PROMOTING PRE-SERVICE TEACHERS' UNDERSTANDING OF DECIMAL NOTATION AND ITS TEACHING

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This paper discusses the results of the first cycle of a design experiment aimed at improving Indonesian pre-service teachers' understanding of decimal notation and at having them participate in a new pedagogy that they can use in future teaching. Classroom observations, written responses to learning activities along with pre-test and post-test data will be discussed to describe how their understanding evolved. Findings suggest substantial improvement in being able to decompose a decimal number into place-value related parts. However the results also showed that many pre-service teachers did not grasp base ten structures in decomposing decimals and the models have not been used as a thinking tool. The next cycle of the design experiment will address these issues.

INTRODUCTION

Studies investigating pre-service teachers understanding of decimal numeration reveal that misconceptions persist in this group (Putt, 1995; Stacey, Helme, Steinle et al., 2001). Weak understandings of place value coupled by weak notions of the magnitude of decimals are amongst the indicators of problems in decimal numeration. Putt (1995) in his investigation of pre-service teachers' knowledge in ordering decimals found problems in understanding equivalent decimals such as 0.7, 0.70, and 0.700. He also noted that some pre-service teachers' interpretation of a decimal number is limited to a single representation. The fact that pre-service teachers in their future employment may share their misconceptions with children underscores the need to improve pre-service teachers' understanding in decimals.

An analysis of some Indonesian commercial textbooks (e.g., Listyastuti & Aji, 2002a, 2002b) indicates reliance on extensive use of syntactic rules based on whole numbers to teach decimals. The approach to teaching and learning decimals is very symbolic and no attention is given to creating meaningful referents such as concrete models to help students make sense of the place value structure of decimal notation. The models for learning decimals presented in the textbooks are the more symbolic models such as number lines, emphasising positions of points rather than lengths of lines. Research in Western countries has shown that this approach does not develop well-connected understanding of decimals. Hiebert (1992) argued that "A greater investment of time would be required to develop meaning for the symbols at the outset and less emphasis would be placed on immediate computational proficiency" (p. 318). Furthermore he contends that having meaningful interpretation of decimal notation will enhance performance in computation skills. Current thinking in Indonesia, influenced by the Freudenthal realistic mathematics education, accepts

2006. In Novotná, J., Moraová, H., Krátká, M. & Stehlíková, N. (Eds.). *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 5, pp. 385-392. Prague: PME.

that improvement in mathematics education will come by increasing emphasis on developing meaning and moving away from teaching based only on rules, and through adopting new teaching methodologies, such as group work, which encourage students to construct mathematical ideas together.

Despite extensive studies of decimals in other countries (Irwin, 1995; Peled & Shabari, 2003; Steinle & Stacey, 2003), a study of teaching and learning decimals in the Indonesian context has not been carried out. Considering the above approach to learning decimals, it is posited that Indonesian pre-service teachers' knowledge in decimals will be limited and not well-connected. Hence, this study intends to develop a set of appropriate learning activities on decimals to promote a conceptual understanding of the topic for pre-service teachers in Indonesia and to strengthen their ideas about how to teach the topic. The study follows a design research methodology adhering to Gravemeijer's account (2004), whereby a set of instructional activities for a specific topic is devised through a cycle of design, teaching experiment and retrospective analysis. The starting point for devising the instructional activities is taken from the existing knowledge of the students and by hypothesizing their learning trajectories.

The small section of the study that is reported in this paper is from the first design cycle and examines one set of activities designed to explore meaningful interpretation of decimals in terms of place value. Stacey (2005) contends that full understanding of the meaning of decimal notation includes the ability of interpreting a decimal number in terms of place value in several ways based on additive and multiplicative structure of decimals. Realizing the importance and the challenge of introducing the use of concrete materials in learning decimals into Indonesia, this study uses concrete models to assist pre-service teachers improve their understanding of decimals and to provide them with ideas for teaching decimals to children.

METHODOLOGY

Participants and Procedure

Two groups of pre-service teachers, i.e. pre-service primary and pre-service secondary attending Sanata Dharma University in Yogyakarta, Indonesia participated in this study. The pre-service primary teachers undertake a two-year diploma program run by elementary teacher training department, whereas the pre-service secondary teachers enrol in a 4-year bachelor of education program run by mathematics and science education department. It should be noted that the nature of their participation and teaching intervention for both groups were the same. Two lecturers carried out the whole activities within 5-6 meetings of 100 minutes each, during August-October 2005. The researcher took an observer role, and directed the video-taping. One lecturer taught both pre-service primary and pre-service secondary cohorts while the other only taught one of the pre-service primary cohorts.

The 3 activities discussed in this paper were carried out in 2 meetings, where the pre-service teachers worked on the activities in groups (4 - 6 pre-service teachers in each group). A total of 30 groups were involved in these activities, 11 from pre-service secondary cohort and 19 from pre-service primary cohort. Two groups of pre-service secondary and 3 groups of pre-service primary teachers were videotaped during these sessions. The selection of groups to be videotaped was based on their consent to be videorecorded and their communicative skills in expressing of their thinking.

The meetings involved very limited number of lecturing, which was a clear departure from normal practice for both lecturers and students. Pre-service teachers work in groups discussing and finding solutions for the tasks together. The lecturers' role in this study is more as a facilitator for delivering activities and leading group presentations and discussions. The rationale for choosing this mode of delivering the activities is to encourage active participation of pre-service teachers in constructing meanings for themselves. It is expected that they will be able to explore more ideas this way, and get firsthand experience of new methodologies for their future career.

In these meetings, two concrete models were used: a concrete model called Linear Arithmetic Blocks (LAB) (See Figure 1 below), and a number expander. These two models are both new for participants in this study. These models have been explored in prior studies on teaching and learning decimals (Stacey, Helme, Archer, & Condon, 2001; Steinle, Stacey, & Chambers, 2002) and suggested as powerful models in learning decimals. LAB represents decimal numbers by the quantity of length (not metric length such as metres and centimetres). It consists of long pipes that represent a unit and shorter pieces that represent tenths, hundredths, and thousandths in proportion. Pieces can be placed together to create a length modelling a decimal number and can be grouped or decomposed (for example to show 0.23 as 2 tenths + 3 hundredths or as 23 hundredths). A number expander, although a concrete model, works on the symbolic representation. It displays the extended notation of a number in different ways as can be seen in Figure 2 below. The use of two models in these learning activities, one with a physical representation and the other using the symbolic representation of number, appeared to be consistent with the goal of constructing meaningful understanding of decimal notation in terms of place value.

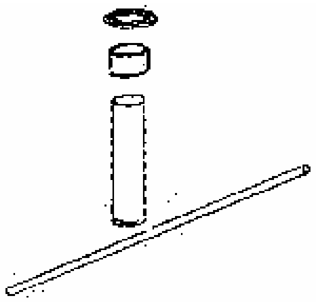


Figure 1: LAB pieces

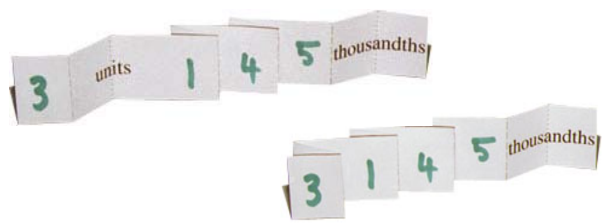


Figure 2: Various expansions of 3.145

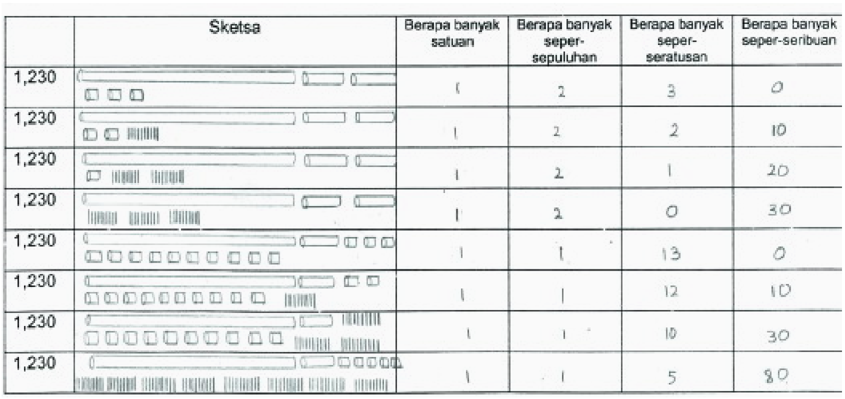
As part of the design research, pre-service teachers took written tests and some of them participated in individual interviews, which were conducted before and after the teacher intervention. Prior to the teaching intervention, 136 participants sat in pre-test and after the teaching intervention, 129 participants sat in post-test. In this paper, the data is drawn from the pre-test, post-test, and observations of groups during meetings, selected because of its relevance to the learning goals and hypothetical learning trajectories related to the activities being reported on.

RESULTS AND DISCUSSION

Activity 1 – Decomposing decimals

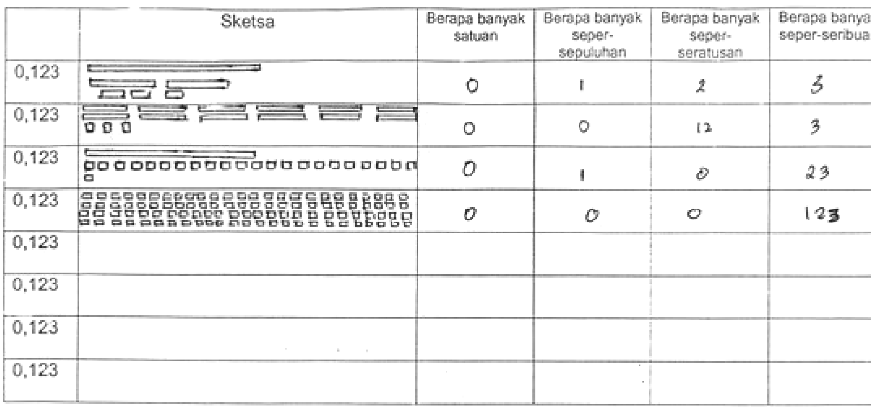
The first activity involved decomposing two decimal numbers, i.e., 1.230 and 0.123 in terms of place value as shown in Figure 3a and 3b. For each decimal number, columns to draw sketches of the LAB representation of the numbers as well as columns to decompose the number in up to 8 ways into ones, tenths, hundredths, and thousandths are presented. By encouraging pre-service teachers to sketch the representations of the numbers, it is expected they will use the LAB model (introduced at the previous meeting) to assist them structuring their solutions.

We found that most groups could find 5 or more different ways to express 1.230 or 0.123. However, their sketches reflected different mathematical understandings, which can be categorized as showing 10-structure, 5-structure and no-structure. Sketches with 10-structure and no-structure are presented in figure 3a and b below. Note that in Indonesian context, we use a decimal comma instead of a decimal point.



	Sketsa	Berapa banyak satuan	Berapa banyak seper-sepuluhan	Berapa banyak seper-seratusan	Berapa banyak seper-seribuan
1,230		1	2	3	0
1,230		1	2	2	10
1,230		1	2	1	20
1,230		1	2	0	30
1,230		1	1	13	0
1,230		1	1	12	10
1,230		1	1	10	30
1,230		1	1	5	80

Figure 3a: structure of 10



	Sketsa	Berapa banyak satuan	Berapa banyak seper-sepuluhan	Berapa banyak seper-seratusan	Berapa banya seper-seribua
0,123		0	1	2	3
0,123		0	0	12	3
0,123		0	1	0	23
0,123		0	0	0	123
0,123					
0,123					
0,123					
0,123					

Figure 3b: no structure

Unfortunately, of 30 groups, only 6 groups reflected the 10-structure in their sketches. Four groups showed a combination of structure of 5 and 10-structure in their sketches with dominant 5-structures, and 20 groups showed no structure. This finding suggests that even though most groups could complete many possible alternatives for decomposing decimal numbers, they did not emphasize base ten structures in their solution, which is very important for teaching.

The researcher also observed that most groups did not work with the LAB model when sketching decimal representations. Instead they found solutions arithmetically by using addition, subtraction, multiplication, and division. Prior learning experiences in decimals with heavy emphasis on symbolic manipulation might cause them to be more comfortable working on the problems arithmetically. The fact that models have not been used as a thinking tool indicated that the use of LAB model has not been well integrated in this activity, and provides a challenge for the next design research cycle.

Indications of impact of the activity 1 will be discussed by comparing pre-service teachers performance in pre-test and post-test items in a pair of item described in Figure 4. For each of the item, four alternatives are sought.

Pre : 0.375 =	… ones +	… tenths +	…. hundredths +	…. thousandths
Post: 0.753 =	… ones +	… tenths +	…. hundredths +	…. thousandths

Figure 4: Pre-test and post-test items investigating decomposition of decimals

In this case, we looked closely at pre-service teachers who sat in both pre-test and pos-test (N=118) comprised of 51 pre-service secondary and 67 pre-service primary. In analysing the number of correct answers, we categorize blank answers as wrong answers. Comparison between pre-test and post-test performance showed that both cohorts made an improvement. Figure 5 below represents the percentage of pre-service teachers in each cohort with the number of correct alternatives. Pre-service secondary showed an improvement as can be noticed from the decrease in the number of answers with incorrect alternatives and the increase in the number of answers with four correct alternatives. Similarly, pre-service primary also showed an improvement after the teaching sessions.

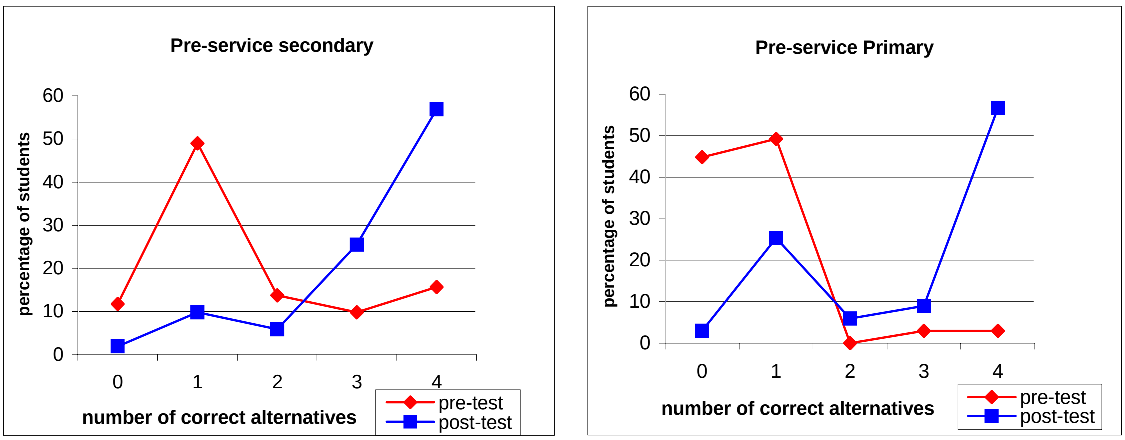


Figure 5: Comparison of Figure 1 pre-test and post-test item

Apparently pre-service secondary had a stronger performance in pre-test compare to pre-service primary cohort. However both cohorts started with about 50% students giving only one correct alternative, i.e., 0.375 is 0 one + 3 tenths + 7 hundredths + 5 thousandths. Interestingly, both cohorts also had almost the same percentage of students answering four out four alternatives at the post-test. This result suggested that both groups gained advantage from the teaching intervention. Pre-service primary cohort showed high improvement as the average number of correct answers given was 0.72 rose to 2.92 out of 4. Meanwhile the average number of correct answers of pre-service secondary rose from 1.67 to 3.26 out of 4.

About 16% of pre-service primary cohort gave 4 blanks in the pre-test, which contributes to the higher percentage of wrong answers in pre-service primary pre-test as can be observed in Figure 5. In contrast, we found no answers with 4 blanks from pre-service secondary cohort.

Some responses to this item in the pre-test suggested a lack of place value understanding as indicated by simply re-ordering the decimal digits, for instance, 0.375 = 5 one + 7 tenths + 3 hundredths + 0 thousandths, 0.375 = 0 one + 5 tenths + 7 hundredths + 3 thousandths, or 0.375 = 0 one + 7 tenths + 3 hundredths + 5 thousandths. Fortunately the number of such responses dropped in the post-test.

The improvement from pre-test to post-test might be considered as a logical consequence after pre-service teachers' participation on activity 1. However, evidence from their responses to other post-test items leads us to believe that some of pre-service teachers also translated these ideas of decomposing decimals into their ideas for future teaching. In response to a post-test item asking ideas to help children to solve 0.3:100, one pre-service teacher used extended notation of 0.3 = 0 + 3 tenths = 0 ones + 0 tenth + 30 hundredths = 0 ones + 0 tenths + 0 hundredths + 300 thousandths and then by dividing 300 thousandths by 100, getting 3 thousandths or 0.003. Similarly another pre-service teacher suggested the use of LAB model to represent 0.3 not using 3 tenths but using 300 thousandths.

In another post-test item asking their ideas to help students determining the larger decimals of 0.7777 and 0.770, 19 pre-service teachers mentioned they will use extended notation of decimals to help children see that 0.7777 is larger than 0.770. Even though only a minority of pre-service teachers responded this way, these results are encouraging and the future design cycle will aim to extend this effect.

Activities 2 & 3: Comparing models & reflections on future teaching

In Activity 2, pre-service teachers were introduced a number expander, to help them check their decomposition of decimals. Following that, they were asked to compare the LAB and the number expander as models. Concerning the relationships between LAB and a number expander, two groups mentioned that both models were related to Bruner's representations. These groups classified LAB as an enactive model and the number expander as a symbolic model. Another group also pointed out that LAB represented ones, tenths, hundredths, and thousandths in concrete ways whereas the number expander represented them in more symbolic ways using numbers and verbal names. Three groups linked their comment directly to the previous activity and pointed out that both models can “show” how to express a decimal number in different ways.

In activity 3, pre-service teachers were asked to write about their new experiences and their ideas for future teaching of decimals gathered from these activities. In their reflection, many of them pointed out that they had not learnt about different ways of decomposing a decimal number before. Their experience was limited to one form of decomposing 0.123 as 0 one + 1 tenth + 2 hundredths + 3 thousandths.

They also pointed out the fact that this was their first experience of using concrete models in learning decimals. Their experiences with models in these activities have inspired them to find other models that can help them to teach decimals in more concrete ways as expressed in the following quotes:

> “This is the first time we use manipulatives in learning decimals so we become more creative in finding new ways of teaching decimals, for instance using paper strips, or plasticine with similar principle to LAB. We also found a concrete way of finding the place value of decimals and we can use concrete manipulatives such as LAB and the number expander later in our teaching, which we haven’t used before. We also experienced new approaches in our learning by finding solutions by ourselves, sharing among different groups, and getting feedbacks from the lecturer”.

> “We learnt that decimals which used to be taught only using numbers, in fact can be represented with concrete materials so that students can actively involved in learning and understand better. For me, in comparing decimals such as 0.123 and 0.1231, I used to round it and concluded that 0.123 = 0.1231 but after this, I know that 0.1231 > 0.123 because when I use LAB to compare them, I can see that 0.1231 is longer than 0.123”.

> “Before we introduce how to do decimal computation, we try to introduce decimals using an LAB. The goal is to help them to understand the meaning of a decimal number, not only be able compute with decimals. Using that model, a student can explore their ideas in a more active way so then they can do calculation problem more easily”.

The first and third comments suggest that the role of models in helping them to create meaningful interpretation of decimals. All comments expressed the contribution of models in creating an active learning atmosphere. However, we could not find any *specific* ideas of how the models will be incorporated in ideas for teaching decimals except for the second quote.

CONCLUSION

The evidence from the study of the first cycle for these activities signified the importance of constructing a meaningful interpretation of decimal notation in terms of place value. The finding suggested the need for the next cycle to design activities where the physical use of the model is a more integrated part of the pre-service teachers’ activity, so they do not answer questions simply by relying on previously learned syntactic rules. Even though most pre-service teachers noted the important role of the models in their reflections and suggested that they will use them for their future teaching, only a limited numbers of pre-service teachers can explicitly express their ideas using models. We hope the design for the next cycle will improve these aspects of the activities and there will be more pre-service teachers who can link their experiences with the activities with their improved ideas for teaching decimals.

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