GRADE 5/6 TEACHERS’ PERCEPTIONS OF ALGEBRA IN THE PRIMARY SCHOOL CURRICULUM

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Although there has been a recent push towards having an active “early algebra” curriculum in primary school mathematics classrooms, many curricula still leave formal consideration of algebra until secondary school. Nevertheless, there are many aspects of mathematics in the primary school that prepare students for later algebra study. This research used a questionnaire and interview with 14 Grade 5 and 6 teachers to determine their views and knowledge about algebraic aspects of the primary curriculum. As a group, the teachers had only a limited sense of how the mathematical activities they utilise in the classroom build a foundation for later work in algebra. Furthermore, although they were generally good at recognising the correctness of students’ solutions, they did not seem to engage deeply in the students’ reasoning, and varied in the views of the value to be placed on some responses.

INTRODUCTION AND BACKGROUND

The past decade or so has seen an increased interest in the place of algebraic activities in the primary (elementary) school. This is partly because of the importance to algebra of generalisable structural arithmetical understanding (e.g., the distributive law, the quasi-variables of Fujii & Stephens, 2001), together with the results of studies (e.g., Blanton & Kaput, 2004; Warren, 2005) suggesting that “young children can do more than we expected before” (Lins & Kaput, 2004, p.64).

Bednarz, Kieran and Lee (1996) highlight that three of the basic ingredients of school algebra are the generalisation of patterns (such as number patterns or geometric patterns), the generalisation of numerical laws, and functional situations. Kieran (1996, 2004) groups these together under the heading generational activities, because each involves the production of some object of algebra: an equation relating quantities, a description or relation capturing the generality of a pattern, or a rule that describes some general numerical behaviour (see Kieran, 2004, pp.22-24). She points out that while there has been a focus on developing facility with manipulation (something that most adults recognise from their own secondary school algebra experiences, and which many might characterise as being algebra), she also emphasises the need for a conceptual understanding of the objects of algebra, and suggests that “noticing structure, justifying and proving have been sorely neglected in school algebra” (p.31). More recently, she has stressed the importance to “early algebra” of analysing relationships, generalising, noticing structure, and predicting, as these are ways of thinking that are foundational for conventional “letter-symbolic algebra” (Kieran, 2006, p. 27).
One class of generational activity sometimes conducted with primary school students involves pattern recognition, often based on a visual design that “grows” iteratively in a sequence. Blanton and Kaput (2004) looked at very young children’s ability to describe functional relationships and found evidence for co-variational reasoning, or keeping track of how one variable changes with respect to another. Warren (2005) found that Grade 4 children are capable of thinking functionally, and can describe—at least in visual terms—how a pattern is generated. Not surprisingly, describing a design according to its position in the sequence is harder than describing the progression from one design to the next. Rossi Becker and Rivera (2006) found similar results with Grade 6 children, and examined how figurative reasoning usually results in greater success than reasoning with numerical quantities alone.

In Australia, where the preparation of primary teachers through university courses usually involves the content and pedagogy of the primary school and little formal study of secondary mathematics, most primary teachers have little expertise in algebra apart from that encountered in their own secondary schooling. With this in mind, and with the belief that primary school students should be engaged in activities that prepare them for algebraic thinking (in all the senses identified by Kieran), we will examine, to a small degree, the algebraic beliefs and knowledge of primary school teachers. The present research looks at two questions: first, what aspects of primary school mathematics do teachers think serve as preparation for high school algebra; and, second, what features do they focus on in students’ responses to a pattern recognition question?

**METHOD**

This investigation was part of a larger study investigating the pedagogical content knowledge of Grade 5 and 6 teachers in an Australian state. The 14 participants were volunteers, whose teaching experience ranged from 2 to 22 years. The study involved a questionnaire, interviews, and the observation and video-taping of a few lessons. The data considered here came from the questionnaire and follow-up interview conducted at the beginning of the study. The 17-item questionnaire covered a range of mathematical content and pedagogical issues. Teachers responded to the questionnaire in their own time, with no restriction on accessing resources, and the follow-up interview allowed the researchers to probe for elaboration and clarification. The interview questions thus varied from teacher to teacher, depending on their written questionnaire responses and the time available for the interview.

Two questions from the questionnaire are the focus of this investigation. The first asked teachers “What aspects of the primary school mathematics curriculum do you think prepares students for Algebra in secondary school?” The second item, shown in Figure 1, immediately followed the first and provided teachers with an algebraic patterning item, together with sample correct and incorrect student responses. They were asked to indicate how they might respond to the students, and to discuss the role of such items in the primary school.
Data from both the written questionnaire responses and the teachers’ interviews have been combined, and are not distinguished except where changes in response occurred. The first question and the last part of the second provide information about the teachers’ understanding of algebra’s place in the curriculum. Responses to the first part of the patterning question provide insights into the teachers’ understanding of algebra itself, and how they recognise and address their students’ understanding.

RESULTS

The primary school curriculum as a preparation for secondary school algebra

The aspects of the primary school curriculum that prepare students for algebra in secondary school that were mentioned by the teachers are shown in Table 1, together with the number of teachers mentioning each one. In some cases it is not clear what teachers meant by certain suggestions, and the interview did not pursue this due to time constraints or a focus on other items. It is conceivable that teachers may have meant “patterning activities” by “problem solving”, since that kind of activity often arises under the guise of problem solving, although three teachers specifically mentioned both. “Magic squares” are possibly a particular kind of missing number activity, in which students determine the numbers that belong in the empty spaces of a magic square having constant row and column totals.
Many of the topics mentioned were what might be expected, although as can be seen no topic was mentioned by more than half of the teachers. The most common response was “missing number” problems, an activity with an “unknown” symbol. Such problems resemble the “solve” symbolic manipulation tasks that many regard as typical of high school algebra, despite the fact that such problems are often solved by arithmetic approaches rather than algebraic ones (see Filloy & Rojano, 1989). Teachers also mentioned pattern generation activities and order of operations as important, with two teachers, who may have experienced similar professional development in pre-algebra ideas, making specific mention of the distributive law.

All but three of the teachers mentioned two or more topics, although only four could list five or more. One of the teachers made no suggestions, writing “I honestly don’t know as I didn’t really do secondary school maths and never had any success at maths in secondary school”. Another of the teachers with a limited response indicated that she felt the curriculum introduces algebra in secondary school and so very little is done at primary school level. In her interview, however, she expanded on the importance of functions and number patterns as a transition to secondary schooling. She also commented about a question we had included on a student quiz, which involved generalising a pattern in order to predict the 100th term. She said “Probably I wouldn’t go to what’s a hundred of that. I might go to five or something, so they can work it out, but not necessarily that generating patterns, which I think is something I’d like to explore.” This suggests she may have a limited awareness that asking for big-numbered terms can force students to generalise properly, rather than just iterating through the first few terms.
Responses to the second part of the patterning question provided further insight into teachers’ perceptions of the purpose of this kind of activity. Nine of the 14 teachers explicitly stated that such activities lead to algebra, with one of these suggesting that it can help with the introduction of symbols. Two of these nine also emphasised affective issues, indicating that if students encounter such activities in primary school they will not be as “fearful” about algebra in high school. Nine of the teachers (six of whom had mentioned algebra) viewed pattern recognition and description as important for mathematics or real life in general, listing one or more of generating, creating, seeing, or developing rules for patterns as significant skills for students. Six of the teachers also emphasised the thinking skills that such activities develop, with another three mentioning that the activity can develop problem-solving skills.

Although the teachers were not asked about the extent to which they conducted patterning activities in their own classrooms, most of the teachers’ comments implied or explicitly stated that they used such tasks. One of the Grade 5 teachers, however, said that although she had seen this kind of question she had not used them. Some teachers also commented on their students’ engagement and facility with these activities. One wrote “I don’t think it is a particularly easy concept to develop and children need time to ‘play’ with ideas and understandings”. Three of the teachers made specific reference to higher ability children, with one saying

Because I think that that’s what makes the brainy kids. That’s what makes the kids who are very good at maths, I believe that they’re actually fantastically fast at picking up the pattern, using patterns they’ve used previously, and applying them.

**Algebraic understanding evident in responses to the patterning item**

We now examine the understanding of algebra and student reasoning evident in teachers’ responses to the first part of the patterning item. From the questionnaire results, eight of the teachers correctly recognised the erroneous response from Student D, with a further three realising the error in the interview, in one case prompted by the interviewer. Two of the teachers did not comment on correctness, and the final teacher—who had not listed any pre-algebraic aspects of primary school mathematics in response to the first question—indicated that he thought “all of these students are correct in their own way”. He said that he liked D, but added that “with all of them they need to explain them to me”.

The teacher who had commented about students needing time to develop facility with pattern activities was one of the very few who gave a detailed description of how she would respond to the students, which she did in such a way that she clearly revealed how well she understood their reasoning. Her responses were written as if she was talking to the student concerned:

**A:** Yes, this formula appears to work. Well done! How did you come up with it?

**B:** Yes, this formula does work. Is there an easier way of writing it? If you are multiplying by 3 and then later subtracting the number of triangles, isn’t it the same as saying (number of triangles) x 2 + 1?
C: Yes, this formula does work. There are quite a few steps to it. Is there an easier way of writing it?

D: While this formula works for the first triangle it does not work for the rest of the pattern. Let’s test it out. I think you have created this formula because adding a new triangle to the pattern requires two more matchsticks. So, in theory it sounds like it should work. How could we use multiplication in our formula?

E: Yes, that will work if you know what the preceding number is. However, what if we want to work out how many matchsticks are used to make 23 triangles? I think we need to find a quicker formula that does not mean we have to work out the preceding 22 triangles.

Only one other teacher really indicated an understanding of the thinking that might have led to Student D’s response. Nobody else showed serious engagement with the details or possible derivations of the formulae produced by the students—with the exception of E—despite most stating that A, B, C and E work. Presumably they checked the outputs of the formulae against their own data. All of the teachers gave some indication of which responses they thought were “better” than others, although they varied in what they meant by “better”. Nine of them commented about responses B and C being “complicated” or “less efficient” in comparison to A, and expressed a hope that students—or at least their more able students—would be able to find an “easier way”. One of them, however, actually rated B and C more highly because of the complexity. Only one teacher said that she valued having multiple solutions and that “you need to be able to see that they’re all valid, logical paths”. She did not, however, suggest that exploring the equivalence of such solutions might be significant (see also APPA Group, 2004).

Six of the teachers commented that they liked the fact that Student D, though incorrect, had tried to express a rule in a relational way, with some of them even preferring this to Student E’s correct, but non-equational, response. In fact, reactions to Student E’s solution were polarised, with some regarding it as good and others as poor. The eight teachers who seemed to regard it negatively commented that it was “too simplistic”, that the student “can identify a pattern, but they’re not necessarily patterns within relationships […], they need to relate it to the number of triangles”, or that the student “hasn’t taken time to find a rule that is more detailed”. One of the teachers suggested that Student E would struggle with more difficult problems, although he has “got the basics”. In contrast, other teachers felt it was good that E had spotted the pattern so quickly, and that it was “nice and simple”. In fact, at least one of the teachers actually seemed to struggle herself with the covariational ideas expressed in responses A, B and C, and appeared much more comfortable with Student E’s solution. Only four of the teachers actually explained the limitations of Student E’s response in terms of the relationship being given iteratively. They pointed out that you cannot work out the answer for a given number of triangles without knowing the preceding term.
Finally, in light of the work of Rossi Becker and Rivera (2006), we note that none of the teachers’ comments suggested an appreciation of the fact that a formula or description of a relationship usually depends on the way the pattern is perceived.

**DISCUSSION AND CONCLUSION**

Before discussing the results, we must note there were many other questions on the questionnaire and interview, and that these items were late in the questionnaire. This may have affected the quality of the teachers’ responses. Furthermore, the items and questions did not specifically target some of the aspects we have discussed, such as the effect of perception on pattern description. The results should be viewed with these limitations in mind, although we can still draw some important conclusions.

Among these teachers, at least, there is some awareness of the kinds of algebraic ideas that can be fostered in the primary school. Unfortunately although all but one could contribute at least one reasonably strong pre-algebraic concept (missing number problems, patterns work, order of operations, importance of the equals sign, realising letters can stand for a number, or formula substitution), only six could list three, and the fact that no more than half of them listed any one of the topics is also of concern. The only time teachers explained how a given aspect contributed to algebra learning was in the fairly obvious case of the missing number problems.

For some teachers, this limited perspective may be due to their own educational history, as evident in the explanation from the teacher who could not list any content areas. One of the older teachers, who explained that her mathematics teaching had undergone a transformation as part of professional development she had undertaken, explained that as far as the patterning item was concerned that “I’ve not had a lot of experience with that sort of thing, but I think it’s something that we really need to get the kids doing … I didn’t get that sort of thing when I was at school”.

There are positives and negatives to note in the teachers’ engagement with the patterning item, too. Most were able to recognise the correctness or otherwise of the students’ responses, and could offer reasons for their judgements of which responses were better. What was missing, though, was a deep engagement in the mathematical and algebraic underpinnings of the activity. Again, this may be a consequence of educational background, and the experience and training the teachers may have had (or, more likely, may not have had) in conducting this kind of task.

Given the power of pattern recognition and similar generational activities for developing facility with algebraic concepts—whether noticing structure, generalising, developing functional reasoning, using symbols, or even manipulating symbols—it seems that more needs to be done to help teachers understand what the key aspects are and how they contribute to the understanding that needs to be developed in the secondary school. It is also evident that teachers may need more guidance to help them use generational activities in an effective way in the classroom. This would allow them to make better judgements about the correctness, derivation, value, and problematic
aspects of different approaches that students might take, and provide better assistance to students as they engage with the task.

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References


