TECHNICAL SCHOOL STUDENTS´ CONCEPTIONS OF TANGENT LINES

Valéria Guimarães Moreira, Márcia Maria Fusaro Pinto

Universidade Federal de Minas Gerais. Brazil.

This paper reports ongoing research investigating how students´ experiences with the notion of tangent line in different lecture courses at a technical school are being integrated to the related mathematical concept. Focusing on a technical course case study, we examine how aspects of the notion of tangent line are related to features of the context in which they are being produced. In addition, we discuss whether it is appropriate, from the perspective of situated learning, to account for practices of school mathematics in the various lecture courses in the curricula as distinct school mathematics practices, or, distinct communities of practices of (school) mathematics.

INTRODUCTION

Our study investigates aspects of the mathematical notion of tangent line which emerge from practices in vocational classes at a technical school. Research questions emerge during the development of a two years research project involving four mathematics undergraduate students as researchers[1]. Supported by Vinner (1991) and Tall and Vinner (1981), we partially reproduce research already conducted by those authors, though referring to students in their first course on Calculus in our own country. The notions of *concept image* as the whole cognitive structure associated to a mathematical notion and *concept definition* as the form of words used to designate a mathematical concept oriented our data collection and analysis. From our results, we became specially interested in those related with the notion of tangent line. Having attended a Calculus lecture where the lecturer presented the mathematical notion as the limit position of secants to a curve at a point, interviewees were invited to draw a tangent line to a curve. The students responded giving explanations involving movement that had occurred in their physics lectures. Their procedures included 'adjusting a circular arc at the point', instead of perceiving the curve locally straight as we generally suggest in our Calculus course. In addition, they spoke freely about the 'centre of a curve at a point' when referring to the centre of the adjusted circular arc.

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These evoked images remind us of mathematical notions which are met, but only, later in advanced mathematics, which we never took into account when teaching the first year Calculus course.

Being aware of students' technical school backgrounds and examining some of their technical design papers, we conjectured that those students' earlier images related to curves and tangent lines may not have been made explicit by their teachers or by any instructional material. Similar responses occurred in many students, suggest that they could have been shared in a technical design activity. It appears for us that an investigation of students' evoked concept image and learning should account for students' practices and a learned curriculum other than to be restricted to the one taught by the teachers or explicit in instructional materials. In adittion, the shared images also highlight a social dimension in their practices. Concerned with students' learning in those terms, we refer to Lave and Wenger's theoretical framework in situated learning and suggested by it, we discuss the practices of school mathematics.

Our interest in this study is twofold. On the one hand, we become acquainted with aspects of students´ notion of tangent line we are not aware of, and which we in general leave untouched in our mathematics lectures. On the other hand, we reflect on our current educational reform-based curriculum for technical schools, examining if it has improved in supporting the role of school mathematics in the vital part of the curriculum – the specific technical courses.

The research as a whole is concerned with school mathematics practices (as we are perceiving them) in regular secondary school courses such as mathematics, physics and technical design, and in specific technical courses in Mechanics, Electronics and Highway Systems, when directly or implicitly approaching the concept of tangent line during the development of the course.

In this paper, we attempt to clarify what we understand as diversity in school mathematics practices. We also present partial results from data analysis, collected through classroom observations and interviews with groups of students from a Highway System technical course case study. Results indicate that students seems to reaffirm their prospective professional culture through school mathematics practices which are close to those already observed and described in workplaces.

RESEARCH FRAMEWORK

Lave and Wenger developed their theory focusing on practices out-of-school. They see learning as developing in practices and as part of a process where people´s identities are developed in participating in communities of practice (Lave and Wenger, 1991). Researchers do not consider as unproblematic a recontextualization of such a theory in formal institutions. With such aim, Wimbourne and Watson (1998) characterize communities of practices expressing their beliefs that many of their features could be re-signified in schools. These refer to developing an identity, supported by a social structure of the practice, with a common purpose and shared ways of behaving, language, habits, values, and tool-use. They consider less obvious that in school classrooms the participants will constitute the practice, given that in most mathematics lessons 'the teacher is not engaged in learning mathematics' (p.94) and also that all participants would see themselves engaged in the same activity, 'because pupils' participation is often passive' (p.94). Adler (1998) expresses her view that the learning of mathematics at school is a specific practice; that mathematics is learned through the language in use in classroom, and also the learning of mathematics would include acquiring, recognizing and developing specific ways of using language. We agreed to guide our investigation into school mathematics practices in different courses on the following features: lecture course goals in approaching mathematics; classroom practices, including the role played by the teacher and students during the activities; didatical materials, from which students may incorporate aspects from the concepts even if they are not made explicit by the teacher and students during the lessons; the mathematical language in use, meaning the mathematical symbolic language and representations evoked by students and the teacher. In general, these are built during the activity and highlight behaviour, language, habits, values and shared tools, which we understand as part of developing an identity.

METHODOLOGY

The whole study is a qualitative research taking place at a technical (secondary) school from March to August, 2003. Methods of data collection are non-participant observation of eight regular course classrooms, analysis of students´written responses to a questionnaire handed by the researcher at the end of each course classroom observation, and semi-structured interviews with eight groups of six students from each regular course, selected on basis of their responses to the questionnaire.

The eight regular course classrooms were previously selected and field notes were taken when activities referred to the notion of tangent line. Some activities were recorded in video. Our questionnaire partially reproduced a previous research instrument (Vinner, 1991) and results are interesting but will not be detailed in this paper. We interviewed groups of students as an attempt to account for students' shared images and to relate them with the context where they were apparently being produced.

The case study we report in this paper focuses on a 4 hours a week course classroom Project, at the beginning of the second year of the Highway System course. 19 students are attending the course. All of them are in their final year of secondary school or had just graduated. Video recording and field notes were taken during 4 weeks observation.

THE HIGHWAY SYSTEM CASE STUDY

The Project course syllabus consists of the development of an activity of plotting roads on topographic maps. Plotting procedures were learned in previous courses. In this same academic term, students are also implementing a project in placement and locating a proper road. In order to discuss school mathematics learning and practices in the Project course, data analysis presentation is organised by relating our observations to *classroom practices*, *didactical materials*, *mathematical language in use*. *Goals in approaching mathematics* in this context are left for a last section in the paper.

Classroom practices

The course activities run in a special class, with individual work desks for students, facing the blackboard and the teacher's desk. The teacher conducts the activity giving initial instructions and discussing procedures he believes as necessary to implement the activity. He does not stay in classroom during the activities, which does not interfere in the students´performance as a whole. When in doubt, students consult each other, constantly discussing the task, which could be nearly described as a collaborative work. They often meet in small groups around some student´s work desk, looking for an agreement, or for explanations from those who had already concluded the activity, or part of it. Mathematics mainly refers to calculations of coordinates to locate the cambers on the road. Checking results may involve the whole class in a discussion. In general, they attempt to get a common result, adjusting their calculations whithin an admissable error.

Didactical materials

A topographic map, pencil, rule, compass, scientific calculator, no text book or other written intructions; a table with data and specific measurements for plotting the cambers, constructed as course work in a previous course.

The mathematical language in use

The teacher explains the procedures to plot the road on the blackboard, paying special attention to the cambers of the roads. At the beginning of his speech, he takes for granted students' awareness of the notion of tangent line. Interrupting his own explanation, he addresses the whole class as follows:

> You all know what a tangent line is, don't you? We have a circle [he draws a circle on the blackboard while speaking], or, it doesn't really matter if it's a circle or another circular picture, okay? Any curved picture. It could be a circle, it could a spiral, it could be any curved picture, she would know better how to speak what a curved picture is [he refers to the researcher, facing her], it could be an ellipse. If you have a line which touches this picture [he draws a tangent line to the circle he had drawn on the blackboard] at a single point, only a single point, we say this line is tangential to the curved picture, isn´t it? And in case of roads, what we are going to do is to take these lines and *adjust them with* a curved picture, in this case a circle, or an arc of a circle …

Notice that images evoked by the teacher relates the notion of tangent line to the notion of tangent line to a circle. He also emphasizes the idea of a tangent line as a line which 'touches the picture at a single point'. He makes an attempt to discuss the concept generically when observing that 'it doesn´t really matter if it´s a circle'; though turning it back specific, when referring to a curve as any 'circular picture'. We may also suggest his unawareness of the mathematical discourse when referring to 'circular pictures or curved pictures'. He definitely contextualizes his discourse and his definition within his own practice when naming the procedure of plotting the tangents by using a technical jargon such as 'adjusting them [the tangent line] with' the curve.

Students' evoked images are observed during the interview. Victor, Carla, Laura, Linda, Daniel and Hilton were handed the questionnaire they had previously responded. They were invited to comment on its first question, which asks

> Write down what a tangent line is.

Follows the conversation:

> Victor: Mine [tangent line or definition of tangent line] are straight lines that once stretched crosses at a single point.
>
> Interviewer: Sorry?
>
> Victor: They are stretched straight lines, umm, that crosses at a single point, making an angle which will be used to define the kinds of cambers which will be plotted in a project.

Notice that Victor is not intimidated by the interviewer's implicit comment on his answer nor by the interviewer's mathematical background he knows as a mathematician. He continues, supporting his concept definition on procedures arising from his technical practice. In respect, devising tangents before defining the curve could be thought as an interesting aspect of his evoked images, reminding us of other mathematical notions such as integral curves.

Victor continues his explanation responding to the interviewer, who insists that he clarify his thoughts. Asking for other students' help, Victor is assisted by Carla and Laura

Carla: Oh, like this. I believe like this, in the context of, like, geometry, in actual mathematics, it is just the one [the straight line] which touches a single point at a curve, at a circle and everything. In the case of plotting roads they would be, like, it is straight lines, it is, it is like it is not in the field, it is like imaginaries. They would be the procedures you develop, they cross each other and they are also called tangents these procedures that you carry out referring to, in topography, when plotting roads, we use them as tangents.

Laura: Yes, then you go there and plot the points, carry out the procedures and then they cross each other and they are considered tangent, for us, you know, in the case of our course.

Both Carla and Laura suggest a distinction between the 'actual mathematics' learned in mathematics classroom and the mathematics reconstructed with their practices, referred to by Laura as (the mathematics) 'in the case of our course'. Carla also suggests a distinction between the 'actual mathematics' and her technical practice observing that, in practice, tangents are 'imaginaries'. We conjecture that, for her, this is not the case in geometry or in 'actual mathematics'. This could suggest a distinction between practising mathematics and using mathematics in another practice that poses a mathematical instrument as imaginary (or abstract?) when dealing with applications, but not when practising the actual mathematics. Linda joins the conversation, being assertive about her mathematical meaning for the word tangent.

Linda: What I´d learnt about this term in my seventh grade is that it [the tangent] touches at a unique point of the circle, isn't it? This is what I´d learnt there, since mine …

Victor: Yes, tangent is that.

Victor had no choice other than agreeing with Linda's mathematical notion. The interviewers' intervention provokes a collective construction of a *concept definition* of tangent, situated in the students' practice of locating highways

Interviewer: Is that the same tangent, when you are plotting highways?

Linda: In the case of, of…

Victor: … plotting …

Carla: …cambers, in the case of whom would be the tangents …

All interviewees at once: …the external tangents …

Daniel: …tangent in such different way would be, as I say, a straight line, isn´t it?

Linda: Yes, they would be straight lines, isn´t it? Straight lines which crosses each other.

In turn, Linda had no choice other than accounting for the notion of tangent line as reconstructed by the group.

DISCUSSION

The episode presented above and students' comments on curves and tangent lines suggested our research questions share common aspects. Other than making explicit well-known images, such as a *tangent touches a curve at a unique point*, they indicate that a context of technical design practices may reinforces such ideas, supported by the notion of tangent to a circle as a special case. Or, rephrasing, it seems that such images naturally emerge from procedures of students' technical design practices, through identifying (arcs of) curves and arcs of circle. In our case study, this fact is already implicit in students' and teacher´s speech. Ideas may not be matching with ours, Calculus teachers, when we present curves to students as intuitively locally straight (and we might be aware of it); but the mentioned procedures naturally bring to bear other mathematical notions, such as centre of curvature or 'given the tangents, determine the curve', that may enrich discussions in our classrooms.

From our analysis of classroom practices we argue that both teacher and student voices appear as legitimated during the whole process. In fact, a correct procedure is bounded by general agreements among the students, in a similar fashion as described by researchers examining the use of mathematics in work place (see Magajna, 1998). Both students and the teacher are constituting the practice, all of them engaged in the same activity, suggesting that classroom practices could attend to those features of communities of practice considered by Wimbourne and Watson (1998) as less obvious in school instituitions. In this case, supported by the researchers' comments, these aspects certainly distinguish the school mathematics practices in the Project course from many other practices in mathematics classrooms.

Language used in the classroom gives clear indication that both students and teacher re-signify mathematical meaning with their practice, modifying the mathematical discourse. From Adler (1998), observation of a distinction between the learning of a language and acquisition of a language, we suggest that our students are acquiring language, in the structure of their practice. In fact, they seem to be 'borrowing' a concept name from the mathematics practice and collectively transforming its meaning to indicate its use in a technical procedure. Therefore, goals in approaching mathematics in this technical classroom seems very different from those in our mathematics classrooms. In this sense, we may think of a diversity of school mathematics practices in school, in particular, in a technical school.

And why does it matter at all? We may well assume a perspective, as students and workers seems to do, that there is an actual mathematics practice which is different from their practice, meaning that what they practice, is not mathematics practice. Sometimes, they apply some mathematics, learned at school. But if we simulate thinking on their practice as re-structuring school mathematics practice, or, as a distinct community of practice of mathematics, we may rethink our vocational and services courses and perceive them as much more complex than a simple context for modeling or applying mathematics. Maybe we are far from understanding what happens with the implementation of our mathematics in other practices.

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