THE DISCURSIVE CONSTRUCTION OF MATHEMATICAL THINKING: THE ROLE OF RESEARCHERS’ DESCRIPTIONS

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A variety of perspectives on the nature and role of discourse in the teaching and learning of mathematics have been developed and applied in recent years. The conduct of research in mathematics education can also, however, be viewed from a discursive perspective. In this paper, I draw on discursive psychology, which has been described as an anti-cognitivist, anti-realist, anti-structuralist approach to discourse analysis and psychology. Based on this perspective, I examine discursive features of a research paper on mathematical thinking to argue that, within the mathematics education research community, researchers’ descriptions of students’ behaviour and interaction make possible subsequent accounts of mathematical thinking, rather than the other way around.

DISCURSIVE PERSPECTIVES ON THE TEACHING AND LEARNING OF MATHEMATICS

A variety of perspectives on the nature and role of discourse in social life have influenced research within mathematics education in recent years. Some research has drawn on sociological theories of interaction, including interactional sociolinguistics and symbolic interactionism to explore the social organisation of mathematics classroom discourse, highlighting, for example, the conventions and norms that arise (Yackel and Cobb, 1996). A related body of work has drawn on sociocultural theory to argue that, in mathematics, talk is ‘almost tantamount to thinking’ (Sfard, 2001, p. 13; Lerman, 2001). Such studies have, for example, attempted to trace the processes of socialisation through which students learn to use mathematical discourse and to do mathematics (e.g. Zack and Graves, 2001). Others have been more interested in the specific nature of interaction in mathematics classrooms. This work includes studies that draw on social-semiotic perspectives to explore, for example, the nature and role of mathematical texts and of intertextuality in mathematics education (e.g. Chapman, 2003). Similarly, others have emphasised the situatedness of mathematical meaning within classroom discourse (Mosckovich, 2003). Some researchers have turned to post-structuralism to examine the processes through which mathematics, teachers and students are positioned or constructed by mathematical discourses (e.g. Brown, 2001). More recently, discursive perspectives have led to new insights into the relationship between mathematics classroom interaction and wider political concerns, such as, for example, the role of different languages in multilingual settings (Setati, 2003).

In general, the various approaches summarised above have sought to understand different aspects of mathematical thinking (which, for this paper, I will use to also
encompass mathematical learning, meaning and understanding). These approaches generally highlight a central role for language, symbols and interaction in mathematical thinking. Conducting such research is also, however, a discursive process, involving, for example, the production and interpretation of various kinds of texts, such as tape-recordings of interviews, lesson transcripts or field notes. The discursive nature of this process has received less attention within the field of mathematics education. In this paper, I explore one particular aspect of the research process: the published research paper. To do so, I will draw on ideas developed in discursive psychology, which are summarised in the next section. These ideas highlight, amongst other things, the role of description in constructing both mind and reality. Taking one research paper (Sfard, 2001) as an example, I explore the role of description in the construction of mathematical thinking in published accounts of research. I conclude by discussing some possible implications for research in mathematics education.

DISCOURSE PSYCHOLOGY

Discursive psychology (e.g. Edwards, 1997; Edwards and Potter, 1992) has been described as offering an anti-cognitivist, anti-realist, anti-structuralist account of the relationship between discourse and cognitive process, such as thinking, meaning or remembering [1]. In the context of research in mathematics education, these points have the following implications (for which I have drawn particularly on Edwards, 1997):

- **Anti-cognitivist**: entails a shift from a focus on ‘what happens in the mind’ (as an individual mental process) to how ‘what happens in the mind’ is done through discursive practice (as a socially organised process); thus, the nature of mathematical thinking or meaning, for example, are jointly produced through interaction.

- **Anti-realist**: reality is seen as being reflexively (and so relativistically) constituted through interaction. Thus, in any given situation, mathematics or mathematical cognitive processes are not pre-given, but are brought about through talk. Rather than mathematical meaning, for example, being pre-determined by words, symbols or diagrams, participants read such meanings into these things through their interaction.

- **Anti-structuralist**: following the preceding point, mathematical meaning and the organisation of mathematical interaction are situated, both in time and in place, emerging from preceding interaction, rather than in standard, predictable ways.

As Edwards (1997, p. 48) points out, this perspective is to some degree related to socio-cultural approaches to psychology. Both approaches recognise the central role played by social processes, culture and language in the development of the human mind. For much research influenced by sociocultural theory, however, the aim is to understand how the mind works, even if mind is constructed through participation in
society. Discursive psychology, by contrast, is more interested in how ideas like ‘mind’ are constructed in particular situations. The difference, Edwards argues, is broadly between ontological and epistemological concerns:

In discursive psychology, the major sense of ‘social construction’ is epistemic: it is about the constructive nature of descriptions, rather than of the entities that (according to descriptions) exist beyond them. (Edwards, 1997, pp. 47-48)

In this approach, therefore:

Mind and reality are treated analytically as discourse’s topics and businesses, the stuff that talk is about, and the analytic task is to examine how participants descriptively construct them. (Edwards, 1997, p. 48)

In the case of research in mathematics education, concerns with mathematical thinking, for example, or the nature of mathematics, would be treated as discursive constructs. I have sought to use these ideas to analyse various examples of mathematics classroom interaction (e.g. Barwell, 2001). My aim was to understand how school students and teachers jointly constructed mathematical understanding, thinking and learning. In this paper, however, I am interested in how mathematical thinking is constructed through the research process itself, particularly in research publications. To facilitate this inquiry, in the next section I examine one published paper by Anna Sfard.

DESCRIPTION AND MATHEMATICAL THINKING: AN EXAMPLE

The paper I have selected (Sfard, 2001) concerns the relationship between discourse and mathematical thinking. The paper is interesting, in that it compares two ways of viewing mathematical thinking: the cognitivist, learning-as-acquisition approach and a communicative, learning-as-participation approach. Sfard sees these two approaches as complementary (p. 49). One strand within the paper involves the presentation and discussion of an exchange between a pre-service teacher and a 7-year old girl, reproduced below (see Sfard, 2001, p. 19). In what follows, I examine this strand of the paper from the perspective of discursive psychology.

Teacher: What is the biggest number you can think of?
Noa: Million.
Teacher: What happens when we add one to million?
Noa: Million and one.
Teacher: Is it bigger than million?
Noa: Yes.
Teacher: So what is the biggest number?
Noa: Two millions.
Teacher: And if we add one to two millions?
Noa: It’s more than two millions.
Teacher: So can one arrive at the biggest number?
Noa: Yes.
Teacher: Let’s assume that googol is the biggest number. Can we add one to googol?
Noa: Yes. There are numbers bigger than googol.
Teacher: So what is the biggest number?
Noa: There is no such number!
Teacher: Why there is no biggest number?
Noa: Because there is always a number which is bigger than that?

In discussing this exchange, initially from a cognitivist perspective, Sfard writes:

Clearly, for Noa, this very brief conversation becomes an opportunity for learning. The girl begins the dialogue convinced that there is a number that can be called ‘the biggest’ and she ends by emphatically stating the opposite: ‘There is no such number!’ . The question is whether this learning may be regarded as learning-with-understanding, and whether it is therefore the desirable kind of learning. (Sfard, 2001, p. 19)

This paragraph is a description of what happened in the conversation. The description is plausible. Nevertheless, the description constructs various aspects of mathematical thinking on the part of Noa. In particular, Noa is constructed as being ‘convinced’ that there is a biggest number at the start of the conversation. Sfard describes Noa’s penultimate contribution that there is ‘no such number’ as ‘emphatic’, also implying a degree of conviction. The use of ‘emphatic’ is linked to the use of an exclamation mark (!) in the transcript, adding to the reasonableness of the description. Noa is also described as having produced opposing statements. These statements are implicitly interpreted as a chronological shift, which is, in turn, called ‘learning’. ‘Being convinced’ and ‘learning’ are aspects of mathematical thinking that are, however, read into the conversation through the description. By juxtaposing two of Noa’s statements and describing them as opposites, the description makes possible, for example, the interpretation that learning has taken place.

Later in the same article, Sfard offers, by way of contrast, a more discursive perspective on the same extract:

…much of what is happening between Noa and Rada may be explained by the fact that unlike the teacher, the girl uses the number-related words in an unobjectified way. The term ‘number’ functions in Noa’s discourse as an equivalent of the term ‘number-word’, and such words as hundred or million are things in themselves rather than mere pointers to some intangible entities. If so, Noa’s initial claim that there is a biggest number is perfectly rational. Or, conversely, the claim that there is no biggest number is inconsistent with her unobjectified use of the word ‘number’: After all, there are only so many number-words, and one of them must therefore be the biggest, that is, must be the last one in the well ordered sequence of numbers…Moreover, since within this type of use the expression ‘million and one’ cannot count as a number (but rather as a concatenation of numbers), the possibility of adding one to any number does not necessitate the non-existence of the biggest number. (Sfard, 2001, p. 46)
Again, Sfard offers a plausible account, with the aim, in this case, of resolving the puzzle raised earlier, of how Noa comes to be ‘convinced’ of opposing ideas in the space of a short conversation. Again, however, Sfard’s description constructs various forms of mathematical thinking on the part of Noa. A key feature of the description is the idea that Noa is interpreting the word ‘number’ as ‘an equivalent of the term ‘number word’’. Based on this description, Sfard is able to provide a rational account of Noa’s utterances. Indeed, the later part of the above paragraph is devoted to setting out the linguistic and mathematical basis for that rationality, which amounts to a reading of Noa’s mathematical understanding in the earlier stages of the exchange. Thus, the nature of the description is intimately related with the argument that Sfard is pursuing. By setting out a particular version of what is happening in the conversation, Sfard makes available particular inferences about Noa’s (cognitive) interpretations, which in turn fit in with Sfard’s larger argument that the conversation represents an example of discursive conflict:

…both interlocutors seem interested in aligning their positions. The teacher keeps repeating her question about the existence of ‘the biggest number’, thus issuing meta-level cue signalling that the girl’s response failed to meet expectations. In order to go on, Noa tries to adjust her answers to these expectations, and she does it in spite of the fact that what she is supposed to say evidently does not fit with her use of the words the biggest number. (Sfard, 2001, p. 46)

The notion of discursive conflict stresses the clash of habitual uses of words, which is an inherently discursive phenomenon. In our present case [of Noa], we could observe a conflict between the two interlocutors’ discursive uses of the words ‘number’ and ‘bigger number’. While aware of the fact that the teacher was applying these terms in a way quite different from her own, Noa was ignorant of the reasons for this incompatibility. In this case, therefore, the girl had to presume the superiority of her teacher’s use in order to have any motivation at all to start thinking of rational justification for a change in her own discursive habits. (Sfard, 2001, p. 48)

The notion of discursive conflict originates, perhaps, in the first paragraph above, an account of the conversation in largely discursive terms; that is, in terms of cues, repeats and alignments. Even in this description, however, a degree of intention is read into Noa’s behaviour: she ‘tries to adjust her answers’, for example. This reading is then overlaid, however, with a more cognitively oriented account of ignorance, presumptions, motivations and thinking. Again, then, it is the nature of the description that makes possible the inferences about Noa’s mathematical thinking.

DISCUSSION

My purpose in examining Sfard’s paper in such detail is not to challenge her argument. Her exploration of the idea of cognitive conflict and her proposal of the alternative idea of discursive conflict are interesting developments and likely to be valuable for research and teaching. Rather, I am interested in how mathematical thinking is discursively constructed through the process of doing and communicating research in mathematics education.
My discussion of Sfard’s paper particularly highlights the importance of descriptions of mathematical behaviour in constructing mathematical cognition. As Edwards (1997, pp. 37-43) has argued in the case of cognitive psychology, such descriptions play an important role: they make available particular interpretations of cognitive processes, whilst shutting out alternatives. Sfard’s descriptions, for example, build in cognitive or discursive conflict, which can then be made explicit. More generally then, in written research reports, descriptions of mathematical behaviour are likely to be shaped to suit an author’s wider argument concerning mathematical thinking and learning.

Of course many authors acknowledge their subjectivity, in the sense that they make it clear that their analyses are interpretations. Indeed Sfard, in her article, compares two such interpretations. Commonly, such acknowledgements are, however, based on the argument that, by using an explicit theoretical framework and giving sufficient detail about how the data were analysed, readers can make up their own minds about the trustworthiness of the analysis. This kind of argument is based on a desire to get at an objective cognitive process that can be interpreted, however indirectly and tentatively, through participants’ behaviour. Students say things to their mathematics teachers and the task is to suggest what or how the student is thinking, learning or understanding. I am not making the reasonable point that language does not offer a ‘window’ into the mind. Research from a broadly socio-cultural perspective (including that of Sfard) recognises the complex role of language and culture in mathematical cognition and in the research process and generally accepts this point (see, for example, Lerman, 2001). Nevertheless, such research is attempting to say something about minds and their relationship with the world.

There are a couple relevant points, then, that are commonly accepted by qualitative researchers relevant to this issue. Firstly, it is accepted that no description can be perfectly accurate. Secondly, it is accepted that all descriptions include some degree of bias or subjectivity on the part of the author. Both these points, however, assume that the problem is, simplistically put, one of accuracy. My point is slightly different: it is that the descriptions themselves constitute mathematical cognition as it is theorised and conjectured about in publications about mathematics education. So Sfard’s descriptions of Noa’s conversation with the teacher make possible the ideas about cognitive and discursive conflict that Sfard wishes to discuss (as much as the other way around).

IMPLICATIONS AND QUESTIONS

Providing descriptions of the interaction or behaviour of people engaged in doing or learning mathematics is an integral part of much published research in the field. There are, however, particular ways of writing such descriptions that are specific to the kind of writing found in research papers, as opposed to, say, newspaper reports. Moreover, whilst I have highlighted the role of description in the construction of mathematical thinking in written research reports, other parts of the research process are likely to draw on other practices. Interviews, for example, can be seen to construct
mathematical thinking in particular ways. Similarly, discussions between researchers or informal conversations at PME meetings may also contribute in different ways to the construction of mathematical thinking. An interesting question then arises of how these different discursive practices work together to produce accepted accounts of mathematical thinking within a wider community. It is also interesting to consider how descriptions work in different ways at different points in the process. The transcript of Noa and her teacher, for example, can be seen as another form of description, perhaps produced prior to analysis and the writing of a research paper. The structuring of such a transcript is also implicated in the construction of mathematical thinking that arises, as with, for example, the role of the exclamation mark I pointed out earlier.

The practices I have highlighted are likely to be familiar to members of the PME community. PME members, however, are not the only people interested in mathematical thinking; it is also of interest to, for example, mathematics teachers, government advisors, students or textbook writers. The National Numeracy Strategy of England and Wales (DfEE, 1999), for example, includes descriptive lists of what ‘pupils should be taught to know’, thereby constructing mathematical thinking as something that can be listed and broken down into small components to which teachers can be held accountable. In this case, therefore, descriptions are used to construct mathematical thinking in rather different ways to those found in research papers. This observation leads to a more general question. If the discursive practices of mathematics education research are implicated in the construction of mathematical thinking within the community of researchers, and if different discursive practices are used to construct mathematical thinking in different ways in different communities (such as those of teachers of curriculum writers), what is the relationship between them?

Given the layers of reflexivity inherent in the argument I have made in this paper (including, for example, my use of descriptions of Sfard’s work), it would not make sense to suggest, for example, that researchers should be more careful when producing their own descriptions. Every description in a paper concerned with mathematical thinking is implicated in its construction. Two courses of action are perhaps possible, each equally valuable. One is for researchers to be aware of the practices through which their accounts of mathematical thinking are produced. The other is for further research to uncover more of the practices of our community, so that that awareness can be enhanced.

NOTE

1. The three ants are derived from a response given by Margaret Wetherell as part of a UK Linguistic Ethnography Forum colloquium at the annual meeting of the British Association for Applied Linguistics, Bristol, 15-17 September 2005.

REFERENCES


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