This paper presents a study on students' frustration in "bridging" mathematics courses, which they have to take as prerequisites for admission into university programs of their choice. The paper focuses on two possible sources of frustration: (a) students' perceived irrelevance of mathematics for their future studies and professions and (b) their dependence on teachers for the validity of their solutions.

INTRODUCTION

Universities experience an influx of mature students who return to school to continue their studies or re-orient their professions. Candidates are often required to take pre-university level mathematics courses as a condition of their admission ("bridging courses"). These are courses in elementary algebra, functions, and one-variable calculus. Many students experience frustration in these courses (FitzSimons & Godden, 2000: 28). There is a need to know more about the sources of students' frustration.

Two years ago, I embarked on a study aimed at a better understanding of frustration in such prerequisite mathematics courses, using a questionnaire and interviews. In this paper, I will focus on an analysis of two possible sources of the frustration: students' perceived irrelevance of mathematics for their future studies or professions and students' dependence on teachers for the validity of their solutions.

The questionnaire for the study is available at http://alcor.concordia.ca/~sierp/.

RELATIONS WITH RELEVANT LITERATURE

The study is situated in the intersection of two research areas: affect (e.g., Hannula et al., 2004) and adult mathematics education (e.g. Coben, 2000). The population I am most interested in are "mature" students (in my university, this means 21+ years old and having spent some time away from formal education).

In mathematics education research, "frustration" is often mentioned in the context of descriptions of affect in problem solving, but the concept is rarely an object of study in itself. An exception is the work of Handa (2003). In the psychological literature, frustration is defined in various ways, depending on the adopted theory of emotion. According to Mandler (1975: 164-4), frustration is a kind of negative emotion aroused upon encountering an obstacle to satisfying one's needs, goals or expectations, which interrupts the ongoing activity. Mandler refers to previous laboratory research on frustration in claiming that the more alternative strategies are available immediately following the interruption, the greater the chances for relaxation of stress.
This is an important aspect for me: my hypothesis is that technical knowledge of a mathematical method without its theoretical justification, is not sufficient to provide those "alternative strategies" at the time of getting stuck on a problem. Frustration thus remains unresolved and may lead to abandoning the task. Yet, the bridging courses tend to focus on the teaching of rigid techniques: one technique for a given type of problem. Using the terminology of Chevallard's theory of teacher's practice (2002) and in particular, his distinction of "moments of study", these courses merge into one the moments of "first encounter" with a task, the "technical moment" and "institutionalization", and reduce the mathematical practice to its practical-technical bloc with little or no attention to the technological-theoretical bloc.

The perceived irrelevance of mathematics, and dependence on teachers for the validity of solutions, often linked with lack of interest for the validity, are well known "affective variables" in the literature. Perceived relevance of mathematics is known to be an important factor in career choice. For students' dependence on teachers in learning mathematics, researchers blame the traditional classroom culture, which appears to teach students not to be responsible for the mathematical validity of their solutions (e.g., Schoenfeld, 1989; Lampert, 1990; Stodolsky et al., 1991). Lack of interest in validity has been reported by several authors (e.g. Evans, 2000: 179).

THEORETICAL PERSPECTIVE

A model of emotion. There was a time when it was common to reduce the learner's organism to only one system – the cognitive system – and to study its various states. But approaches that integrate cognitive, emotional and sometimes also socio-cultural processes are more frequent today. Comprehensive and dynamic models of the psychology of emotion are needed, which made Scherer's (2000) process component model of emotion attractive for mathematics educators (e.g. Op 't Eynde, 2004). In this model, the functioning of the cognitive system is seen as always interacting with other organismic systems: the arousal of the autonomic nervous system, the expression of the motor system, the action tendencies of the motivational system and the feeling of the monitor system. In periods of emotional calm, the interaction is low. In the process of emotion, the functioning of the systems becomes highly interdependent and interrelated.

A model of adult learning. For the study of affect in adult mathematics education, Evans and Wedege (2004) proposed to adapt Illeris' model of learning (2003): "1) All learning includes two basic processes: an external interaction process between the learner and his or her social, cultural and material environment, and an internal psychological process of acquisition and elaboration; 2. All learning includes three dimensions embedded in a societally situated context: the cognitive dimension of knowledge and skills, the emotional dimension of feelings and motivation and the social dimension of communication and co-operation."

The perspective of didactics: taking into account the institutional context. Scherer’s theory of emotion is an abstract psychological theory that does not study relations between emotions and the situations in which they occur. Illeris’ model reminds us
that in studying emotions in a learning situation from a didactic point of view – which is the point of view I wish to take in my study - the characteristics of the situation must be factored in. That is, the basic unit of study is meant to be not a just a psychological subject, but a didactic situation in which he or she participates. The situation is characterized by certain mathematical and didactic organizations and by the positions of the participants in relation to these organizations (Chevallard, 2002). A student's frustration, when externalized, changes the situation into a very difficult one: communication is interrupted, the learning process is broken, and neither the teacher nor the student can go on in the same way. Different pedagogical and didactic interventions, other learning strategies have to be tried.

The choice of the interventions and learning strategies is constrained by the institution in which the didactic situation takes place. In my study, I treat the mathematics bridging courses as an institution, with its own "action arena" (course outlines, classes, exams, instructors, students) and "exogeneous variables" (e.g. rules of passing the courses) (Ostrom, 2005: 15). They are an institution within an institution (defined, among others, by the university rules of admission into academic programs). Students often blame their frustration on how these two institutions are interconnected. They knock on the door of, say, a school of business, and it turns out that the doorkeeper is a mathematician. Some students consider this rule of the game to be unfair (as we know from interviews with students). How can a mathematician decide if they "belong" to the school of business or not? Also, the content of the courses, they way they are run and students are evaluated, depend a lot on the mathematics department's resources, needs and goals, as well as on the moral commitment of faculty to values that they consider both natural and important (Douglas, 1986). Knowing these resources, needs, goals and values may help to explain students' frustrations and assess the possibilities of reduction of the institutional sources of these frustrations.

THE RESEARCH INSTRUMENT

A questionnaire was designed and sent to about 800 students enrolled in the bridging courses; 96 responses, 63 from mature students and 33 from non-mature students were obtained. Interviews with 6 students were also conducted.

The questionnaire items were inspired by existing instruments (e.g., Haladyna et al., 1983; Schoenfeld, 1989) and experience of teaching the bridging courses. The organizing principle for the choice of items was to cover the respondent's appraisal, action tendencies and feelings (Scherer, 2000), from the positions of Person or member of a society at large, Learner of mathematics, Student in the bridging courses and Client of the university institution. As Person, the respondent was addressed in items about gender, age, time spent away from education and also in the open item about math: "Complete the sentence, 'Math is…". As Learner, the respondent was asked to agree, disagree, or remain neutral to statements about, e.g. the difficulty of mathematics, or learning habits (e.g. "I need the teacher to tell me if I am right or wrong"), and to express preferences with regard to certain mathematical statements,
thus revealing his or her action tendencies in front of certain mathematical tasks. As Student – the respondent is in the position of a subject of the school institution, who has certain obligations vis-à-vis of this institution: doing homework, taking tests. The item, "At university one is expected to be an autonomous learner", or "I did not work hard enough in the course" addressed the respondent as Student. Items such as, "I would rather not take the course if I had a choice", "I will never use most of the material we covered in the course", or "In the course, there was little feedback on my performance" addressed the Client of the university institution, who asks for services, pays for them and has the right to evaluate their quality. The questionnaire included open items (e.g. explain why you like or dislike math, why you think you have succeeded or failed a course, complete the sentence "Math is…"), which left the choice of the position to the respondent.

Responses were analyzed using simple descriptive statistics and the psychological and institutional perspectives outlined above.

RESULTS

The following abbreviations will be used: "ms" for mature student(s), "nms" for non-mature student(s).

Being forced to take a bridging math course and feeling unhappy about it. The respondents were generally "enthusiastic about coming back to school" (84% of all 96 students agreed with this statement in item 9; 95% of 63 ms did so and even 64% of the 33 nms agreed, although this question was not for them, really, since they never left school). Respondents were not so enthusiastic about taking math courses, though.

"Math is extremely discouraging when you are forced to take it as a prerequisite. If I were going for a major in math, then I would understand that the course is necessary. However, in the commerce program, there is nowhere near as much or as difficult math as I have just taken. I also have another year of math prerequisites to take in order to get into the program I want. If I fail math, I don't get into commerce. So I feel math is the only thing that's stopping me from getting into the program I want…. I am currently spending 20 hours of studying math outside of class and I got 30% on my midterm. I'm starting to think that I'm the problem, and that's very discouraging." (Respondent # 39, ms)

About 2/3 of respondents (65% ms and 73% nms) reported taking the course because the academic advisor told them to (item 65) – we can consider these students to have been "forced" to take the course. Over 59% of all (54% ms, 70% nms) also agreed with "I'd rather not take this course if I had a choice" (item 13). Counting students who felt both forced to take the course and unhappy about it, we got 45% all, 40% ms and 55% nms. Mature students thus appeared to be more accepting of the situation than non-mature students. Not liking mathematics did not appear to be the main reason for students' reluctance: only 35% of the 57 students who would rather not take the course expressed their dislike of mathematics (in item 66 "I don't like math" or in completing the sentence, "Math is…", item 76) (38% ms, 30% nms). It was
slightly more likely to be a reason for ms than for nms. Therefore, sources of students' frustration with the bridging courses must be sought elsewhere than in their dislike of mathematics.

**Perceived uselessness of the mathematics courses.** Students expressed their doubts about the relevance of the math courses for them by agreeing with "I'll never use most of the material we covered in this course" (item 64), in their explanations of why they don't like mathematics (item 66) and in completing "Math is..." (item 76). We found that 44% of all students, 38% ms and 55% nms expressed this view at least once. Again, mature students appeared to appreciate mathematics more often than the non-mature students.

While uselessness was invoked in frustration, usefulness of math was not the main reason for being pleased with math. Of the 41 ms and 13 nms who said they liked mathematics, only 6 ms and 2 nms attributed it to some usefulness. One popular reason for liking mathematics was liking to solve problems and experiencing the "awesome feeling" – as one student put it – of finding "the correct answer" (17 students in all, 13 ms and 4 nms).

Unfortunately, it appears that students knew they got "the correct answer" not by checking or testing it themselves, but relying on teachers or books to tell them if they were right or wrong.

**Dependence on teachers for the validity of solutions.** Two thirds of the respondents agreed with "I need the teacher to tell me if I am right or wrong" (65% ms, 70% nms).

The questionnaire also contained items (74 and 75) where two kinds of solutions (labeled "a" and "b") to linear inequalities with absolute value were given and students were asked which one they liked better. Item 74 was about $|2x - 1| < 5$ and item 75 about $|2x - 1| > 5$. Solutions "b" were based on the properties of inequalities with absolute value (e.g. $|x| < t$ iff $-t < x < t$). We will call them "theoretical". Solutions "a", which we will call "procedural", resembled the procedure, commonly used in high schools, where the solution of the inequality is reduced to the solution of two equations. The procedure contains many rules of deriving the solution of the inequality from the solutions of the equations, and students often have trouble remembering them and applying correctly. In item 74, both solutions were correct. In item 75, solution "a" ended with an incorrect result.

We chose these particular problems because, in my experience of teaching the bridging courses, some students displayed a remarkable resistance to adopting the theoretical approach, loudly protesting and arguing for the procedural one, which they "have always used", so "why would they have to forget what they have already learned". Thus, we hoped this question would provoke some stronger feelings in respondents and make them open up their hearts in responding to open items.

There was a very clear preference for the procedural solutions: 69% all, 65% ms and 76% nms chose solution a in item 74; 62% all, 52% ms and 79% nms chose the
incorrect solution \(a\) in item 75. Solution \(b\) in item 74 was chosen by 19\% all, 21\% ms and 15\% nms; in item 75 solution \(b\) was chosen by 20\% all, 24\% ms and 12\% nms. Very few students explained their preference of any type of solution by its being correct; "clearer", "simpler" were the common reasons. In item 75, only 3 ms and 1 nms explicitly remarked that solution \(a\) is incorrect. And even then, the assessment was not based on the fact that the set proposed as the solution did not satisfy the given inequality.

DISCUSSION

Students' perception of the prerequisite mathematics courses as irrelevant for their future studies and professions, and their disregard for mathematical validity are understandable. The courses are focused on techniques: simplification of algebraic expressions, solving equations of various degrees and types, differentiation and integration, solving typical word problems. It may be hard for the students to understand the purpose of achieving mastery in solving problems such as: "Factor completely: \(6a^2 x^3 y + 10 a^3 x^4 y^4 + 14 a x^3 y^5\)" given on a final examination in a course required as a prerequisite for admission into, among others, psychology. It may be impossible for them to assess the validity of their solution as well. In the course, there is no theory of irreducibility of polynomials, so "completely" cannot have the technical meaning of "irreducible". How can the students know when to stop factoring? What if the given expression were: \(2ax^3 y^5 + 6 a^2 x^3 y^3 + 2 a^2 x^4 y^3 + 6 a^3 x^4 y\) ? How would the students know that the factorization \(2 a x^3 y (y^4 + a(3 + x)y^2 + 3a^2 x)\) is not complete and try to reduce it further to \(2 a x^3 y (y^2 + ax) (y^2 + 3a)\) ? The correctness of the answer must be decided by the teacher's verdict and not a mathematical proof. To succeed in the course, students must learn the rules of an institutional game. They cannot figure these rules out otherwise than empirically, by looking at the teacher's reactions to their solutions. That's why they "need the teacher to tell them if they are right or wrong" so badly.

At the same time as agreeing with "I need the teacher to tell me if I am right or wrong", many respondents (70\%) agreed that, at the university, one is expected to be an autonomous learner (item 38). I consider this discrepancy between, on the one hand, their vision of the ideal university student, as well as their probable sense of control over their lives as adults who just made an important decision (returning to study), and, on the other, their lack of personal agency as learners (Bandura, 1989) as a deep source of their frustration with the mathematics courses.

The word "deep" is used to reflect the fact students did not explicitly mention lack of control as a source of frustration. They didn't, because, maybe, this could feel like admitting to the loss of self-esteem, and maintenance of self-esteem is known to be very important for mature students (FitzSimons & Godden, 2000: 19-20). Students could maintain a positive self-esteem by relinquishing control (Sedek et al., 1998) and making the teachers responsible for their learning. If the teacher "cannot explain well" (a frequent complaint in the interviews), he or she is responsible for students' mistakes. This approach to the management of one's emotions could be interpreted as
a shift from the position of Learner (who is responsible for the results of one's
cognitive activity) to the position of a Client, who is indignant when the expected
services are not delivered. Failure is easier to accept if one doesn't see oneself as the
problem, but can blame an institution. As argued above, to some extent, the
institution is to blame, indeed.

FURTHER CONSIDERATIONS

Institutions are difficult to change. They are based not only on conventions and
rational rules of economy, but also on values that are considered "natural", and
therefore impossible to change without destroying the world in which they live
(Douglas, 1986: 46). Any attempt at changing or developing an institution in a certain
desired direction must therefore be based on a thorough understanding of what
constitutes its stable "genetic code" (Ostrom, 2005) and what are the things that can
be changed without jeopardizing its existence.

In the case of the bridging mathematics courses at my university, the problem is to
find if there is a possibility of making room for just enough theory to allow students
to develop some minimal autonomy relative to the validity of their solutions. Without
this autonomy, these courses are, indeed, irrelevant for their future studies, because
knowledge learned this way is not open for further development; it is good only – at
most – for passing a final examination.

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