TOWARDS THE DEVELOPMENT OF
A SELF-REGULATED MATHEMATICAL PROBLEM SOLVING
MODEL

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Various models of Mathematical Problem Solving (MPS) have been suggested since 1957; however most of them did not take place within a structure of a theory. In this paper we focus on the theory of Self-Regulated Learning (SRL) in order to develop a satisfactorily comprehensive model of MPS, especially adjusted to the principles of the aforementioned theory. After examining the relevant literature on both theories of MPS and SRL, a first draft of the model was developed. Two research studies were conducted in order to check and validate the mapping of the model so as to become a useful tool for upper primary school students while they are working on process mathematical problems. The final version of the model is expected to constitute a powerful tool for independent and student-guided problem solving.

THEORETICAL BACKGROUND

The theory of Self-Regulated Learning (SRL) encourages debate about imperative changes in current teaching practices (Boekaerts, 1997), stressing the necessity for the transmission of responsibility of learning from teacher to student by providing tools for independent learning with which children can take charge of their own learning and seek after lifelong learning (Tanner & Jones, 2003). Mathematical Problem Solving (MPS), as one of the most valuable aspects of math lessons (Bruder, Komorek, & Schmitz, 2005) and as one of the most difficult tasks primary school students have to deal with (Verschaffel, De Corte, Lasure, Vaerenbergh, Bogaerts, & Ratinckx, 1999), appears to be a challenging area for MPS and SRL researchers. Since the 1980s their main concern has been to conduct studies aiming at improving students’ self-regulation skills in MPS; however, most of the experiments in MPS were not closely related to a specific theoretical perspective on self-regulation (De Corte, Verschaffel, & Op’t Eynde, 2000) but aimed at improving certain aspects of self-regulation, such as metacognition. There is a strong need to suggest an integral model of SRL adjusted to MPS procedures. Therefore, the aim of this paper is to gather and discuss the relevant literature in order to suggest an integral and comprehensive model of SRL adjusted to primary “process” MPS procedures and to report on research to validate the model. A “process” problem is the one that can be solved using more than one strategy or “process” (LeBlanc, 1982) and thus it is considered as even more difficult for young children.
Models of Self-Regulated Learning

Various models of SRL were studied and compared in terms of their suitability to be applied to the MPS procedures. Some of these were the model of cyclical phases of self-regulation (Zimmerman, 2004), the six component model of SRL (Boekaerts, 1997), the model of cognitive, metacognitive and resource management strategies (Pintrich, 1999), and the four-phased model of SRL suggested by Winne and Hadwin (1998; in Winne & Perry, 2000). The models were compared according to four parameters that were determined as crucial to the process of MPS. The first parameter was the visual aspect of the model which was explained in terms of hierarchy and cycling. Hierarchy is tantamount to the order the solver proceeds through while trying to solve a mathematical problem; for instance, first reading the text of the problem and then obtaining an answer. Cycling can be interpreted as going through the procedure from the beginning, by rereading the text, checking for understanding and so on. The second parameter has been the incorporation of the, crucial to MPS, strategy-use aspect (Posamentier & Krulik, 1998). By strategy-use is meant not only the MPS strategies (e.g. finding a pattern) but also the use of SRL strategies (e.g. distinguish relevant from irrelevant data). The third parameter that was set was whether the model was taking into account the theory of motivational beliefs, since it is suggested that it promotes the use of self-regulated learning strategies (Pintrich, 1999; Marcou & Philippou, 2005). The fourth parameter was whether the model presents SRL as an aptitude or event (Winne & Perry, 2000). Aptitude SRL entails the ability to demonstrate SRL behaviour in various domains whereas event SRL develops only during one particular event (Winne & Perry, 2000). In the present quest, SRL will be studied as an event, since its application will be restricted in one domain, namely process mathematical problems and not learning in general.

The models that appeared to satisfy most the parameters set for SRL models were the ones proposed by Zimmerman (2004) and Pintrich (1999). Zimmerman (2004) describes children’s development of academic self-regulation from a social-cognitive perspective proposing that students’ academic effectiveness depends on their use of key SRL strategies and their beliefs about the effectiveness of those processes. This procedure is happening in three cyclical phases: forethought, performance and self-reflection. Zimmerman’s model includes hierarchical and cyclical structure, it incorporates motivational beliefs, views SRL as an event, and it includes strategic aspects. A key point of Pintrich’s (1999) theory is the use of SRL strategies, cognitive, metacognitive and resource management. Cognitive learning strategies are the rehearsal, elaboration, and organisational strategies (Weinstein & Mayer, 1986; in Pintrich, 1999). Some examples are the recitation of information (rehearsal), explaining of ideas to a fellow student (elaboration) and selecting, outlining and organizing the main ideas using a network (organisational). Self-regulation of cognition is considered in many studies (e.g. Panaoura & Philippou, 2003; Tanner & Jones, 2003) as a basic dimension of metacognition; thus Pintrich names these as metacognitive strategies and are used for planning, monitoring, and regulation of cognition. Examples of metacognitive strategies are skimming a text and generating
questions before the actual reading of it in order to activate any relevant prior knowledge (planning), self-questioning to check understanding and to inform whether or not a goal is being achieved (monitoring), going back and rereading a piece of complicated text, and reviewing aspects of one’s work (regulating) in order to “…bring the behaviour back in line with the goal” (Pintrich, 1999; p.461). Finally, the resource management strategies are the time and study environment control strategies, the effort regulation strategies, peer learning and help seeking. For example, peer learning implies the students’ willingness to collaborate with their peers to reach inside of what they cannot attain on their own and help seeking enables students to identify when they are not able to proceed further and so find the appropriate source of assistance (e.g. teachers, peers).

Models of Mathematical Problem Solving

A procedure of studying and comparing the models of MPS (see Table 1) was also followed. Some of the models were the well-known four-step model of Polya (1957), the three-stage problem solving strategy suggested by Schoenfeld (1985), the four-stage-cognitive regulation strategy for MPS of Lester, Garofalo and Kroll (1989; cited in De Corte et al., 2000), and the five-step cognitive self-regulatory strategy of Verschaffel et al (1999). Given that the new model is to apply to primary age children, the parameters set for the comparison were the number of stages, the terminology of each stage of the problem, the social context and MPS content (e.g. types of problems) in which the model of MPS was implemented. After the comparison, it appeared that a combination of Shoenfeld’s (1985) and Polya’s (1957) model would fit better the proposed model.

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Table 1: Models of Mathematical Problem Solving

Towards the development of a self-regulated MPS model

The proposed model emerged after comparing the models of both theories of SRL and MPS and selecting not their “best” aspects but the most suitable for the purposes of this study. Many of those aspects were combined in order to construct a model, as comprehensive as possible, for primary school students, and applicable to process...
mathematical problems, typically most difficult for Cyprus primary teachers and students. The following figure summarizes what has been argued so far.

**Figure 1: MPS as a cyclical SRL event (based on Zimmerman, 2004)**

SRL strategies are the cognitive, metacognitive and resource management strategies suggested by Pintrich (1999), whereas MPS strategies are the strategies that can be applied on MPS per se, such as making a drawing, intelligent guessing and testing, finding a pattern, and working backwards (Posamentier & Krulik, 1998). The next step was to try to delve more deeply into each phase by describing the actual SRL strategies that can be used within each phase.

**THE FIRST STUDY**

The first study, carried out in the UK, was designed to check the mapping of our model and to allocate the SRL strategies in each of the three phases of the model. Five students, one of year 4, two of year 5 and two of year 6 were given in written form a set of three process problems. Children decided to work as a group and were asked to solve at least two of the three problems by writing down details of their work and thinking aloud. The researcher’s role was restricted to observing the students without interfering. The session lasted for 40 minutes and was audio-taped.

After transcribing and analysing the audio-taped session, the results confirmed the cyclical and hierarchical structure of the model for both problems. The analysis contributed also to the allocation of each strategy in each phase. For example, highlighting or underlining key-words was observed in the Reading and analysing the text phase, whereas the reviewing aspects of their work was observed in the Looking back phase. The strategies that were not observed, such as time and study environment control, were excluded from the model. After the mapping of the model was revisited, there was a need to conduct a second study in order to evaluate the new structure of the model and to estimate its value as a tool, used by both teachers and students during classroom practices.
THE SECOND STUDY

A primary school in Cyprus was selected to participate in this study. Five teachers of year 4, 5 and 6 were taught about the SRL theory and the new model and asked to proceed to a teaching intervention in order to implement the theory and the model in real classroom settings. More specifically, the aims were to receive feedback from the teachers about the efficiency of the model as tool of teaching MPS, to investigate the impact of the model on students’ behaviour and to validate the structure of the model.

The teaching intervention was implemented in at least three lessons within two months, according to which the regulation of the learning process is gradually passed from the teacher to the students. Two or three students from each class worked in mixed ability groups on process problems for about 30 minutes, before and after the teaching intervention. Since young children find it difficult to express their thoughts about their cognitive and metacognitive ability (Panaoura & Philippou, 2003), clinical interviews were conducted with students, so as to observe the use of SRL strategies as these appear naturally within the context of MPS. The interviews were video-taped so as to seek for and detect any possible changes in students’ behaviour. The researcher, as clinical interviewer, presented the problem, modified questions if the child seemed to misunderstand them and challenged answers to test the strength of the students’ conviction. These were achieved mostly by asking questions like “how did you do that”, “can you do it loud”, “how did you figure that out”, “can you show me how you did it”, and “how do you know”.

Results

Teachers stated that the model can indeed be a powerful tool for both teachers and students in MPS, since it “puts an order to students’ thinking and to teachers’ teaching”. However, they recommended that some of the strategies could be combined to one strategy (e.g. distinguish relevant from irrelevant data with finding key-words) and that the one-way arrows should be replaced by two-way arrows since children can oscillate between phases while working on problems. In order to check the impact of the model on students’ behaviour, the behaviour of each group of students was examined by producing time-line graphs, a procedure similar to the one used by Schoenfeld (1985).

![Figure 2: Time spent in each phase of the model before the teaching intervention](image-url)
Figures 2 and 3 demonstrate the time that was spent in each phase of the model for one group of students of year 4 before and after the intervention, while working on isomorphic problems.

As can be observed from the above figures, students in the pre-teaching phase tend to spend more time in the Performance phase and no time to the Self-reflection phase, contrary to their behaviour during the post-teaching phase in which they spend more time in the Forethought and Self-reflection phases. In other words, students spend more time to read, analyze and understand the text of the problem and on verification processes in order to review and correct their work. Furthermore, as can be seen on Figure 2, students, after reading the text, decided to follow a certain approach and then stuck to it without trying to change it. However, after the teaching intervention, as shown on Figure 3, students were oscillating between analysing the text and performance, indicating their effort to find the most suitable approach to tackle the problem. Another interesting result was that students, after the teaching intervention, appeared to have developed a SRL language while talking to each other. Excerpts from their talking, such as “I am not sure if this is correct…I think we should leave out the irrelevant information” and “we need to check it…to go back” demonstrate that the model had an impact on students’ language. Taking into consideration the teachers’ suggestions and the analyses of the clinical interviews, the structure of the model was critically revisited and evaluated. For reasons of space the full model will be shown at the research presentation.

**DISCUSSION**

The impact and efficiency of the model can not be clearly decided after being implemented for the short period of three lessons within two months. Although there were some very positive indications of its suitability in the second study, there is a need to investigate the impact of the model when this is being implemented in teaching for a longer period of time, perhaps for a whole school year. In this case, it will be possible to examine its impact on students’ ability in MPS, as well as on students’ motivational beliefs. As mentioned before, primary students’ use of SRL strategies was found to be positively related to their motivational beliefs concerning MPS (Marcou & Philippou, 2005). This implies that teaching students how to
effectively use the SRL strategies while tackling mathematical problems can have an impact on their self-efficacy, task value and goal orientation beliefs (Pintrich, 1999). A new research study aiming to investigate the impact of the revised model, when it is implemented for a longer period of time by more experimental classes, is now being designed. The results will shed further light on whether teaching MPS according to the revised model can help primary school students become self-regulated problem solvers.

References


Marcou & Lerman
