MATHEMATICS: A HUMAN POTENTIAL[i]

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The learning of mathematics is a human potential. To scientifically support students' abilities to learn requires understanding mathematics learning in the absence of teaching as well as learning that occurs in response to teaching. I briefly describe work that aims at explicating the mechanism(s) by which learners learn from their mathematical activity and reflective processes in the context of task sequences designed to support their learning.

I start with the premise that human's have the potential for mathematical reasoning, knowledge, and communication[ii]. As such, mathematical potential should be realized through education. However, what is the appropriate relationship between the science of education and the development of this human potential? I begin with an analogy.

Joseph Chilton Pearce (1992) described how western medicine developed a protocol for delivering babies that included putting the mother in a supine position, administering drugs to the mother, and putting the baby in the nursery following delivery. These techniques dramatically reduced the mother's role in the birthing process and resulted in the baby's need for resuscitation at birth -- frequently by being held upside down and spanked. Western scientists studied primitive cultures and found that babies delivered by their mothers, usually at home, did not need resuscitation, had lower incidence of infant mortality, and smiled and showed signs of intelligence up to two months earlier than babies born in Western hospitals. Pearce's point was not that we should return to home births without trained professionals, but rather that the proper role of medical science is to understand, support, and enhance the natural processes that are part of the species inborn abilities, and be prepared for medical emergencies that might occur. Mothers have inborn abilities to birth a baby and nurture it after birth. The role of medical science is to foster and support optimal expression of these abilities.

A similar story can be told of mathematics education. Students were brought into schools and told and shown new material that they were supposed to learn. Students' ability to solve problems and to exhibit conceptual understanding was unimpressive. Issues of how to motivate students became a frequent topic of concern. The system was treating students as if they had no ability and motivation to learn. Attention to children before they go to school reveals that these children are engaged in a continual process of rapid learning that includes two incredible achievements: learning to speak at least one language and developing a concept of number. Have we as educational professionals understood, supported, and elicited this incredible ability to learn? Are we teaching mathematics in a way that fosters students' flexible use of their full

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complement of intellectual resources to solve problems and build more complex understandings?

To scientifically support students' abilities to learn requires understanding mathematics learning in a way that is a useful basis for mathematics teaching. For explanations of mathematics learning to be useful, they must account for learning in the absence of teaching as well as learning that occurs in response to teaching. Teaching can be thought of as the intentional creation of carefully conceived opportunities for students to use their learning abilities to develop powerful mathematical concepts and reasoning in relatively short order.

> The difficulty of studying learning—and teaching—lies, in my view, in the fact that it demands the study of the processes by which children come to know in a short time basic principles … that took humanity thousands of years to construct. (Sinclair, 1990, p. 19)

Sinclair's comment can be understood as pointing to the need to harness students' potential through attention to the social and the cognitive aspects of learning. Two main theoretical frameworks have provided a foundation for thinking about mathematics teaching and learning: socio-cultural theory, based on the work of Vygotsky, and constructivism, based on the work of Piaget. Each frames an inquiry into the question of supporting students' ability to learn mathematics. Socio-cultural perspectives have focused on the human processes of learning that derive from viewing learners as social beings interacting in cultural settings. This perspective has highlighted learning through participation in groups, the appropriation of cultural tools, the negotiation of meanings, and the characteristics of learning communities (e.g., classrooms) that foster mathematics learning.

Bereiter (1985) comment can be seen as an important bridge between studies done from these two perspectives:

> How does internalization take place? It is evident from Luria's first-hand account (1979) of Vygotsky and his group that they recognized this as a problem yet to be solved. (p. 206)

Constructivist perspectives background some of the social issues and focus on the internal processes of the learner as culturally established knowledge and functioning are constructed as individual abilities. Piaget's construct of *assimilation* provides a way to think about which students can appropriate what knowledge under what circumstances. Research based on a constructivist perspective has provided information on how learners reason at different stages of learning particular concepts, providing a rudimentary map of the conceptual terrain.

My recent work[iii] focuses on understanding how students construct mathematical concepts through their own mathematical activity. This is in line with the goal of understanding, supporting, and enlisting students' abilities to learn. The rationale is that if we understand how students construct new abstract concepts through their activity, we can generate a set of principles for the design and sequencing of mathematical tasks. A constructivist perspective provides the principal framework for this investigation. Our work is oriented by Piaget's (2001) claim that the development of more complex understandings evolves through the learners' activities and their inherent ability and tendency for reflection. *Reflection*, which is often not conscious, is the natural tendency and ability for learners to identify commonalities in their experience (von Glasersfeld, 1995).

Bereiter (1985) observed.

> The areas in which instruction has proved most uncertain of success have been those areas in which the objective was to replace a simpler system by a more complex one. (p. 217)

Current thinking in mathematics education embraces the posing of mathematical tasks as an integral part of the teaching/learning process. However, what informs the design and sequence of mathematical tasks? At least in the areas where mathematics teaching has been "most uncertain of success," a scientific approach to the design of task sequences is needed. Towards this end, our work is oriented by the goal of explicating the mechanism(s) by which learners learn from their mathematical activity and reflective processes.

In order to study these learning processes, we have adapted a teaching experiment methodology with individual subjects. In these teaching experiments, the role of the researcher/teacher is restricted to posing problems that are part of a designed task sequence, negotiating the meaning of the problems, probing the subject's thinking, and asking for justification of the subject's actions and statements. The researcher does no direct instruction, gives no hints or suggestions, and asks no leading questions. The methodology is aimed at allowing the researchers to have consistent access to the learner's activity and to minimize the influences of others on the learner's thinking.

Of course, it is never possible to study human activity and learning independent of socio-cultural factors. Thus, it is always important to have socio-cultural lenses, as well as other lenses (e.g., affective), ready at hand during the interpretation of data. Our strategy is to minimize social interaction while studying a variety of learners as they learn a variety of mathematical concepts. This strategy does not eliminate the social aspect of thought, language, interaction, and tool use, but rather makes the students' activity more prominent as compared to the impact of the interaction between student and researcher.

In our first, empirical study of this type, we worked individually with 3 prospective elementary teachers to develop understandings of division of fractions. These subjects developed an understanding of the meaning of division of fractions and reinvented a common denominator algorithm for division of fractions based on understanding the invariance among quotients across changes in the (common) units of the divisor and dividend.

For development of the algorithm, the task sequence began with the student drawing diagram representations of division-of-fractions word problems using rectangular wholes. This starting point was selected based on our anticipation that the students would be able to solve the problems in this form (without any instruction on this during the study) *and* that this student resource could be a useful basis for reinvention of the algorithm with understanding. This choice of a starting point is consistent with the Realistic Mathematics Education principle of guided reinvention (Gravemeijer, 1994).

The successful learning by the students, in response to the sequence of tasks, provides evidence of the powerful effect of a carefully engineered sequence that fosters students' abstractions. Further it provides one case that we were able to analyse in terms of the process by which the students came to that abstraction and the related key aspects of the task sequence. Many more such examples with different age students learning different mathematical concepts are needed in order to have an adequate data set based on which we could elaborate a mechanism(s).

I offer a glimpse of the data from Erin's reinvention of the common denominator algorithm. Erin was asked to draw diagrams to solve first division-of-fraction word problems and then context-free problems. When she was quite competent in doing so and explaining her work, she was given additional context-free problems with large denominators (23/25 ÷ 7/25, 7/167 ÷ 2/167) and asked to not draw a diagram, but "anticipate what you would get if you would draw it." For two consecutive problems of this type, Erin was not able to determine an answer directly, but was able to solve the problem by narrating step by step the diagram drawing process she would have used. These problems were followed by a third problem that had the same numerators as the second problem (7/103 ÷ 2/103). Erin immediately gave the answer of "3 ½." Erin had anticipated the commonality in her activity. From this point, Erin had a curtailed strategy for solving division of fractions problems that she could explain and justify upon request. Her explanations demonstrated her understanding of the invariance among quotients across changes in the (common) units of the divisor and dividend.

Humans have an amazing ability to develop new mathematical abstractions. Greater understanding of their abilities to learn mathematics can result in design principles for curriculum development and related principles for instruction that use and supports learners' learning abilities.

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[ii] *I make no claim that this potential is innate. Rather, this is an observation about human's of normal intelligence living in mathematically sophisticated cultures. It seems useful to think about these abilities as being a combination of biology, development, and socio-cultural influences.*

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