed to develop multiple representations simultaneously over subsequent lessons through different activities, such as visual sorting, manipulative drawing and matching, while the Chinese teacher tried to develop the concept of linear equations (symbolic representation) and graphs of linear equation (graphic representation) through solving problems (daily verbal problems and symbolic linear equations) by making use of tabular and numerical representations. Thus, the use of multiple representations appears to be an instructional goal for the U.S. teachers, while she is intended to use multiple representations as a means for students’ understanding of linear relations. For the Chinese teacher, the multiple representations were used as a means to understand linear equations.

The finding that the U.S teacher tried to treat all four representations equally and develop them simultaneously through different activities may explain why the U.S. students preferred to choose concrete strategies, drawing representations both for fostering understanding of concept and also for applying knowledge. However, the Chinese teacher paid more attention to developing symbolic and graphic representations by treating numerical and tabular representations as tools for developing other representations. This finding may explain why Chinese students preferred using symbolic and abstract representation to solve problems (Cai, 2005; Cai & Lester, 2005).

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References


