A TEACHER’S METHOD TO INTRODUCE STORY-PROBLEMS: STUDENT-GENERATED PROBLEMS

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The paper analyzes the method of a third-grade teacher to elicit the generation of story-problems on the part of the students throughout the school year. The teacher triggered the students’ creativity and encouraged them to use their personal experiences and their imagination to make stories and to take into account numerical relations to ask numerical questions related to their numbers. The teacher first started with a story from her own personal experience, generated two questions from her data, and invited the students to generate new questions. Students answered all questions operating mentally with the numbers and explaining their ways of operating using natural language and idiosyncratic symbolizations. The teacher’s activity provided her with an opportunity to assess the students’ conceptualization of numbers and their flexibility to operate with them in ways different from the use of traditional algorithms. Simultaneously, students were able to self assess their own understanding of numbers and their conceptual progress. By the end of the school year, students were willing to attempt the solutions of story-problems that surpassed the difficulty of textbook story-problem for their level.

THEORETICAL CONSIDERATIONS

Documents from different educational organizations have advocated a synergistic method for the teaching of mathematics: Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), Everybody Counts (NRC, 1989), Professional Standards for Teaching Mathematics (NCTM, 1991), A Call for Change (MAA, 1991), Assessment Standards for School Mathematics (NCTM 1995), Principles and Standards for School Mathematics (NCTM, 2000). This method encourages a conceptual and collaborative interaction between teacher and students that brings to the fore the complementarities between individuality and partnership, subjectivity and intersubjectivity, idiosyncrasy and conventionality, teaching and learning. Teacher and students working in collaboration create a classroom environment in which all feel active participants in a community where listening, interpreting, expressing, explaining, and justifying by means of language and other signs naturally emerge and are expected. That is, communication becomes a dynamic semiotic interchange in which teachers and students give and take. We agree with Freudenthal that in this semiotic interaction students’ understanding undergoes “a succession of changes of perspective, which should be provoked and reinforced by those who are expected to guide them” (1991, p. 94).

The synergy in the semiosis of the interchange puts on the shoulders of teachers a new set of expectations and obligations with themselves and the students. Teachers need to focus more on the students’ emergence and evolution and mathematical
meanings and their acts of understanding. In order to do so, they have to learn to note their own personal actions and those of the students; they have to learn to assess and interpret what they note and to map out projected lines of action. Likewise, students also have to learn to do so according to their own needs and self-developed goals. As von Glasersfeld (1995) points out, abstracting and assimilating as acts of understanding must be carried out by the individual because they involve, under all circumstances, the persons’ own experiences. In summary, a synergistic/collaborative method of learning and teaching requires both the students’ involvement in co-constructing their own mathematical understanding and the teachers intentions to teach in harmony with the students’ current knowledge making their actions correspond to their interpretations of the students’ means of acting.

**METHODOLOGY**

**Teaching experiment**

The teaching-experiment methodology, as initially conceptualized by Steffe and colleagues (Cobb and Steffe, 1983), encompasses the social interaction between student and teacher; although it focuses only on the analysis of the arithmetical activity of the students. When this methodology is extended to the classroom and when the intersubjectivity of the participants is considered in tandem with the arithmetical activity of the students, it focuses not only on the analysis of students’ arithmetical reasoning but also on the teacher-student and student-student actions and interactions (Cobb and Yackel, 1995, Yackel, 1995, Cobb, 2000).

Following a classroom teaching-experiment methodology similar to that of Cobb, and colleagues, we conducted a yearlong whole-class teaching experiment with a group of third graders (5 girls and 9 boys); these students attended an elementary school that was considered to be an at-risk school.

The teaching experiment in which the third graders participated consisted of daily classroom teaching in which the third-grade teacher taught the arithmetic class and the researcher was a participant observer. Dialogical interactions between teacher and students and students themselves characterized each teaching-learning episode. In each episode, the teacher made an effort to “see” students’ interpretations and solution strategies from their own perspectives. Consequently, to understand the students’ conceptualizations and their own interpretations of arithmetical tasks, the teacher had to interpret, in parallel, both her own arithmetical actions and those of the students in order to sustain a meaningful dialogical interaction by closely following the students’ acts of understanding.

In the prior school year, the teacher participated in a summer camp with teachers and children using the above methodology. Subsequently, the teacher and the researcher team-taught the arithmetic class to her fourth-grade class. Most of the time, the roles of teacher and researcher were reversed; the researcher took the role of the teacher and the teacher was the participant observer. The teacher also participated in theoretical courses, taught by the researcher, to further her understanding of what was
happening in the classroom. In both school years, the researcher and the teacher engaged in daily conversations about the nature and purposes of the arithmetic tasks to be posed to the students, the students' numerical strategies, the students’ use and creation of signs, and the mediating role of those signs in the students’ conceptualizations.

**Classroom Mathematical Activity**

The arithmetic instruction these students received in the prior school years could be characterized as traditional in the sense that students were expected to perform arithmetic computations using only conventional algorithms explained to and modeled for them by the teacher. Used, as they were, to hear first the teachers’ explanations, the students, at the beginning of the school year, waited for specific directions when given an arithmetic task. But when the teacher did not respond to their expectations, they started to rely on their own reasoning. A common practice in the teaching experiment was to pose oral and written arithmetic tasks for the students to solve mentally. After allowing appropriate time, the teacher proceeded to the whole-group discussion in which students wrote, explained, and justified their solution strategies and ways that enabled them to find their answers. The rest of the students were expected both to listen carefully to the solution and to express their agreement or disagreement with justification. Soon, students began to take on responsibility for their own thinking and the classroom social and socio-mathematical norms (Yackel and Cobb, 1996) began to change. The classroom dialogical interactions improved as the students felt that their contributions were taken into account and their solutions were validated and accepted by the other members of the class.

**Instructional tasks and data collection**

The research team generated, in advance, the instructional tasks for the teaching experiment. However, some tasks were modified and new ones were generated to accommodate the cognitive needs of the students. In general, the generation of instructional tasks and the research activity co-evolved in a synergistic manner. The guiding principle for the preparation of instructional tasks was both to facilitate students’ broad conceptualization of natural numbers in terms of different units and the representation of these numbers using numerals and number words. The arithmetical tasks used allowed the students freedom to symbolize and to explain, in their own ways, their numerical thinking, their numerical relations, and their solution strategies.

To analyze the evolving classroom arithmetical activity, the lessons were videotaped and field notes were kept on a daily basis. The task pages, students' scrap papers, and copies of overhead transparencies used by the students were also collected. All data was chronologically organized.
Story-problems

Although all the arithmetical tasks were created and orchestrated by the researcher, the teacher initiated a yearlong exploration of the teaching of story-problems and the researcher became her collaborator. This exploration was then coordinated with the other tasks already planned for the teaching experiment. The teacher first started by modeling the generation of a story-problem using her own life experiences and after generating some questions about her data she invited the students to generate other questions. Then, she challenged the students to generate their own story-problems and to write them down. During her language arts class, the students edited the problems and wrote them on decorated paper provided by the teacher. The teacher collected the problems and made transparencies. Each student was supposed to do two things: (1) to solve his/her own problem and (2) to pose the problem to the class. Each student took the role of the teacher, posed the problem to the class, interpreted the solutions of his/her peers, and tried to find whether or not the other students agreed with the solution strategies presented and why. All the fourteen problems were solved in this manner. The process took 10 classes (two weeks) of 50 minutes each. The same process was repeated two more times during the school year, in Christmas and Valentine’s Day. Finally, from the end of February on, the teacher posed the students story-problems that surpassed the difficulty of the third-grade textbook story-problems; in addition, they were willing to undertake the solutions of those problems individually. During the problem-solving sessions, the classroom interactions between teacher and students followed the pattern of interaction established throughout the teaching experiment but it was enhanced due to the students’ increasing self confidence in their numerical ability.

At the end of March, the researcher organized the problems generated by the students and the solutions and explanations given to each problem and made a bounded notebook entitled “Our Story-problems”. This notebook was presented to class and placed in the classroom on the shelf designated by the teacher for the students’ projects. It was surprising to see the students use their free time to go over the problems and the written solutions. It was even more surprising to see them start mathematical conversations based on the comparisons of the solutions to particular problems. Since story-problems were also part of the arithmetical tasks prepared for the teaching experiment, we were encouraged to pose to the students more challenging problems than we would have had if the student-generated problem activity had not been initiated by the teacher. This is to say that the teaching experiment co-evolved with the initiative of the teacher.

In what follows, we analyze the teacher’s method; the structure of some of the story-problems generated by the students; and the merits of this type of method to motivate students’ creativity in their conceptualizations of number, numerical relations, symbolizations, and operations with numbers.
ANALYSIS

The first problem was interactively generated by the teacher and the students. Based on her children’s Halloween experiences, the teacher wrote the following story on the board. “My three daughters Mary, Kate, and Megan went trick-or-treating last night. Mary collected 11 candy bars, Kate 6, and Megan 15. I want to know: (a) how many candy bars Kate and Mary collected? And (b) how many more candy bars did Megan collect than Kate?” The students answered these questions operating with the numbers mentally. Different solution strategies were presented to the class. Then, the teacher invited the students to generate more questions using the data given. They asked: (a) how many candy bars did they collect altogether? And (b) how many more candy bars did Megan and Kate have than Mary? The teacher wrote the questions on the board and, again, students answered these questions mentally in different ways.

The majority of the problems generated by the students were about addition and subtraction although multiplication was also implicit in some of them.

1. I had 10 candy bars. I got 30, and then I got 40 more. I got 40, then I got 30, then I got 40 more. Will you add it all up?

2. I spent $50 dollars on my sister’s Ruth gift, $50 dollars on my sister Ashley, $100 on my Mom, $2000 dollars on my Dad, and $3000 thousand dollars on my brother. How much money did I spend altogether?

3. I had $100 dollars. You had only half of what I had and then we got 14 dollars. How many dollars do we have together?

4. Adam saw a parade with 63 cars in it. Fourteen of the cars were red. How many cars were not red?

5. Deidre works 147 days each year. How many days does she not work each year?

6. I had 300 pieces of candy. I gave 30 pieces of candy away. How many pieces do I have left?

7. I had $2000 thousand dollars. I got Mrs. Ludlow a very special gift that cost $900 dollars. How much money do I have left?

8. My mom had one thousand dollars and she wanted to buy a gift for me. Now she has only $125. How much money did she spend?

9. Kristal wants to buy a TV for $157 dollars. She has $29 dollars. How much more money does she need?

10. I have one thousand friends. Another person has only three friends. How many more friends do I have?

11. I have two thousand pieces of candy and my Mom has ten pieces. How many more pieces of candy do I have than my Mom?

12. I have one 100 slices of pizza and my brother has one 1000. He gave away 99 slices. How many slices of pizza does he have left?
13. I got 24 candy bars. I gave 4 bars to my Dad, 5 to my Mom and 11 to my brother. How many bars do I have left to eat?

14. I made one hundred cookies. I gave two cookies to each of the fifteen people in my family. Santa came and ate two cookies. Then his seven reindeer came and ate two cookies each. How many cookies do I have now?

15. If I eat three fourths of the apple pie, how much pie is left?

16. I have thirty pizzas. Melinda ate thirty halves. How many halves are left? How many pizzas did Melinda eat? How many pizzas are left?

17. I had one hundred sixty dresses. I gave you half. How many dresses did I give you? How many dresses do I have now?

For the most part, students used fairly large natural numbers and solved the problems mentally. This was not surprising to us given our emphasis on mental computation and on the conceptualization of number as units of units. Students expressed their solutions in writing using conventional notation or idiosyncratic diagrams in order to present them to the other students. Since the conceptualization of fractional units was also one of the main guiding principles of the study, it did not surprise us the students’ initiative to use simple fractions in their story-problems. Even though the stories were simple and tied to the students’ imagination or personal experiences, the wording of the questions and the quality of some of the questions seem to be very advanced for third graders. Only a few times did they use the words “left” in subtraction problems and “altogether” in addition problems; in contrast, other complex expressions were used to frame their questions about addition and subtraction problems.

Some subtraction problems had to do with the complement of a set (problems 4, 5, 6, 7, 8, and 12) as well as the comparison of two sets (problems 9, 10, and 11); other problems required a sequence of subtractions, or a sequence of multiplications, additions and subtractions (problems 13 and 14); another problem presented extra information (problem 12). Other problems required a good conceptualization of simple fractions of continuous wholes (problems 15 and 16) while another required the conceptualization of fractions of discrete wholes (problem 17). Although the majority of the problems had only one question, some of the problems had more than one question (problems 16 and 17). The analysis of students’ solutions to these problems and the role of their idiosyncratic diagrams and symbolizations in their understanding are presented elsewhere.

Given the emergence of story-problems with multiple questions, students were presented with problems of that sort. One example of such problems is the following. Sunny school has 250 students, Hoover school 150, Lincoln school 350, and St. Joseph school 300. Which school has the most students? Order the schools according to the number of students. How many more students attend Lincoln than Hoover? How many fewer students attend Hoover than St. Joseph? What is the difference
between the number of students attending Lincoln and Hoover? How many students attend these schools altogether?

Problems of this type were consecutively extended to problems about the populations of several cities as well as the population of several states. The population was given in millions. Similar questions were asked, using different ways, to linguistically indicate the operations of addition and subtraction. These types of problems helped the students to focus their attention on the type of unit while making sense of the size or magnitude of the numbers.

Another type of problem given to the students was called “riddle problems”. These problems introduced multiplication to the students and they solved them using double counting. Two examples of this kind of problems are the following:

I am a two digit number. I am a number between 25 and 35. I am a multiple of 3 and 9. What number could I be? Why?

I am a two digit number. I am a number between 20 and 60. I am divisible by 1, 2, 3, 4, 6, 8, 12 and 24. What number(s) could I be? Why?

Another type of problem was called “project problems”. These problems, as all the other problems, were not given to the students to perform only one particular operation and therefore they were challenged to think and to develop a sense for numbers and numerical relations. The following is an example of one of the easier problems:

(a). It takes Carol 7 minutes to ride a mile. How long does it takes for her to ride 3 miles?

(b). Todd rode 2 miles to the store. The trip took about 12 minutes. About how long would take him to ride a mile?

(c). One day, Todd and Carol rode bikes to the school. Todd lives half a mile away. It took him 4 minutes to get there. Carol lives one mile away. She got there in 10 minutes. Who rode faster? How do you know?

(d). Carols’ family has 2 cars and 5 bikes. How many vehicles do they have in all? What fraction of the vehicles are bikes? What fraction of the vehicles are cars?

CONCLUSIONS

These third-graders’ generation of story-problems indicates their progressive understanding of number, their ability to encapsulate imaginary and real life experiences into arithmetical contexts in order to establish numerical relations of interest to them. The students also indicated both their ability to incorporate their emerging understanding of fractions and their ability to take on more challenging story-problems. The level of these story-problems was beyond the level of well classified textbook story-problems requiring only one arithmetical operation to answer only one question following a procedure already modeled for the students by the teacher. The story-problems generated by these students motivated them to attempt the solution of more challenging story-problem subsequently posed to them.
The teacher’s initiative to encourage the students to generate their own story-problems prompted the early introduction of story-problems planned for the teaching experiment and accelerated the students’ symbolic activity and the dialogical interactions among the students and between the students and the teacher.

References


