This paper presents some preliminary results of a larger study that investigates the relationship between students’ conceptual understanding of fractions, students’ motivation and their social context (teachers’ practices in the mathematics classroom and students’ socio-economic status). Data were collected from 302 sixth grade students through a questionnaire comprised three Likert-type scales measuring motives, goals and interest, and a test measuring students’ understanding of fractions. A hypothesized model connecting students’ understanding of fractions and different motivational constructs was next tested. Findings revealed that students’ understanding of fractions and their interest in mathematics were influenced by their fear of failure, their self-efficacy beliefs, and their mastery goals.

BACKGROUND AND AIM OF THE STUDY

The relationship between students’ achievement and affect has recently attracted increased interest on the part of mathematics educators (see e.g., Hannula & Pehkonen, 2004; Breen, 2004). In a sense it was a response to educational psychologists who have investigated factors that promote and undermine affective constructs like students’ motivation and beliefs (Pintrich, 1993). Current studies on the relationship between motivation and achievement tend to highlight the multidimensional and situational nature of the construct of motivation. According to this contextual perspective, the efficacy of motivational determinants to predict the performance and achievement of individuals may vary according to culture, the contexts they are called on to act (mathematics or language), their personal characteristics etc. (Buffard & Couture, 2003).

Four basic theories of social-cognitive constructs regarding student motivation have so far been identified: achievement goal orientation, self-efficacy, personal interest in the task, and task value beliefs (Pintrich, 1993). In this study we conceptualise motivation according to achievement goal theory, arguably one of the dominant theories in the field of motivation today (Zusho et al., 2005). Achievement goal theory was developed within a social-cognitive framework and focuses on students’ purpose of task engagement and how this goal orientation influences the way students approach, engage and respond to achievement situations (Elliot & Church, 1997).

Two particular goals have recently been emphasized in the literature, namely mastery goals that focus on learning and understanding, and performance goals that focus on the demonstration of competence. Recently, there has been a theoretical and empirical distinction between performance-approach goals, where students focus on how to outperform others, and performance-avoidance goals, where students aim to avoid looking inferior or incompetent in relation to others (Elliot & Church, 1997).
Goal orientation research suggests that a mastery goal orientation is associated with positive achievement beliefs that lead to adaptive educational outcomes. More specifically, the limited research related to students’ mathematics achievement revealed that the adoption of mastery goals was associated with a positive pattern of engagement that included challenge seeking and persistence in the face of difficulty (Kaplan et al., 2002). Moreover mastery goals were positively related to students’ ability in problem solving strategies and to students’ achievement in mathematics test (Kaplan et al., 2002). Contrary to the adaptive nature of mastery goals, performance-avoidance goals were found to be associated with test anxiety, low achievement and avoidance of help seeking in mathematics classroom (Elliot, 1999; Kaplan et al., 2002). The findings regarding performance-approach goals are mixed and show both positive and negative effects. This motivational orientation was related to self-efficacy, positive attitudes towards the task and positive relations between performance-approach goals and grades (Elliot, 1999). Other studies however found that the adoption of performance-approach goals was positively related to maladaptive outcomes such as experiencing negative affect in response to a difficulty and challenge, using low-level learning strategies, and attributing failure to low ability (Kaplan et al., 2002).

Recently there is also an increased emphasis into the antecedents of these three achievement goals. Particularly the hierarchical model of motivation developed by Elliot & Church (1997) argues that the three achievement goals appear to mediate the relation between achievement motives; in particular the success approach motive (need for success, self-efficacy) and the motive to avoid failure (fear of failure), and select achievement and motivational outcomes. More specifically, Elliot & Church (1997) found that the need for success was associated with the adoption of both mastery goals and performance-approach goals, while the fear of failure was linked to both performance-approach and performance-avoidance goals. These goals were differentially related to academic outcomes; the mastery goals predicted students’ interest, while performance-approach goals were related to actual performance (Elliot & Church, 1997).

Although there are numerous studies investigating the relationships between achievement goals and specific motivational constructs or achievement (Kaplan et al., 2002), relatively few studies have tried to test causal models that combine students’ achievement motives, their goal orientations and actual achievement as well as their personal interest in mathematics. Most importantly, very few studies in this research area refer to primary school students in the context of mathematics teaching and learning.

In this respect the purpose of the study was:

- To test the validity of the measures for the six factors: fear of failure, self-efficacy, mastery goals, performance-approach goals, performance-avoidance goals, and interest, in a different social context.
• To examine the relationships between students’ achievement motives (fear of failure, self-efficacy), achievement goals (mastery, performance approach and performance avoidance goals) and outcomes (students’ interest in mathematics and mathematics achievement).

• To test a causal model that examines the relationships between students’ achievement motives, achievement goals and achievement outcomes.

METHOD
Participants were 302 sixth grade students, 137 males and 165 females from 16 intact classes of an economically homogeneous school district. All participants completed a questionnaire comprised of three scales measuring: a) achievement motives (fear of failure and self-efficacy), b) achievement goals (mastery, performance-approach and performance-avoidance), and c) outcomes (interest). Specifically, the questionnaire comprised of 35 Likert-type 5-point items (1- indicating strong disagreement and 5 strong agreement). The five items measuring Self-efficacy were adopted from the Patterns of Adaptive Learning Scales (PALS) (Midgley et al., 2000); a specimen item was “I’m certain I can master the skills taught in mathematics this year”. Students’ fear of failure was assessed using nine items adopted from the Herman’s fear of failure measure (Elliot & Church, 1997); a specimen item was “I often avoid a task because I am afraid that I will make mistakes”. The five-item subscale measuring mastery goals, as well as the five-item measuring performance goals and the four-item measuring performance-avoidance were adopted from PALS; respective specimen items in each of the three subscales were, “one of my goals in mathematics is to learn as much as I can” (Mastery goal), “one of my goals is to show other students that I’m good at mathematics” (Performance-approach goal), and “It’s important to me that I don’t look stupid in mathematics class” (Performance-avoidance goal). Finally, we used Elliot & Church (1997) seven-item scale to measure students’ interest in achievement tasks; a specimen item was, “I found mathematics interesting”. These 35 items were randomly spread through out the questionnaire, to avoid the formation of possible reaction patterns.

To investigate the relationship between the above motivational and social factors and the students’ understanding of fractions a three-dimensional test was also administered, each dimension corresponding to each of the levels of conceptual understanding- interiorization, condensation and reification- proposed by Sfard (1991). The tasks comprising the test were adopted from published research and specifically concerned the measurement of students’ understanding of fraction as part of a whole, as measurement, equivalent fractions, fraction comparison (Hannula, 2003; Lamon, 1999) and addition of fractions with common and non common denominators (Lamon, 1999). We developed tasks corresponding to each of Sfards’ conceptual levels. In this paper, however, we report only on the relation among measures of students’ motives, goals and social factors, and achievement.
FINDINGS

With respect to the first aim, the students’ responses were subjected to exploratory factor analysis, which resulted in a six-factor solution, explaining 54.80% of the total variance. All loadings were high and statistically significant, ranging from .45 to .86. The six factors corresponded to students’ achievement motives, goals and outcomes as were described in the questionnaire, with one exemption: the fear of failure factor was split in two parts. This finding supports the construct validity of the questionnaire used to collect data on pupils’ motives, goals and outcomes. Factor scores for each dimension were estimated by calculating the average of the items that comprised each factor. Table 1 presents the mean scores, standard deviations, and Cronbach’s alpha coefficients for each of the six factors.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Mean</th>
<th>SD</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest</td>
<td>3.85</td>
<td>.89</td>
<td>.89</td>
</tr>
<tr>
<td>Mastery goals</td>
<td>4.52</td>
<td>.46</td>
<td>.71</td>
</tr>
<tr>
<td>Performance approach goals</td>
<td>3.08</td>
<td>.93</td>
<td>.80</td>
</tr>
<tr>
<td>Performance avoidance goals</td>
<td>2.85</td>
<td>.93</td>
<td>.51</td>
</tr>
<tr>
<td>Fear of failure</td>
<td>2.20</td>
<td>.78</td>
<td>.66</td>
</tr>
<tr>
<td>Self efficacy</td>
<td>4.09</td>
<td>.62</td>
<td>.71</td>
</tr>
</tbody>
</table>

Table 1: Means, Standard Deviations and Cronbach’s alpha coefficients of the six factors identified by exploratory factor analysis.

The Cronbach’s alphas were found quite high (ranging from .66 to .89) for five of the factors, while alpha was low for the factor performance-avoidance goals. The latter result might be partially attributed to the fact that the factor comprised of only four items and partially to cultural difference between USA, where the scale was developed and Cypriot students. Specifically, one of the traditional trends in Cypriot schools provides that most students attend private coaching institutions, or pay for private coaching at home, particularly whenever they believe that they run the risk to fail. So, instead of avoiding a subject, they would most probably try to find ways (i.e. private mathematical lessons) in order to approach a task efficiently.

Table 2 presents the correlations between the variables. Mastery goals were positively correlated with self-efficacy (.467) and with both outcomes, strongly with interest (.470) and less strongly with achievement (.180). On the other hand it was found to be negatively associated with the fear of failure (-.358). Performance-approach goals were not related to fear of failure but they were positively related to self-efficacy (.208). The fear of failure motive was also negatively related to interest (-.440) and to students’ achievement (-.278) while it was also negatively related to self-efficacy (-.358). Lastly, the achievement motive self-efficacy was also positively related to achievement (.208).
Table 2: Correlations for the Variables.
* Correlation is significant at the 0.05 level. **Correlation is significant at the 0.01 level.

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Fear of failure</td>
<td>-</td>
<td>-.421**</td>
<td>-.358**</td>
<td>.018</td>
<td>-.440**</td>
<td>-.278**</td>
</tr>
<tr>
<td>2. Self-efficacy</td>
<td>-</td>
<td>-</td>
<td>.467**</td>
<td>.208**</td>
<td>.470**</td>
<td>.208**</td>
</tr>
<tr>
<td>3. Mastery goals</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.106</td>
<td>.470**</td>
<td>.180**</td>
</tr>
<tr>
<td>4. Performance goals</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.171**</td>
<td>-.095</td>
</tr>
<tr>
<td>5. Interest</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.135*</td>
</tr>
<tr>
<td>6. Achievement</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

In order to examine the antecedents of achievement goals and the consequences of the adoption of achievement goals, multiple regression analysis was performed. The regression of mastery goals on the antecedents self-efficacy and fear of failure revealed significant main effect (F = 49.755, p < .001) for both self-efficacy, $\beta = .384$ p < .001 and fear of failure, $\beta = -.197$, p < .001. Specifically, students who felt high fear of failure were more likely not to adopt mastery goals. Students who believed that they could master mathematics if they tried hard (high self-efficacy) were more likely to adopt mastery goals. Regressing performance goals on the antecedents (self-efficacy and fear of failure) revealed moderate effect for fear of failure $\beta = .128$ and p < .05 and significant effect for self-efficacy $\beta = .262$ and p < .001. Students who felt high fear of failure were more likely to adopt performance-approach goals. That is their focus was on the demonstration of competence relatively to others. Students who had high self-efficacy beliefs they were also likely to adopt performance goals.

The regression of interest on achievement goals was also significant F = 46.047, p < .001. Particularly the regression of interest on mastery goals revealed significant main effect, $\beta = .457$, p < .001, while the regression of interest on performance-approach goals revealed modest significance ($\beta = .123$ and p < .05). The adoption of mastery goals, that is focusing on the development of competence and task mastery led to advanced interest in mathematics. The same results revealed moderate effect for performance-approach goals. Students who focused on the demonstration of competence relatively to others, showed an interest in mathematics.

The regression of mathematics achievement on achievement goals revealed moderate effect on both goals (F = 7.128, p = .001) with mastery goals to have more effect ($\beta = .192$ and p = .001) than performance-approach goals ($\beta = -.115$ and p < .05). Students whose focus was on the development of their competence and task mastery were more likely to achieve higher conceptual understanding of fractions than students who held low mastery goals. Students, whose focus was on the demonstration of competence relatively to others, were more likely to have lower conceptual understanding of fractions than students who held low performance goals.

Structural equation modelling was also applied to test the relationships between students’ achievement motives, achievement goals and achievement outcomes using EQS (Hu & Bentler, 1999). Particularly, the causal model suggested by theory and
practice claims that the relation of motives (self-efficacy and fear of failure) to outcomes (interest and achievement) is mediated by the achievement goals. Three types of fit indices were used to assess the overall fit of the model: the chi-square statistic, the comparative fit index (CFI), and the root mean of square error of approximation (RMSEA). The chi square index provides an asymptotically valid significance test of model fit. The CFI estimates the relative fit of the target model in comparison to a baseline model where all of the variable in the model are uncorrelated (Hu & Bentler, 1999). The values of the CFI range from 0 to 1, with values greater than .95 indicating an acceptable model fit. Finally, the RMSEA is an index that takes the model complexity into account. An RMSEA of .05 or less is considered to be as acceptable fit (Hu & Bentler, 1999). As reflected by the iterative summary, the goodness of fit statistics showed that the data did not fit the model well ($\chi^2 = 71.64$, df = 6, p < .000; CFI = .791 and RMSEA = .19).

Subsequent model tests revealed that the model fit indices could be improved by adding paths joining directly students’ fear of failure to interest and achievement and students’ self-efficacy to interest. The model that emerged after these modifications had a very good fit to the data ($\chi^2 = 3.66$, df = 3, p > .30; CFI = .998 and RMSEA = .027). Figure 1 shows the model that emerged, as well as the path coefficients among the six factors. The following observations arise from Figure 1. Students who felt high fear of failure had low interest in mathematics and low mathematics achievement. Students that held high self-efficacy that is, the students who believed that they could master mathematics if they tried hard enough, they had high interest in mathematics. Mastery goals were based on fear of failure ($-.197$) and were strongly based on self-efficacy (.384). Performance-approach goals were moderately based on fear of failure (.128) and self-efficacy (.262). Students who felt high fear of failure had high performance goals.

Interest was predicted directly by mastery goals (.266), by performance-approach goals (.102), by self-efficacy (.218) and by fear of failure (-.255). The mathematics achievement was predicted directly by mastery goals (.105), by performance goals (-.102) and fear of failure (-.239). It can be claimed from the model in figure 1 that
mastery and performance-approach goals served as mediators of the direct relationship between self-efficacy and mathematics achievement.

DISCUSSION

The present study is within the framework of the ongoing discussion about the relationship between students’ motivation and achievement. Factor analysis in conjunction to other studies (Elliot & Church, 1997; Zusho et. al., 2005) did not support the trichotomous: approach-avoidance-achievement goal conceptualization. Specifically, the data did not support the distinction in three different achievement goals (mastery, performance-approach and performance-avoidance goals). This may be partially due to cultural differences between environments, or to variable samples’ age; in the present study participants were just above 11 years of age, while in other studies the samples consisted of college students. Another possible cause of this phenomenon may be the limited number of the items that measured the performance avoidance goals.

The strongest predictor of students’ achievement and their interest in mathematics was students’ fear of failure. The model shows that fear of failure had a direct effect on students’ achievement and to their interest in mathematics, and indirect effect on both variables via mastery and performance goals. The negative effect that the fear of failure, or fear for mathematics had on students’ achievement is stressed in many studies (Breen, 2004; Elliot & Church, 1997). However, unlike the results of this study, in Elliot’s & Church’s study (1997) fear of failure appeared in the causal model to have only an indirect effect on students’ achievement and interest.

Consistent with previous research findings, (Elliot & Church, 1997; Zusho et al., 2005) path analysis investigating the antecedents of each of the two goal orientations revealed that mastery goals were predicted by self-efficacy and performance goals were predicted by both, fear of failure and self-efficacy.

Path analysis investigating the consequences of achievement goals adoption for the outcomes, interest in mathematics and achievement in mathematics revealed that performance-approach goals facilitated interest in mathematics but proved to have negative effect on students’ achievement in mathematics. Elliot & Church (1997) and Zusho et. al., (2005) found in their studies that performance-approach goals had positive influence both on interest and on achievement. In addition, the data of this study revealed that mastery goals facilitated both students’ interest and mathematics while Zusho et. al., (2005) in their study found that mastery goals facilitated only students’ interest.

Beta values were poorer in this model than the models in the other studies (Elliot & Church, 1997; Zusho et al., 2005). One explanation is what Hannula & Pehkonen (2004) support that the casual relationship between achievement and affective constructs is problematic and varies according to students’ age. Moreover, it may well be that other factors like teacher’s practices and the students’ socio-economic
status have a stronger impact on the different motivation and performance outcomes that students in this age adopt, factors that this study will investigate further.

References


