INVESTIGATING MATHEMATICS LEARNING WITH THE USE OF COMPUTER PROGRAMMES IN PRIMARY SCHOOLS

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The purpose of this paper is to report on the development of a methodology designed to investigate the way in which primary school students learn mathematics with the use of computer interactive programmes. Our work is part of a Mexican project devised to enrich teaching and learning in the classrooms with the use of multimedia resources. Our approach is guided by enactivism, a theory for learning about learning (Reid, 1996) in which multiple perspectives are used and where methods are continuously being refined in the process of doing research. Learning is investigated through the observation of classroom cultures and of students’ mathematical actions. Illustrative examples show how our way of working is helping us in looking at how the computer programmes are being used in the classrooms.

INTRODUCTION

Enciclomedia is a large-scale Mexican project that has been devised with the purpose of enriching primary school teaching and learning by working with computers in the classrooms. An electronic version of the mandatory textbooks that are used in all primary schools in Mexico is being enhanced with links to computer tools designed to help teachers with the teaching of all subjects. As members of the Mathematics group in Enciclomedia, we create resources and strategies which can help teachers and students in their teaching and learning of mathematical concepts. An additional and extremely important part of our work is to investigate how students learn mathematics as they use the computer tools that Enciclomedia provides them with. The purpose of this paper is to report on the development of ways of working that can enable us to characterise the learning of mathematics with Enciclomedia. To begin with, we consider some theoretical ideas about the learning of mathematics; in particular, with the use of computer interactive programmes. We also give a brief description of the interactive programmes that are being used in mathematics lessons. Later, we discuss the way in which the learning of mathematics with these programmes can be investigated. We talk about the approach we have taken in Enciclomedia and the methods we are developing for our project. In addition, we give some examples of the results we have obtained with our investigation so far.

SOME IDEAS ABOUT THE LEARNING OF MATHEMATICS

What do we mean by ‘learning mathematics’? How can we investigate the way in which students learn of mathematics with the use of a computer? Our theoretical ideas about mathematics learning are based on enactivism, a theory of knowing which considers learning as effective or adequate action (Maturana and Varela, 1992). Our minds are seen as ‘embodied’ and cognition as ‘embodied action’. These
ideas of ‘embodiment’ entail two fundamental senses: on the one hand cognition is seen as ‘dependent upon the kinds of experience that come from having a body with various sensorimotor capacities’ and on the other, individual sensorimotor capacities are considered to be ‘themselves embedded in a more encompassing biological and cultural context’ (Varela, 1999, p. 12). The first meaning of embodiment locates cognition in our bodies, and prevents us from thinking about it as an abstract notion that is detached from our everyday experience. The second situates our learning in a wider social and cultural context.

In enactivism learning occurs when individuals interact with each other, changing their behaviour in a similar way. In a particular context or location, the participants create together the conditions that will allow actions to be adequate. Learning outcomes cannot be predetermined or predicted, but the criteria for the adequateness of actions are, specified by teachers and students. As members of a particular community interact with each other, patterns of behaviour are created, constituting what in this paper we call a classroom culture (see Maturana and Varela, 1992).

With these ideas in mind is that we are interested in investigating the learning of mathematics with Enciclomedia by looking at the actions that take place in the classroom and that we consider to be effective.

**Learning mathematics with computer tools**

From an enactivist perspective, the use of computer tools is part of human living experience since ‘such technologies are entwined in the practices used by humans to represent and negotiate cultural experience’ (Davis et al., 2000, p. 170). Tools, as material devices and/or symbolic systems, are considered to be mediators of human activity. They constitute an important part of learning, because their use shapes the processes of knowledge construction and of conceptualization (Rabardel, 1999). When tools are incorporated into students’ activities they become instruments. Instruments are mixed entities that include both tools and the ways these are used. For this reason, instruments are not merely auxiliary components or neutral elements in the teaching of mathematics; they shape students’ actions and therefore they are important components of the learning processes:

> Instruments constitute the means that shape and mediate knowledge and our registers of situations and because of that they exert an influence that can be considerable... they influence the construction of knowledge (ibid, 1999, p. 204)

Every tool generates a space for action, and at the same time it poses on users certain restrictions. This makes possible the emergence of new kinds of actions. In that sense, the use of a tool can contribute in the opening of the space of possible actions for the learner (Rabardel, 1999). The influence that tools exercise on learning is not immediate. Actions are shaped gradually, in a complex process of interaction. Instruments are not given, they do not exist in themselves, and they do not imply a predetermined way of working. Rather, people incorporate tools into their activities and they shape them as they use them (ibid, 1999). Solving mathematical problems with the use of computer programmes is closely related to the tools available, and
students need, on the one hand, adequate actions related to the mathematics involved and, on the other, actions that are effective in relation to the use of the tools themselves. In the classrooms, students construct meanings through their actions which are contextualised in phenomenological experience, that is, in a process of social interaction and with the guide of the teacher (Mariotti, 2001).

The purpose in Enciclomedia, is to develop programmes which can broaden students’ experiences with mathematics. We have developed different kinds of programmes; they vary, for example, in the kinds of interactivity they promote and in the types of problems they pose to the users. So far we have developed programmes related to different mathematical concepts or areas such as fractions, probability, area, perimeter and proportionality. The programmes are closely related to the activities in the students’ textbooks, but they are mostly thought of as spaces for mathematical exploration. They usually provide the users with something they would not get if they used the textbook alone. For example, programmes give the students immediate feedback on their actions on the computer, and they often simulate situations that are difficult to recreate or experience in the classroom, such as large number of occurrences of random events. Many also show different representations of the same concept, such as numeric and graphic, which are linked together in the programme.

The investigation of the way students learn mathematics as they use these programmes in their classrooms is a crucial part of the process of development of the tools themselves and one we are addressing through this work.

SOME IDEAS ABOUT METHODOLOGY

The choice of methods used in our investigation of mathematics learning is also inspired by the enactivist approach. ‘Enactivism, as a methodology [is] a theory for learning about learning’ (Reid, 1996, p. 205). Research is considered to be a way of learning, and therefore researchers are seen as individuals developing their learning in a particular context. From an enactivist perspective, researchers interpret the world in a particular way, influenced by their previous experiences. In addition, in the process of doing research, researchers influence and shape the context in which they are immersed (ibid, p. 206). The interdependence of context and researchers makes the research process a flexible and dynamic one. Research does not occur in a linear fashion; rather, it is seen as a recursive process of asking questions. The work reported in this paper is only the first part of a complex process of interaction and development of ideas. Because of the nature of our work we consider it to be not only research but ‘action research’. We think of our educational initiatives as dynamic suggestions which are under constant modification. The development of the computer programmes in Enciclomedia is an ever-changing process and our work as researchers is also being continuously shaped and modified by our interactions with textbooks, teachers, students and with each other. The methods we have started using to investigate mathematics learning will change in the future according to what we observe in the classrooms and to the feedback we receive from colleagues.
Research questions

We were interested in investigating those activities that we found to be effective as students worked in mathematics problems using Enciclomedia. We wanted to investigate and analyse the way in which the use of the computer interactive programmes contributed in shaping students’ actions, and especially, we wanted to observe the development of actions that could be described as mathematical. In order to do this, we decided to get a sense of the culture of the classrooms and more specifically, to identify mathematical actions that could be observed during the lessons. In what follows we describe in more detail the methods we used, and some preliminary descriptions of what we have observed so far.

INVESTIGATING THE CLASSROOM CULTURE

In order to research the learning of mathematics with Enciclomedia, we contacted a school in Mexico City where we worked with two Year 5 and two Year 6 groups of about 25 students each (aged 11-13). Two of us visited the classrooms at a time and our role was that of participant observers. We helped the teacher in giving general directions on how to use the computer programmes and we walked around the room, making comments or asking questions about students’ work. As we entered the classrooms we contributed in creating certain kinds of classroom cultures – that is, patterns of actions and interactions. When digital technologies are used, these change the way students and teachers interact with each other and therefore particular classroom cultures emerge. Furthermore, the roles of the teacher and of the students change as the culture of the classroom is modified by the use of the programmes.

The classroom cultures we investigated were influenced by the pedagogical approach taken by the national curriculum, which is being followed by the textbooks and by Enciclomedia. In agreement with this, certain activities were explicitly fostered in the classroom while others were discouraged. For example, an attitude of tolerance and respect for others’ opinions was promoted; students were invited to work collaboratively, to ask questions and to participate in discussions. In addition, they were asked to justify their opinions and to work in an orderly fashion.

In order to register the characteristics and the development of the classroom cultures, we carried out detailed observations of students’ actions. We used multiple methods for the collection of data. We used audio recording during the lessons. We recorded whole group discussions as well as interactions that occurred between two or three students and/or between students and teachers or researchers. We also used a video camera, with the purpose of recording, for each lesson the actions of a particular pair of students. So far we have videoed different students on every session.

Additionally, for each lesson, we filled in an observation sheet in which the following aspects of students’ behaviour appear: Active/Passive, Attentive/Inattentive, Working with others/Working individually, Freedom/Constraint, Giving correct answers/Formulating explanations, Understanding/Remembering. These aspects had emerged in a previous study in which they had been helpful in analysing students’
The following examples are taken from the notes written on the observation sheets on several dates. Key words, which are highlighted in the documents, appear in italics:

**Students** are *active* when they work with the programmes. They constantly interact with the programmes and with their peers. They also ask questions and often want to explain or show things to the teacher and researchers.

Many students are eager to participate in whole-group discussions. A few of them are quiet, but all of them are *attentive*. Students get *distracted* when, working with the interactive programme, they cannot solve a problem after many attempts.

*Individual* work seems to be more frequent when students are working with activities from the textbook; when they start exploring the problem with the interactive programme; and when their solutions are giving them unexpected feedback (due to incorrect answers). Students appear to *work in groups* more frequently once they have an understanding of the problem.

The programme (*The Balance*) seems to give students *freedom* to explore with different situations and to experiment with different strategies. The textbook and, at times, the teacher, *restricted* students’ actions.

Most students are looking for *correct answers*. This seems to be reinforced by the teacher who stresses the importance of getting them. The computer programme ‘Perimarea’ gives further emphasis to this approach.

Sometimes students’ explanations include phrases such as ‘that is the way we were taught’ ‘that is how the formula goes’ which indicate *memorisation*. A few students, however, give sophisticated explanations with complex mathematical ideas involved. These explanations are not necessarily correct in a conventional sense.

**Students’ mathematical actions**

When entering the classrooms we were particularly interested in looking at those actions which students performed during the lessons and that could be considered mathematical. With the purpose of identifying these actions, the audio and video tapes we obtained from each lesson were analysed from a different perspective than the one taken when thinking of the classroom culture. In addition, when we observed the lessons, we wrote down, individually, those actions that we thought were mathematical. We used a second observation sheet with the following headings: *Initial mathematical behaviour* (which refers to students’ actions related to mathematics during the whole group introductory discussion at the beginning of the lesson), *Mathematical actions* (those observed during the rest of the lesson, which are related to the mathematical concept(s) in the textbooks’ chapter) and *Other mathematical actions* (they do not explicitly address concepts in that chapter). Particular incidents, where mathematical behaviour is observed, were written at length under each heading. In addition, we have kept records of students’ work with
paper and pencil. Acting mathematically does not necessarily mean, to us, solving a problem in a conventional ‘correct’ manner. We collectively decide on what is mathematical by having discussions in which we talk about our notes, our transcripts from the audio tapes, and about what we observe on the videos. To support our interpretations about mathematical actions, we also read the literature on the teaching and learning of the different areas or mathematical concepts which are being explicitly addressed in each lesson. We use the textbooks to identify these concepts, and to learn about the purpose of the chapters in them. We are working on the development of criteria for identifying mathematical actions, which are not fixed but ever-changing.

For example, the following extract was taken from the notes that were written under Initial mathematical behaviour on the 15/10/2005, which were later contrasted and complemented with the transcripts from the audio tape from the same date:

**T:** What does the word area mean? Use your own words.
**Students:** ‘The centre of the shape’ (S1) ‘The opposite to the perimeter?’ (S2) ‘The part that is not on the edge’ (S3)
**T:** Can you show me? What is the area of the board? (Student touches the central part)
**T:** Anyone else? What is the area of this rectangle? (Student fills in its central part)
**T:** Who can say something different? What does ‘area’ mean?
**S5:** Everything except for the border.

Afterwards, the group worked with the interactive program ‘Perimarea’, where they were supposed to calculate the area for different shapes by counting the squares on a grid that is shown on the screen (see Figure 1).

![Figure 1. Perimarea](image)

We noticed, both during the lesson and on the video from that session, that students were giving the answers by trial and error. They got feedback from the programme; telling them whether they were missing or they had too many square units. By the end of the lesson, it was evident that they had not changed their ideas about the area being the central section of the surface of the shape.

In other lessons, students worked with a programme called ‘The Balance’, comparing fractions and solving problems from the textbook where they had to fill in boxes that represent weights on a scale. The problem in the textbook’s chapter can be reproduced with the programme (Figure 2). Immediate feedback is given, as the programme shows whether the scales are in equilibrium or not. During these lessons, the following mathematical actions were registered (28/11/05):
• Students ask questions such as ‘Why is this heavier than this?’
• Students give explanations about how to equilibrate the balance: ‘fractions have to get smaller’. Other explanations are more sophisticated, for example, a student used graphic representations (of pizzas) on the board to show how a fraction with odd numerator can be divided in two.

• Students compare rational numbers, identifying ‘more or less heavy’ weights.

We noticed that when students first began using The Balance, many also answered with trial and error. However, they gradually refined their strategies and started producing more efficient and systematic methods for obtaining fractions that equilibrated the balance. Using ‘random’ numbers proved to be an inadequate manner of addressing the problem. Their effective behaviour, after a few sessions, was very different from the one we had observed when they used Perimarea.

CONCLUSIONS: SOME REFLECTIONS ON METHODOLOGY AND DIRECTIONS FOR FUTURE RESEARCH

Learning mathematics is a complex process. The introduction of digital technologies in the teaching of mathematics has been considered by some as an answer to the mechanical problems students present when they learn mathematics, allowing for the examination of conceptual understanding. This has proved not to be the case, as tools often introduce different problems and their use generates new sets of questions about student’s learning (Lagrange, et. al., 2001; Laborde, 2004). Investigating the learning of mathematics with the use of computer tools implies addressing the complexity that is intrinsic to learning and devising methods that allow researchers to explore the way in which these tools shape students’ actions. Using multiple perspectives is a feature of the enactivist methodology (Reid, 1996, p. 207) that we have found particularly useful in our investigation. This refers to the exchange of ideas with other researchers and also to the examination and re-examination of different kinds of data. Through the comparison of different events, in different ways, we are able to explain more. Our way of doing research, which is being gradually developed in the practice of creating resources and using them in the classrooms, has allowed us to find differences in the way students’ use different computer programmes. We believe that the careful collection of different types of data, and the discussions we have amongst each other have greatly enriched and strengthened our interpretations. We found that Perimarea, by restricting students’ activities and options for answers, reinforces the students’ tendency to try out responses without giving much reflection to them. The Balance, on the contrary, seemed to invite students to act mathematically, using concepts form the textbooks in a variety of ways. We noticed that students started looking for explanations which could help them interact with their peers and teachers as they talked about their work with the Balance. Patterns of behaviour were shaped;
that is, changes in the classroom cultures could be observed. This is a gradual process, we have been observing these students’ for three months and it is only by detailed observation that we could appreciate the changes in students’ actions.

Our own learning has also been shaped by our work in these classrooms. Our methods have already started being modified. The instruments we are using for data collection are being refined. For example, on observation sheet 2, which we use for noting down mathematical actions, we now want to include a heading in which we specify mathematical actions observed when a student works with a particular computer programme. This with the idea of analysing, in more detail, the way in which students interact with the tools and how they become instruments in that process of interaction. Are students’ mathematical actions with the programmes different from the ones they carry out when they are not working with the computer? How do students interact with the programmes we have not yet investigated? How can we modify programmes like Perimarea so that they invite students to be more reflective? These are the questions guiding us after the first phase of our investigation of the learning of mathematics with the computer programmes we are developing.

References


