MAKING MATHEMATICS MORE MUNDANE –
A SEMIOTIC APPROACH

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The trivial fact that mathematics is a human activity is interpreted by viewing it as a semiotic activity with (systems of) signs and diagrams. Organizing learning as the progressive participation in this social practice with and on signs is deemed to make mathematics more accessible and intelligible and less often a cause of anxiety and frustration.

Under all circumstances mathematics is done and produced by human beings. Any calculation, algorithm, proof, formulation of a theorem, drawing of a diagram, inferring a consequence from assumptions, etc., all this plainly and observably is carried out by somebody. And this somebody always is a member of a social context. Thus, mathematics is deeply and genuinely human. A question, often discussed and never resolved, then is if this human mathematics refers to, or is about, something essentially different from what we human mathematicians produce. As longs as mathematics, like in ancient Egypt or Mesopotamia, consists of a collection of recipes to solve (everyday life) problems this question is not asked. This might be so since the mathematical signs used have natural and immediate referents. But in more detached and generalized settings a kind of desire appears for genuine mathematical objects as referents for the used signs. This desire leads to well-known solutions ranging from Platonism to Empiricism. But, one should be aware that also all these kinds of ontology are devised by human thinkers, even if they postulate extremely non-human origins of mathematics. My first and basic thesis thus is:

- Make the learners aware of the human origin and nature of all of mathematics and of all that is said about mathematics.

To realize this goal, it will be necessary to have the learners do mathematics of whatever kind themselves and also reflect about what they are doing. Historical glimpses will be supportive to this end as well. Besides the notorious failure of so many students in mathematics at all grades and levels, another very detrimental phenomenon is the widespread anxiety and frustration on the part of many learners of mathematics. I consider this to be an extremely unacceptable situation which mathematics education practice and research must strive to change to the better. Anxiety and frustration are closely related to the feeling that one does not understand the mathematics, that it is beyond one’s cognitive or intellectual grasp. Clearly, it is unavoidable that a piece of mathematics will be too complicated and inaccessible for somebody. But this is also the case for puzzles like Sudoku of which nobody is afraid. Of course, in the case of a school subject where one possibly might fail to pass an exam there are many reasons for anxiety. But in mathematics, I think, a specific
feature is that learners consider themselves in principle unable to understand. A notorious non-understanding of this sort concerns \((-1)(-1)=1\) or the imaginary number \(\sqrt{-1}\). In my view, one reason is the following. The common discourse, also in classrooms, suggests that the usage of those signs is determined by the objects (numbers) for which they stand. But those “abstract” objects (even if they existed) cannot be grasped by the learner who then thinks, “It is too abstract for me.”

Mathematics often is experienced as not making sense, as arbitrary and useless, as something for which you need a special aptitude and gift possessed only by a few. Those phenomena of non-understanding, anxiety and fear, feeling of lack of aptitude appear as surprising and unjustified if one considers the widespread and also well accepted usefulness of mathematics (which is not doubted even by those who hate maths). All that is even more surprising if you confront it with the talk about the aesthetics and beauty of mathematics. If one takes this serious then something must be very wrong in how mathematics is presented to, and perceived by, the students. I cannot present a solution to this paradoxical situation. But I will offer some thoughts about principles for a way to alleviate the problems. I start from the widely accepted view that an important human ability is the production and usage of signs of all sorts, linguistic and non-linguistic ones. Much of our individual and social life is regulated and mediated by sign systems. As authors like Vygotskij and Peirce have emphasized: Our thinking and communicating are sign activities. Thus, the design and usage of sign systems is a deeply human quality and activity of which mathematics is a highly specialized and extremely powerful variant. Of special relevance in this context is the Peircean notion of diagram which is defined as an icon of relations which in the well-known semiotic triad may coincide with its object (Stjernfelt, 2000).

One important aspect of mathematics as a sign activity is that it produces symbolic structures that can be used to model situations and processes of many sorts. On the one hand, already available symbolic/diagrammatic structures (like the decimal number system, fractions, differential equations, combinatorial graphs, etc.) of mathematics can be used as functional models to describe non-mathematical situations, to make predictions or to prescribe structures and relations (normative models). All these contexts of applicability on the other hand serve as sources for the design and development of symbolic/diagrammatic structures. In programs like Realistic Mathematics Education those contexts are used with great efficiency for the development of appropriate sign systems within the class-room community. Manipulation of the mathematical structures, mostly in the form of symbolic systems, permits a kind of understanding since one can take the former as explanations for the observed phenomena in the modelled situations. For that it suffices to consider mathematical models as theoretical constructs which operationally simulate (within chosen degrees of accuracy) experiential observations without stipulating any kind of ontological correspondence between the two. This is made very clear by the fact that in many cases very different mathematical structures “explain” the same phenomenon by for instance making similar or compatible predictions. The main consequence for
mathematics education from all that is that mathematics very efficiently empowers human thinking in many diverse areas. And mathematics education has to devise more and better methods to enable the learners to experience in an authentic way this empowerment by mathematical knowledge. This concerns using ready-made sign systems and the design of those in the classroom as well. Students must become aware of the fact that by the use of mathematics you can think and imagine what otherwise is completely unthinkable. It has to be emphasized that this goal can be attained also with simple mathematics. Just think of sociograms for describing or designing social group structures. Of course, not everybody will find that enticing, there is no guarantee for interest and motivation. A special form of this cognitive empowerment is presented by what can be called hypothetical thinking: designing possibilities, analyzing alternatives, answering “What-if-questions”, and the like. This feature is very helpful for planning activities and for evaluating different “futures”. Mathematics permits us to make concrete the assumption of something which is not yet the case and to draw conclusions and consequences from that assumption. And all that can be done together in a group. It is always a shared and social activity which can be scrutinized, doubted and discussed since it is based on or even involves the production, manipulation and interpretation of sign or diagram systems materialized by writing on a sheet of paper or on a computer screen.

The humanistic intentions of such a semiotic approach to learning and doing mathematics are now manifold. Firstly, it embeds mathematics into the general and basic human (individual and social) faculty of sign production and use. Then it emphasizes and makes clear the genuinely human origin and quality of mathematics which one can view as demystifying mathematics. But, most important perhaps, it turns mathematics into a social practice of a great variety of activities, actions and operations with signs presented by inscriptions, mostly on paper. Learning mathematics in such a view is not the acquisition of static knowledge (about objects like numbers or functions) but the progressive participation in the practice of sign activities. Thereby, the meaning of the signs is constituted by their usage within the practice and not by reference to a priori given objects outside and independent of the practice. Even the abstract objects of mathematics like numbers or functions are the emergent product of this sign activity. This does not make learning mathematics easy. Participating in a social practice is demanding in many respects. There are rules and conventions to be followed, many routines have to be acquired and demanding problems have to be solved. There clearly are ways of organizing the classroom which are more compatible with this view than others. And, what I consider being very important, is that the learners become aware of this trait of doing mathematics as a semiotic activity and that there is great value in becoming proficient in operating with the signs of mathematics.

I have so far emphasized the more utilitarian aspect of mathematics as a sign activity by using and interpreting the symbolic structures as models of and models for. But I want to conclude with the possibility of cultivating interest in the sign systems and diagrams as independent of potential interpretations. For school learning this could
mean the exploration of properties and relationships of the symbolic/diagrammatic structures in their own right. In many cases this amounts to recognizing recurring patterns and regularities (like with figural numbers) or to investigate questions of the type “What happens if …?” The learners carry out experiments on objects (inscriptions on paper) according to rules which they know they themselves have designed or could have designed (Peirce speaks of diagrammatic thinking). This kind of activity I consider being an essential part of the social practice of mathematics where consequences of agreed upon conventions are explored, but now within the symbolic or diagrammatic structures of mathematics themselves which there are considered as the objects of mathematics. This can be started with basic number relations and be repeated again and again over the learning process. It is conceivable that through such activity a positive attitude towards mathematics is fostered even when one does not master some parts of the diagrammatic practices. This is based on the assumption that within the semiotic paradigm the activities and processes which might lead to some proficiency are easier to convey and to justify. The mathematical activities based on manipulating and designing inscriptions and diagrams can be demonstrated, observed and imitated, giving mathematical activities an aspect of a handicraft. Mathematics then appears not so much as a mental and individual activity with abstract objects but as a shared and social practice of sign usages. The connection between engagement and successful participation in the social practice should become more transparent, the more it is reflected upon and discussed explicitly in the classroom. Of course, it is necessary that the teacher shares this view. Some suggestions for further and related reading are given in the references.

References


