FRACTIONS IN ADULT'S ELEMENTARY SCHOOL: THE CASE OF LUCINA

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Abstract. *The case of Lucina must be placed at a level of a partial advance achieved in the development of an exhaustive qualitative research recently conclude in a night elementary school of Mexico City, an institution that provides young people and adults with academic formation. The research problem herein discussed is the enrichment of semantic and conceptual contents of fractions in relation to the solution of arithmetic problems that allow the reconstruction of previous vital experiences of the subjects. The case of Lucina is analyzed in light of the application of an exploring questionnaire that preceded to realization of two didactical interview applied to a 41-year-old woman attending to sixth grade of elementary school. The didactical interviews were the main source of information in the study reported herein.*

Introduction

The United Nations (1997) has recognized that one of the most important spaces today is education for adults and the research derived from the same, which have been put aside in the international setting for multiple reasons. In Mexico, more that one million young and adult persons receive elementary instruction in different institutions belonging to the State, which are integrated to the programmed system and to the public open teaching system.

In order to efficiently assist this important field, it is require to investigate the main contents of mathematical education developed in such setting and the learning they propitiate. The use of fractions and the treatment of meanings and concepts herein discussed constitute one of the main milestones of such arithmetical instruction.

Theoretical Framework

Messina (1993), García Carrasco (1997), Jóia (1997), Mariño (1997), Soto (1997) among other researchers recognize that general knowledge and mathematical knowledge to be acquired in school by young people and adults in an elementary formation process must begin as of the accumulation of experiences and knowledge achieved in different social settings, that its, in the other vital spaces such people evolve. This coincides with what Gimeno and Pérez (2000) have characterized as a "critical reconstruction of reality", by approaching to the situations proposed through several teaching treatments.

In this way, young people and adults have a symbolic field loaded with sense for global learning and for the specific follow-up of fractions through posing and solving problems which, from the educative institution, reconstruct scenes of real family, work and community life (ratifying what was previously posed in Valdemoros, 2000). Also, as a reference to the last source, it is worth saying that while natural numbers facilitate varied non-school learning within several cultural spaces, fractions are accompanied with a more limited repertoire of "previous knowledge" (we adopt an expression widely spread by researchers as Mariño, 1997).

We consider that semantic contents, notions and concepts referred to fractions present a great diversity in the field of school solution of arithmetic problems. Therefore, the meanings of **measure, intuitive quotient** (related to sharing situations), **multiplicative operator** and **ratio** are recovered in this study, according to Kieren (1984, 1985, 1988), who attributes several applications of fraction numbers in a concrete field. The meanings of the **part-whole relationship** and the **unit** notion are also retaken since such researcher considers them as fundamental supports of the semantic contents previously indicated. Regarding the **multiplicative operator**, it has mainly been granted with effect of **fracturing operator** according to the contributions made by Freudenthal (1983) and Streefland (1993), attributing it the role of such fraction that is related to the dynamic partitioning of a whole.

We also recovered from Piaget, Inhelder and Szeminska (1966) the acknowledgement of the **part-whole and part-part relationships**, which constitute the structural bases of the fraction concept, which are, respectively, support for the addition and multiplication of fractions.

Reasearch Problem

Based on the aforesaid, we assume a congnoscible focus that privileges the active role markedly carried out by **young and adult people when constructing meanings, notions and concepts of fraction in the field of arithmetic solution problems that recover relevant experiences of life**, being the latter the **research problem** discussed in this study. After having identified such research problem, we formulated the following **question**:

What type of arithmetic tasks can favor, in better conditions, the construction of several meanings, notions and concepts of fraction in young and adult people?

Facing such research question, we explicit the **hypothesis** that it is **in the solution of arithmetic problems where *work*, *community* and *family experiences* are involved by young and adult persons when they are enriched and effectively construct new meanings, notions and concepts related to fractions.**

Method

Methodological Instruments. The questionnaire and the "didactical interview" (under the foundations given on the same in Valdemoros, 1998) were included in the study.

1. Initial Questionnaire. It was **exploratory** and was submitted to the qualitative analysis of its results. It was applied in order to be able to choose in better conditions the subjects of the case study, as to have general information regarding all the young and adult persons integrating the school groups with whom the research was carried out. The problems or tasks comprised in the questionnaire presented a brief text carefully complemented by certain drawings that allowed as a whole to achieve "a reconstruction of several aspects of their life experiences."

This instrument was composed by eight tasks, where the **meanings of the fraction** as **quotient resulting from concrete sharing situations, measure, ratio** and **multiplicative operator** were involved. The **part-whole relationship** was investigated in more specific way through tasks that required its differentiation from the **part-part relationship**. The **unit** was preferably explored around the discrimination of **continuous and discrete wholes**, as well as through the reconstruction task of a discrete whole from the initial identification of one part. Young and adult persons were asked to complete, translate and interpret different information expressed through several representation channels (texts, arithmetic-technical notations, geometric figures, drawings of different nature); they were also asked to carry out diverse partitioning tasks, the recognition of equivalence relationships and the realization of elementary additions and subtractions of fractions.

2. Didactical Interviews. Every subject to be interviewed was chosen for his/her performance in the questionnaire. Two different interviews were carried out to five young and adult persons. These interviews were individual and semi-structured: each of the subjects was submitted to tasks of common design (similar to those included in the questionnaire), so the interview results could be compared to the results obtained in the five cases.

The didactical nature of the interviews was determined by the succession of two moments differentiated in their realization: a) an initial exploring phase, where we tried to determine the advance of each subject by their own means and, b) a final didactical-constructivist phase where the interviewer tried to promote in the interviewed the overcoming of the cognitive difficulties that arose in his/her previous evolution feeding him/her back, but never proposing solutions or obstructing new searches looked for by the subject. The passage from the first to the second moment of the interview was assessed in each situation and depended on having exhausted all the reasonable possibilities of the initial exploration. The interview constituted the main instrument in the development of the case study.

Subjects. The students who solved the exploratory questionnaire were 17 young and adult persons incorporated to fourth, fifth and sixth grades of elementary school, with ages between 14 and 70 years old with different working activities (construction workers, domestic workers, housewives, ambulatory marketers, artisans, etc.). From the five cases submitted to study, we only present the case of Lucina, a 41-year-old domestic worker incorporated to sixth grade of night school and who is near to conclude her elementary education. We chose Lucina because she had started her studies when she was a child, leaving it inconcluded since then and recently retaking it and advancing a little more than a half of her elementary studies in the primary school for adults, where she has evidenced a good evolution in the learning of new knowledge and in the update of the previous knowledge.

The research site. The investigation was developed in a night public elementary school of Mexico City, located in a peripheral neighborhood inhabited by workers from different rural zones of the country. The school is integrated by small groups of

teenagers and adults who correspond to the six grades established in all educational institutions of this type belonging to the public school teaching system.

Qualitative Validation Procedures. Regarding the questionnaire, "cross controls" were applied between two observers since it was an ideal resource for such instrument and did not create resistance in Lucina. In the interviews, in virtue that they were made simultaneously (with the intervention of two researchers and two interviewed adults), it was decided to "triangulate" different processes of solution displayed by the selected woman before analogue arithmetic problems.

Analysis of the Results Obtained in Lucina's Case

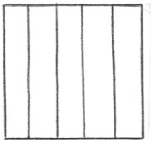
We privileged the follow-up of the representation modes adopted by the chosen woman, as well as the cognitive difficulties detected in the domain of some meanings, the semantic contents that presented a systematic aspect in Lucina and the type of arguments with which she justified the respective processes in problem solving.

- **Results Registered in the Initial Exploring Questionnaire**

Globally, in the realization of the **questionnaire** she evidenced a correct use of fractions, appealing to notations expressed in technical language whenever she was asked to give the solution of the corresponding task in such way.

In an analogue way, Lucina exhibited a careful domain of the equal partition in those arithmetic problems where she had to make the sub-division of a collection in sub-collections, or, the sub-division of a figure given in a pre-established number of parts. However, in the partition problem presented in the questionnaire she was able to adequately carry out the equal partition of the continuous whole, but could not correctly identify the fraction she was required, as it is shown in Figure 1.

A father gives to his five sons and daughters a piece of land like the following:



Please mark in the previous drawing how can it be distributed in such a way that each of the five sons and daughters receive the same. Write the corresponding fraction:

Thus, each son/daughter has ___5/5___ **of the whole piece of land.**

Figure 1. Lucina carried out a careful equal partition, but did not indicated properly the fraction of the whole that corresponded to each of the beneficiaries in the partition of the land.

Regarding the direct recognition of the equivalence relationships between fractions, she evidenced clear difficulties in the management of the same, maybe she made an ambiguous interpretation of the task presented. Figure 2 exemplifies it through the

solution given by Lucina to a simple problem of the initial exploring questionnaire, where she only pointed out one of the subjects involved in the equivalence relationship, but omitted the other subject compromised in such relationship, with which the recognition of the equivalence was not given.

The artisan is covering with red enamel the shadowed parts of these metallic shields:

Shield A Shield B Shiled C

Use arrows to link the following columns:

Shield A	2/4 painted in red
Shield B	4/8 painted in red
Shield C	3/8 painted in red

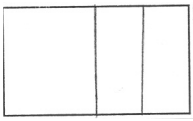
In which shields does the artisan uses the same amount of red paint?
___Shield C___

Figure 2. Lucina solved in an incomplete way an elemental problem of equivalence among fractions in the initial exploring questionnaire.

In general, Lucina expressed proper interpretations of the continuous whole and of the discrete whole. It was notorious that in several tasks she showed her need to highlight the global conformation of the whole in the figure included to the problem. Regardless of what we just observe, Lucina was not able to identify the fractions correctly in a recognition task of the part-part relationships as exposed in Figure 3.

Presumably, this took place because the previous learning had excluded that type of elaborations in school work.

Jose has a rectangular piece of land where he planted flowers in ½ of it. He planted roses in a ¼ of such half.



Indicate in the drawing the part where he planted flowers and the part he dedicated to roses.

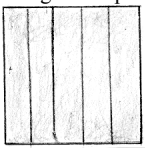
What part of all the piece of land did Jose dedicated to roses? ___3/4___three quarters___

Figure 3. The task of the questionnaire where Lucina was not able to identify properly the part-part relationship.

- **Global Performance of Lucina in the Interviews**

I. During the **first interview** some of the activities of the questionnaire were reconstructed; in order to do so, the interviewer intervene promoting new elaborations and feedbacking the answers as it has been describe in the **Method**. We illustrate the aforesaid with Lucina's re-questionings regarding the familiar piece of land among five brothers (see Figure 4).

A father gives to his five sons and daughters a piece of land like the following:



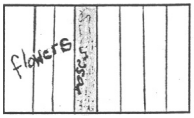
In the previous drawing, how can the piece of land be distributed in order that all five sons and daughters receive the same.
Write the corresponding fraction:
Thus, each son/daughter has ___1/5___ **of the whole piece of land.**

Figure 4. In the first interview, Lucina confirmed the equal partition of the initial questionnaire rectifying the fraction related to each beneficiary of the partition.

In the same interview, other activities of the initial questionnaire were reconstructed appealing to simplifications resources of the tasks causing many difficulties. We exemplify this with Lucina's reconstructions included in Figure 5.

Figure 5 and its contrast to Figure 3 allow the appreciation of the advances that Lucina achieved in the setting of the tasks involving the part-whole and part-part relationship with the subsequent identification of the fraction.

Jose has a rectangular piece of land where he planted flowers in ½ of it. He planted roses in a ¼ of such half.



Indicate in the drawing the part where he planted flowers and the part he dedicated to roses.
What part of the piece of land did Jose dedicated to roses?
_____1/4_____

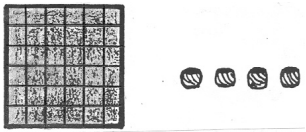
Figure 5. A posteriori of several re-elaboration processes, Lucina re-posed the part-part relationship during the first interview.

Likewise, during the first interview and when reconstructing the task already presented in Figure 2, the interviewed properly recognized the equivalence relationship among the fractions, which is pointed explicitly and in relation to **"shields A and C".**

II. In the **second interview** new arithmetic problems were introduced, where Lucina showed an efficient management of the **fraction as a measure.**

In those situation involving simple proportional variations, the interviewed woman privileged the use of the **fracturing operators**, of the **"dividing by half"** type, which consideration allowed her to fundament the corresponding solution. Figure 6 shows an example of what we just expressed.

Mrs. Martinez knits cloths with crochet needles. For the following cloth, she needed 4 hanks of thread.



She has been asked to make a cloth which size is half the area of the previous cloth. How many hanks does she need to knit it? 2 hanks of thread.

Explain how did you solve it. Thinking that if for the first cloth she used 4 hanks and is going to make another that measures half of it, then it is two hanks of thread.

Figure 6. The interviewed supported her process in the consideration of the fracturing operator when solving a very simple situation of proportional variation.

Before this situation and paying attention to the circumstance that other adults evidenced similar solution processes, we assume that it will be advisable to introduce the elemental didactical treatment of proportions as of a wide use of fracturing operators as initial tools highly loaded with sense.

Conclusions

The case presented has allowed to confirm the hypothesis sustained at the beginning of the study, that is, it **is in the resolution of arithmetical problems where working, community and family experiences are re-posed by adults when they are enriched and effectively construct new meanings, notions and concepts related to fractions.**

In the beginning, through the questionnaire, it was confirmed that the domain of basic semantic contents by Lucina was the key to carry out properly the solution process of the arithmetic problems she was presented to, while some of her cognitive difficulties were detected regarding the part-whole and part-part relationship. The "didactical" interviews gave wide and clear evidences of construction and reconstruction of meanings, notions and concepts of fractions when solving problems that favored the mathematical re-elaborations of her life experiences.

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