

INVESTIGATING PRESERVICE TEACHERS' UNDERSTANDING AND STRATEGIES ON A STUDENT'S ERRORS OF REFLECTIVE SYMMETRY

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This study examined how preservice teachers understand reflective symmetry and what types of pedagogical strategies preservice teachers use to help a student who has misunderstanding of reflective symmetry. It was found that a large portion of preservice teachers has limited understanding of reflective symmetry. It was also revealed that preservice teachers have tendency to rely on procedural aspects of reflective symmetry when helping a student understand reflective symmetry correctly although they recognized a student's misconceptions in terms of conceptual aspect. Furthermore, it was found that even a large portion of preservice teachers who showed sound understanding of reflective symmetry relied heavily on procedural aspects of reflective symmetry. This study has implication to researchers and teacher educators.

INTRODUCTION

It is well-known that the quality of teaching depends on teachers' knowledge of their subject. Since Shulman (1986) declared the missing paradigm in educational research as the study of teachers' knowledge, for two decades, researchers have devoted considerable attention to what teachers should know and be able to do. According to Shulman, it is not enough for mathematics teachers to know mathematics itself well and represent it understandably to students. Teachers should know students' common conceptions, misconceptions, and difficulties when learning particular content, and they should have the specific teaching strategies necessary to address students' learning needs in particular classroom circumstances.

Symmetry is an important concept in mathematics learning. *Principles and Standards for School Mathematics* (NCTM, 2001) sets symmetry as one of the significant geometry concepts. Symmetry is repeatedly stressed grade after grade in school curriculum. Besides, it has been applied to other mathematics strands as well as many other areas including physics.

However, there is not much research of how preservice teachers understand reflective symmetry and what kinds of pedagogical strategies they use in teaching reflective symmetry. Although recently, more research on teachers' pedagogical content knowledge has been widely published, there remains overlooked research on teachers' pedagogical content knowledge, in particular, pedagogical strategies. In addition, it has been reported that teachers commonly lack important strategies necessary to help students overcome those difficulties although teachers have some knowledge about students' difficulties, (Berg & Brouwer, 1991; Magnusson, Borko,

Krajcik, & Layman, 1994; Magnusson, 1991; Smith & Neale, 1989; Krajcik & Layman, 1989).

The purpose of this paper is to investigate preservice teachers' understanding of reflective symmetry and their pedagogical strategies when they help students who showed misunderstandings of reflective symmetry. This study is guided by two research questions:

- (a) How do preservice teachers understand reflective symmetry?
- (b) What types of pedagogical strategies do preservice teachers refer to in order to teach students reflective symmetry?

THEORETICAL FRAMEWORK

In order to understand what and how teachers should know to teach reflective symmetry effectively, this study refers to the *Everyday Mathematics* curriculum, and the van Hiele theory.

Understanding of Reflective Symmetry

The *Everyday Mathematics* curriculum (UCSMP, 1999), the most widely used standards-based reform curriculum in the U.S (Carroll, 1998), was analyzed from first grade to sixth grade. Symmetry in the *Everyday Mathematics* curriculum is introduced in the first grade, built upon throughout grade 2-5, and reintroduced in sixth grade.

In short, the *Everyday Mathematics* curriculum introduce symmetry by folding and matching (first and second grade), connecting the matching points and measuring the distance from each point to the fold line (third grade), using a transparent mirror to discover basic properties of reflections (fourth grade), and reviewing isometric transformations and performing them on geometric figures (sixth grade).

The van Hiele Theory

This study was based on the van Hiele model of geometric understanding. There are five levels, which are sequential and hierarchical. The level and main characteristics are in Table 1. This is general criteria used, not specifically applied to symmetry. This study attempted to come up with levels specific for symmetry.

Level	Characteristic
Level 0: Visualization	Students visually recognize figures by their global appearance. They recognize triangle, squares, parallelograms, and so forth by their shapes, but they do not explicitly identify the properties of these figures.
Level1: Analysis	Students start analyzing the properties of figures but they do not inter-relate figures or properties of figures.
Level 2:Ordering	Students logically order the properties of figures by short chains of deductions and understand the interrelationships between figures.

Level 3: Deduction	Students start developing longer sequences of statements and begin to understand the significance of deduction, the role of axioms, theorems and proof.
Level 4:Rigor	The objects of thought at level 4 are deductive axiomatic systems for geometry.

Table 1. The van Hiele level (Clements and Battista, 1992)

RESEARCH DESIGN

Participants

Fifty-four preservice teachers participated in this study. Thirty-two were in their senior year of the elementary teacher preparation program and twenty-two were prospective teachers with a math major seeking middle and secondary school certification. Preservice teachers in secondary and elementary programs were included in order to obtain a broad range of responses to the study's task, since it has been reported that elementary teachers have limited knowledge of mathematics (Ma, 1999; Ball, 1990; Behr & Lesh, 1991; Simon, 1993). Thus, by including two different groups of prospective teachers with different backgrounds and experiences with mathematics, this study intended to collect richer information about pre-service teachers' reasoning and responses as well as gaining insights by contrasting the two groups' responses to the same task. However, there was no difference between two groups. Therefore, this study did not compare with and contrast to difference of understanding between two groups.

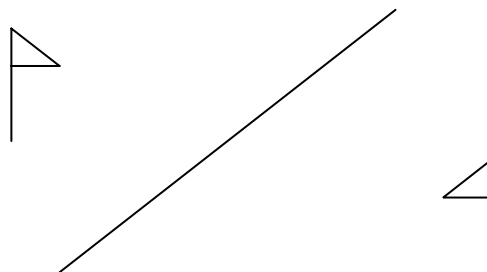
Instrument

Two written tasks were developed, such as *Content knowledge task* and *Pedagogical strategies task*, based on Healy and Holyes' (1997) set of paper and pencil items, Rowan, et al. (2001)'s release items, and the *Everyday Mathematics Teacher's Lesson Guide* (UCSMP, 1999). Both two tasks focus on a common set of invariant properties: the reflected image is the same size and shape as the original, it is the same distance away from the line of symmetry, and is "opposite" or "reversed". When connecting corresponding points, it is the perpendicular line (90° angle) to the line of symmetry.

For content knowledge, two multiple-choice questions and two construct items were developed. Multiple-choice questions contain one "correct" choice and three or four "incorrect" choices. First two multiple-choice items were developed based on the *Everyday Mathematics Teacher's Lesson Guide*. The *Everyday Mathematics Fourth Grade Teacher's Lesson Guide* provides common misconception students are likely to have when learning symmetry. It says, "Although the parallelogram has no lines of symmetry, many people think it does" (p. 742). In addition, the *Everyday Mathematics Teacher's Lesson Guide* from first through sixth grade says that the new figure of symmetry is the same size and shape as the original figure but the opposite of the original object; the distance from one point to the line of reflection is the same

as the distance from its matching point to the line of reflection. Based on these guides, the following questions were developed to assess preservice teachers' understanding. In the first question, preservice teachers were asked to identify the number of lines of symmetry in a parallelogram. In the second question, they were asked to choose the description that most closely matched the properties of reflective symmetry. The third and fourth questions were developed based on Healy and Holyes' set of paper and pencil items. In the third question, preservice teachers were asked to perform reflection. In the fourth question, they were asked to explain their strategies for the third question.

For pedagogical strategies, two construct questions were developed, based on Healy and Holyes' set of paper and pencil item. In Healy and Holyes' article, one student, called Emily, was asked to reflect the left flag to the slanted line. Her response is shown below. In the first question, preservice teachers were asked to identify Emily's errors. In the second question, they were asked to respond about ways to help Emily understand the reflective symmetry.



Data Collection and Analysis

The task went through multiple layers of development which included pilot testing with two volunteers who were interviewed in order to check for possible misunderstandings. The task was then administered as a survey to the entire class in three mathematics methods course sections, two elementary and the other secondary, towards the end of the semester. We only report on the data of the prospective teachers who signed the study's consent form.

For data analysis, responses to the first and second question of content knowledge were graded based on a correct answer. For the remaining questions of content knowledge, two categories were developed based on *the Everyday Mathematics curriculum*: Knowing the properties of reflection (KR) and Creating the reflected image (CR). Each category was recategorized into three or more sub-categories depending on preservice teachers' responses to the questions. The first category (KR) is defined by how preservice teachers explained or identified given reflection problem in terms of the properties of reflective symmetry such as perpendicular line, equal distance, and diagonal line. The second category (CR) is defined by how preservice teachers answered by focusing on just creating the reflected image such as using mirror, flipping, turning the paper, folding and tracing, and coordinating. Three subcategories of KR and six subcategories of CR were identified: perpendicular line (90° angle), equal distance, equal distance and 90° angle (KR); using mirror, flipping,

turning the paper, folding and tracing, coordinating, and matching (DR). Subcategories were coded as PL, ED, EDPL; M, FL, TP, FO, C and MA, respectively. For example, the response of KR to question 3 was coded as PL if students answered to the third question by focusing on perpendicular line or 90° angle, like “by drawing a perpendicular line from points on the figure to the line of reflection.” The response of CR to question 3 was coded as FO if students explained it like “I folded on the line and traced the flag on the back of the paper, then unfolded the paper and copied my trace from the back to the front of the page.”

For data analysis of pedagogical strategies, two more categories were added: Informal Expression (IE), and Misinterpretation (MI). IE is defined by how preservice teachers fail to identify Emily’s errors in terms of either properties of reflective symmetry or doing reflection. MI is defined by how the preservice teachers express lack of knowledge to Emily’s learning difficulties. For example, the response was coded IE if preservice teachers did not rely on or specify Emily’s errors either properties of reflective symmetry or creating the reflected image, like “she is confusing symmetry and mirror reflection.” The response was coded MI if preservice teachers answered, like “I don’t know” or “She is unable to visualize flipping the image over the line.” Table 2 shows categories and subcategories for data analysis.

Category	Subcategory
Knowing the properties of reflection (KR)	PL (Perpendicular line/ 90° angle)
	ED (Equal distance)
	EDPL (Equal distance and angle)
Creating the reflected image (CR)	M (Mirror)
	FL (Flipping)
	TP (Turning the paper)
	FO (Folding and tracing)
	C (Coordinating)
	MA (Matching)
Informal Expression (IM)	
Misinterpretation (MI)	

Table 2. Category and Subcategory

RESULT

Research Question 1: How do preservice teachers understand reflective symmetry?

It was found that a large portion of preservice teachers has lack of content knowledge of reflective symmetry. A large portion of preservice teachers has misconception of reflective symmetry. They misunderstood that the parallelogram has lines of symmetry. They confused symmetry and rotation. When they were asked to explain

how to perform reflection, over half of preservice teachers relied on the procedural knowledge of reflective symmetry such as folding rather than focused on the properties of reflective symmetry.

In the first question, prospective teachers were asked to find how many lines of symmetry are in the parallelogram. Among prospective teachers, forty-one out of fifty-four prospective teachers (76%) answered correctly. Thirteen out of fifty-four prospective teachers (24%) answered incorrectly. Among prospective teachers who gave an incorrect answer, nine out of thirteen (69%) thought that the parallelogram have two lines of symmetry.

In the second question, prospective teachers were asked to choose a choice explaining the properties of reflective symmetry. Among prospective teachers, thirty-five out of fifty-four prospective teachers (64%) answered correctly. Nineteen out of fifty-four (36%) answered incorrectly. In particular, all prospective teachers who gave the incorrect answer chose answer, which says “When the original drawing is rotated, the two drawings match”, as the closest explanation to the line of symmetry instead of a correct answer c, which says “Every vertex is the same distance away from the mirror”. It is revealed that many prospective teachers confused the property of reflection and those of rotation.

In the third question, forty-six out of fifty-four prospective teachers (83%) represented reflective symmetry correctly. Eight out of fifty-four (17%) represented it incorrectly. Figure 1 show examples of incorrect representation of reflective symmetry.

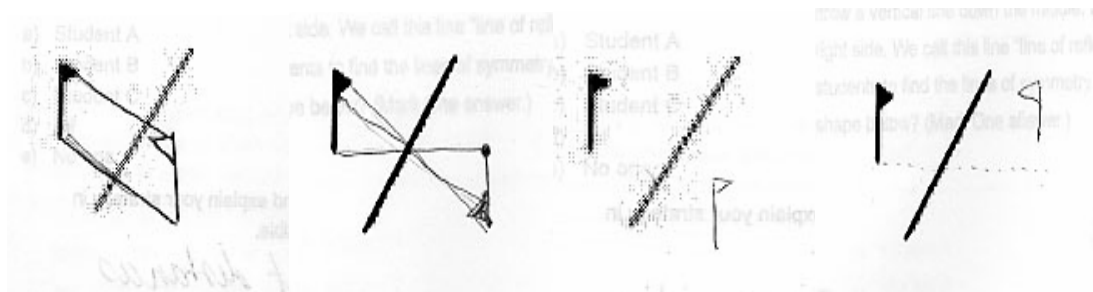


Figure 1. Examples of Incorrect Representations

In the fourth question, prospective teachers were asked to explain how they performed reflection in question 3. Twenty-three (43%) out of fifty-four prospective teachers explained reflection focusing on knowing the properties of reflective symmetry (KR). Twenty-nine (54%) explained it in terms of creating the reflected image (CR).

Research Question 2: What types of pedagogical strategies do preservice teachers refer to in order to teach students reflective symmetry?

It was revealed that regardless of teachers' identification of student's errors, a majority of preservice teachers relied on procedural aspect, such as the activities of doing the reflection, when they instructed students. In this study, preservice teachers were asked to identify one student's errors and respond it, preservice teachers

identified student's errors from the conceptual aspect such as the knowing the properties of reflection (ex. equal distance) more than procedural aspect such as activities of doing reflection (ex. Folding, matching, etc). However, when they instructed students, they focused on procedural aspect more conceptual aspect.

In the fifth question, prospective teachers were asked to find out what Emily's errors are. Thirty-one out of fifty four (56%) prospective teachers identified Emily's errors in terms of knowing the properties of reflection. Ten out of the fifty-four (19%) focused on creating the reflected image. Table 3 shows the specific results.

Type of Identification	Subcategory	Response (N)	Total (%)
Knowing reflection	Diagonal line *	22*	56
	Equal distance	8	
	Distance and angle	1	
Creating reflection	Flipping *	5 *	19
	Matching	1	
	Coordinating	1	
	Inverting	1	
	Rotating	2	
Informal expression		3	6
Misinterpretation		10	19
Total		54	100

Note: * represents the most frequently used subcategory

Table 3. Prospective Teachers' Identification of Emily's error

In the sixth question, prospective teachers were asked to respond to Emily's errors. Twenty-five out of fifty four (46%) prospective teachers instructed Emily referring to knowing the properties of reflective symmetry. Twenty-two out of fifty four (41%) instructed Emily in terms of creating the reflected image. Table 4 shows the specific results.

Type of Strategies	Subcategory	Response (N)	Total (%)
Knowing reflection	Diagonal line*	11*	46
	Equal distance	6	
	Distance and angle	8	
Creating reflection	Flipping	1	41
	Folding *	11*	
	Matching	1	

	Graph paper	2	
	Showing	1	
	Mirror	4	
	Rotating	2	
	Informal expression	1	2
	Misinterpretation	6	11
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	Total	54	100
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Table 4. Prospective Teachers' Pedagogical Strategies

To examine the consistency between prospective teachers' identification to Emily's error and their teaching, pedagogical strategies prospective teachers used were compared with prospective teachers' identification of Emily's errors. It was revealed that frequency of knowing the properties of reflection decreased in pedagogical strategies but frequency of creating the reflected image increased. This shows that even though prospective teachers identified Emily's errors as coming from knowing the properties of reflective symmetry incorrectly, they tried to cope with it by focusing on creating reflection such as folding, flipping, etc.

This study has implications for both teacher educators and researchers. First, this study showed that preservice teachers have limited understanding of reflective symmetry and have tendency to rely on the procedural aspects of reflective symmetry when using teaching strategies. Teacher educators need to put more emphasis on geometry and explore how to help preservice teachers improve their understanding and pedagogical strategies content knowledge of geometry. Second, one of the interesting results is that many preservice teachers confused reflection and rotation and used them in the same situation. When preservice teachers were asked to identify the characteristic of reflection, half of preservice teachers considered the explanation of rotation as that of reflection. However, there is little research why students or preservice teachers confuse reflection and rotation. Future study should specify what difficulties teachers have in teaching reflective symmetry as well as what confused students about the difference between reflection and rotation.

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