

# AN INVESTIGATION OF BEGINNING ALGEBRA STUDENTS' ABILITY TO GENERALIZE LINEAR PATTERNS

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This is a qualitative study of 22 9<sup>th</sup> graders in a public school in California who were asked to perform generalizations on a task involving linear patterns. Our research questions were: What enables/hinders students' abilities to generalize a linear pattern? What strategies do successful students use to develop an explicit generalization? How do students make use of visual and numerical cues in developing a generalization? Do students use different representations equally? Can students connect different representations of a pattern with fluency?

Twenty-three different strategies were identified falling into three types, numerical, figural, and pragmatic, based on students' exhibited strategies, understanding of variables, and representational fluency. Some of the more common numerical strategies include the following: use of finite differences in a table; random or systematic trial and error; or use of finite differences to generalize to a closed formula. Some of the more common visual strategies identified were the following: visual grouping manifesting either a multiplicative or an additive relationship; use of visual symmetry such as seeing concentric or polygonal relationships; visual finite differences; and figural proportioning.

This study is consistent with findings from an earlier study we conducted with preservice elementary teachers (Rivera & Becker, 2003) as well as work done by Küchemann (1981) and Stacey & Macgregor (2000). Overall, students' strategies appeared to be predominantly numerical. In this study we identify three types of generalization based on similarity: numerical; figural; and pragmatic, in accord with findings by Gentner (1989) in which children were shown to exhibit different similarity strategies when making inductions involving everyday objects. Students who use numerical generalization employ trial and error as a similarity strategy with no sense of what the coefficients in the linear pattern represent. The variables are used merely as placeholders with no meaning except as a generator for linear sequences of numbers, with lack of representational fluency. Students who use figural generalization employ perceptual similarity strategies in which the focus is on relationships among numbers in the linear sequence. Variables are seen as not only placeholders but within the context of a functional relationship. Students who use pragmatic generalization employ both numerical and figural strategies and are representationally fluent; that is, they see sequences of numbers as consisting of both properties and relationships. We see that figural generalizers tend to be pragmatic eventually. Finally, students who fail to generalize tend to start out with numerical strategies and lack the flexibility to try other approaches.