

THE FUNCTIONS OF PICTURES IN PROBLEM SOLVING

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In the present study, we assert that pictures serve four functions in problem solving: decorative, representational, organizational and informational. We, therefore, investigate the effects of pictures based on their functions in mathematical problem solving (MPS), by high achievement students of Grade 6 in Cyprus, in a communication setting. A number of tasks were developed and techniques of observation and interviews were conducted for gathering qualitative data from eight students. All kinds of pictures, except the decorative one, were found to be conducive to MPS and the communication process. Findings also suggest that the use of pictures in successful MPS depends on the relationship between the picture and the task (function of picture), and on students' mental abilities.

THEORETICAL BACKGROUND

Bruner (1961) supports that learning proceeds through three levels: the enactive, the iconic and the symbolic. In other words, pictures function as a mediator between the practical and the theoretical formal level of understanding. In the field of mathematics learning and instruction, pictures play an important role as an aid for supporting reflection and as a means in communicating mathematical ideas. Many researchers consider imagistic representations as a fundamental cognitive system for mathematical learning and problem solving (DeWindt-King, & Goldin, 2003), while expert mathematicians as well as mathematics students perceive visual representations as a useful tool in MPS and frequently attempt to use them (Stylianou, 2001).

The process of visualization is considered to be indispensable in mathematics learning and more specifically in MPS. The important role of visualization in MPS is stressed by the findings of recent studies, which consider the ability to form images of mathematical relationships as a necessary presupposition for effective MPS (Brown, & Wheatly, 1997). In the context of MPS, visualization refers to the understanding of the problem with the construction and/or use of a diagram or a picture to help obtain a solution (Bishop, 1989). Researchers argue that the solution of a problem may be accomplished by using either visual representations, or analytic thought processes, or both. The analytic method involves cognitive handling of objects and procedures with or without the use of symbols (Zazkis, Dubinsky, & Dautermann, 1996).

The views supporting the use of visual aids or pictorial representations in mathematics do not diminish the importance of the verbal code of communication in the context of learning tasks or the effect of language on the formulation of thinking, communication and “doing mathematics” (Kieran, 2001). Language and pictures are

considered as two distinct kinds of representation and communication of mathematical ideas, with fundamental differences in regard with their informational content, structure and usability (Schnotz, 2002), which complement each other.

Thomas, & Yoon Hong (2001) suggest that students can interact with a representation, including a pictorial one, in two ways, by observing it or by acting on it. In the present study, we consider that the active use of pictures in the process of MPS may lead students to internal conflicts, which can be meaningful and beneficial in the solution effort. These conflicts appear when the text of the mathematical problem leads to a different solution procedure than the one derived from the use of the picture associated with the problem. However, the use of pictures, images or diagrams may have negative effects due to the obstacles they can create (Bishop, 1989). These kinds of difficulties may be caused by factors, such as the selection and highlighting of some aspects of the picture at the expense of others, the emphasis on irrelevant details and the inappropriateness of spatial arrangement, which may give rise to misunderstanding (Colin, Chauvet, & Viennot, 2002).

Based on Carney and Levin's (2002) proposed functions that pictures serve in a text, this study used a similar categorization of pictures in order to examine the role of each type of pictures in students' performance, in MPS. This study proposes four functions (categories) of pictures in MPS: (a) decorative, (b) representational, (c) organizational, and (d) informational. Decorative pictures do not give any actual information concerning the solution of the problem. Representational pictures represent the whole or a part of the content of the problem, while organizational pictures provide directions for drawing or written work that support the solution procedure. Finally, informational pictures provide information that is essential for the solution of the problem; in other words, the problem is based on the picture.

What is new in this study is the investigation of the relationship between different types of pictorial representations and MPS. Specifically, the purpose of the study was to explore the role of pictures based on their function, in MPS by students of Grade 6, in the context of an experimental model of communication. Three research questions were accordingly formulated: First, which is the effect of each category of pictures on students' MPS performance? Second, which strategies do students use to solve a problem accompanied by each category of pictures? Third, which is the effect of each category of pictures on the communication between students during MPS?

METHOD

The study was qualitative in nature and employed an interaction process, which included both oral and written responses to tasks, based on the communication model introduced by Weber-Kubler (1981). Eight students of Grade 6, presenting high performance in mathematics, participated in the study. Four problems corresponding to the four functions of pictures, mentioned above, were used. The subjects were separated in couples and the administration procedure was repeated four times, according to the communication model shown in Figure 1 below.

The communication model allowed for the observation of the way students work and interact, and helped us determine whether students solve and communicate each problem with or without the use of the picture accompanying it. Specifically, the researcher initially described verbally the text of the problem and the relevant picture to Student 1, who tried to solve the problem. When he finished, he was given the written form of the problem along with the relevant picture and allowed to modify his previous solution or solve the problem again from the beginning, if necessary. Next to this, Student 1 described the particular problem to Student 2, playing the role of the researcher and the previous procedure was repeated, as shown in Figure 1.

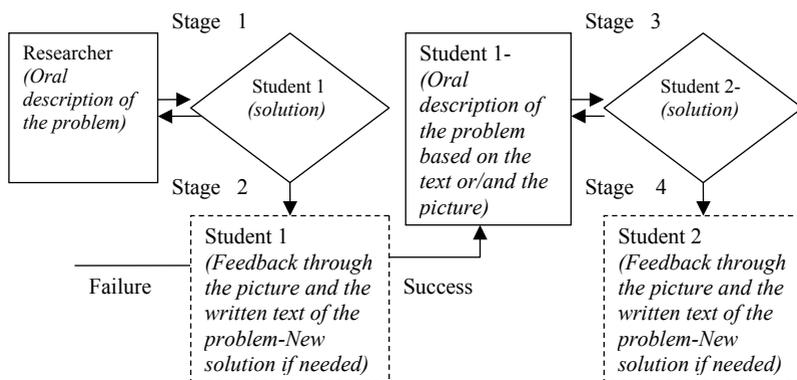


Figure 1: The procedure of the communication model

During the communication process, students could use paper and pencil in order to present their solution procedures, in written form, before and after the use of the picture. They were asked to explain and justify their solution strategies and determine whether the picture was useful for solving the problem, and if so, in what way. Each experiment lasted nearly 80 minutes for each couple and was recorded.

The tasks, which were based on mathematical problems used in recent studies or publications, are the following:

1. *Mrs Brown put her students into groups of 5, with 3 girls in each group. If Mrs Brown has 25 children in her class, how many boys and how many girls does she have?* (Misailidou, 2003)

The picture, which accompanied the problem, was decorative, and just represented a boy and a girl.

2. *Find the weights of the three items: a tetrahedron, a sphere and a cube, based on the information shown in the picture.* (Olson, 1998)

The picture (informational) consisted of all the information of the problem. It represented different combinations of the items on three scales: on the first scale there

were a tetrahedron, a cube and a sphere and the indication was 23 kg, on the second scale there were two cubes and two spheres and the indication was 22 kg, and on the third scale there were two tetrahedrons and a sphere and the indication was 28 kg.

3. *A man planted a tree at each of the two ends of a straight path. He then planted a tree every 2 m along the path. The length of the path is 10 m. How many trees were planted at the path altogether?* (Booth, & Thomas, 2000)

The picture of this problem had a representational role. It represented a path with two trees, one at each end of the path.

4. *Last week 10 campers camped at the mountain. Each day 8 loaves of bread were available for them to eat. This week 15 campers camped at the mountain. How many loaves are there available for them for the day?* (Misailidou, 2003)

The picture had an organizational role for the problem. A horizontal line separated the data concerning the campers and the loaves last week and the corresponding data this week. Above the line, on the left, there were 10 campers arranged in three rows (4 campers in the first and 4 in the second row and 2 campers in the third row) and, on the right, there were 8 loaves in three rows (3 loaves in the first row, 3 in the second and 2 in the third row). Below the line, on the left, there were 15 campers in four rows (4 campers in each of the first three rows and 3 campers in the fourth row) and on the right there was a question mark.

As regards the analysis of the data, the following criteria were taken into account for determining the effect of the pictures on MPS: i) whether students used the accompanied picture by acting on it or by observing it in order to solve the problem, ii) whether they confronted internal conflicts by using the picture, and iii) which method/s (visual, analytic or both) they used for the solution of each problem. In regard with the communication process, it was examined whether students, who articulated the problem to their partners, used elements of the picture in their verbal description.

RESULTS

The effect of the decorative picture on MPS was found to be insignificant, since it did not provide any information or feedback for the solution of the problem. All the students overlooked the use of the particular picture in their solution procedures. In particular, six students used analytic methods, whereas two students applied visualization strategies, by drawing their own pictures or diagrams, which seemed to be more helpful than the decorative picture. For example, S8 constructed five rectangles representing the groups, and wrote inside each one of them the numbers 3 and 2, referring to the girls and the boys, respectively. The following statement made by this student indicates the trivial role of the decorative picture: *“The picture of the problem didn’t help me at all. My drawing helped me more. By drawing the groups, it was easier for me to find the number of boys and girls”*.

On the contrary, the effect of the informational picture was fundamental in problem solving, since it consisted of all the data needed for the solution of the problem and, therefore, was used by all the students. The most common method for the solution of this problem was the trial and error strategy, which was employed by seven students. This strategy involved the active use of the picture as a representational aid, which assisted students to organize their thinking, for example, by noting down next to each item a number indicating its weight. Thus, the use of the picture did not just provide the data of the problem, but it also supported the formation of conflicts between the number combinations proposed by the students, and the correct answers. The only student (S6) who applied a systematic method to find a solution for the problem, by making use of the picture, stated: *"It (the picture) helped me find that one cube and one sphere weigh 11 kg. At the first scale the cube and the sphere weigh 11 kg, so the tetrahedron weighs 12 kg. As for the third scale, since the tetrahedron is 12 kg, 12 plus 12 equal 24, so 4 kg are left, and that's the sphere."* S2 was the only student who used, almost exclusively, the analytic method to resolve the particular problem. By listening to the description of the problem, she wrote down only the indication of each scale. Then, she reached successfully to a solution by applying mentally the trial and error method, without using any picture. Thus, S2 used the informational picture only for collecting the data needed for the solution and not as a means of representation, or feedback for the solution process, like the other students.

The representational picture was used in an active manner by all the students in problem solving. This is attributed to the spatial ability that was essential for the understanding and the solution of the particular problem. All the students employed visual methods by using the given picture, which in most cases caused internal conflicts. Specifically, the initial intuitive way of thinking before the presentation of the picture, involved the application of the operation of division $10:2=5$. By using the picture, students could visualize (actively) the spatial arrangement of the trees at the path, which enabled them to figure out the correct number of trees. In particular, seven students drew five trees at the path and they, subsequently, realized that the distance between the trees was not 2m. As a consequence, they ended by drawing four trees along the path, which was the correct solution. The positive influence of the representational picture was evident in the communication between the students, as well. In particular, during a communication experiment, in the description of the problem to her partner (S2), S1 altered, non-consciously, the data of the problem: *"A man planted 2 trees at each end of a path. Then he planted a tree every 2 m. The length of the path is 10 m. How many trees did the man plant at the path altogether?"* Before the presentation of the picture, based on the incorrect data, S2 conceived the path as a rectangle and doubled the trees of each side. By observing the picture given to her, S2 managed to resolve the problem correctly, without any difficulties. Thus, it can be asserted that the picture facilitated the communication between the students.

The organizational picture had also an important role in problem solving. The use of the picture enabled five students, who had employed additive strategies or inappropriate multiplicative strategies before the presentation of the picture, to use

proportional reasoning by applying correct multiplicative structures. Students who experienced the particular internal conflict made use mostly of visual methods. In particular, they acted on the picture by reorganizing the elements of it, either mentally or practically, so that the 15 campers below the line would be separated into two groups, one of 10 campers and another one of 5 campers, which would be matched with the corresponding groups of loaves (8 and 4). For example, S1 who applied the particular procedure, explained: “10 campers get 8 loaves, so 10 of the 15 campers would also get 8 loaves. 5 campers are left. I’m thinking that if 10 campers get 8 loaves, then 5 campers get half as much that makes 4 loaves”. Nevertheless, for S5 the picture created an obstacle. Influenced by the arrangement of the elements of the picture, he applied an inappropriate pattern in his solution process. Specifically, he stated that 4 campers have 3 loaves next to them, thus the number of loaves could be found by subtracting 1 from the number of campers ($4-1=3$). He subsequently applied the pattern to all the rows of the 15 campers and ended with a wrong solution. Three students focused on analytic methods, since they used directly the correct proportion in their solution processes, without using the picture.

Students’ views in regard with the role of the pictures in MPS depended on the pictures’ contribution in their solution procedures. Specifically, all students stated that the decorative picture could not help in any way; on the contrary, they all recognized that the representational picture had a supportive role in their solution procedures. As for the informational picture, only some students realized that without the picture they could not begin the solution of the problem. This is attributed to the fact that, at first, the picture was presented “orally” to them, so they initially dealt with the problem without using the picture visually. Finally, only the five students who used it in their solution procedures acknowledged the positive effect of the organizational picture on MPS.

In regard with the effect of the different categories of pictures to the communication process, it can be asserted that the need for describing the decorative, representational and organizational picture was trivial. This was evident from the fact that students articulated the problem to their partners, without any reference to the particular types of pictures involved. On the contrary, the description of the informational picture contributed significantly to the communication process, since it provided all the data of the problem.

DISCUSSION AND IMPLICATIONS

Findings of the present study revealed that the representational, informational and organizational picture, but not the decorative one, had a significant effect on MPS. The use of the pictures led frequently students to internal conflicts, which in turn enabled them to find correct solutions for the problems. The decorative picture did not have a substantial role on students’ responses in MPS or the communication process. This finding gives support to Carney and Levin’s (2002) view, that decorative pictures do not enhance the understanding or any application to the text. It

was also found that the arrangement of the elements of the picture may cause difficulties in MPS. This finding is in line with the argument of Colin et al. (2002), who point out that the spatial position of the elements of a picture, may lead individuals to inappropriate inferences.

Most students' strategies involved the use of either visual or analytic methods, focusing on the method that better matched their abilities and the problem or picture involved. The visualization process which was made explicit in two different ways, with students' interaction with the given picture or the generation of a drawing by them, was meaningful and facilitated significantly MPS. Students interacted with the given pictures in two ways, by observing them or by acting on them (Thomas, & Yoon Hong, 2001). Observation was detected mostly in the case of the decorative picture, while active processing was identified mainly in the case of the representational picture. Interaction with the informational and the organizational picture varied; it was active for some students and observational for others. The observed diversity of students' responses concerning the visualization process may be attributed to the interaction between the student and the stimulus, which in the present study refers to the problem and the picture involved. This interactive relationship depends on students' preferences, and their spatial and visualization abilities, i.e. their competence to recall, generate, choose and operate appropriately with the visualization (Bishop, 1989).

As for the communication process, the primary means of communication was the verbal use of language. The organizational and representational pictures had a helpful, supportive and complementary role, and aided in providing precisions, details and feedback for the text and its oral description. The decorative picture had no effect on communication, whereas the informational picture was not only useful, but also essential and substantial in problem solving.

To sum up, from the present study, it is clear that effective problem solving, which makes use of pictures, depends on the relationship between the picture and the problem (function the picture serves in the problem) and the students' previous knowledge and abilities. Therefore, it would seem important that teachers select pictures based on their function in MPS. Another implication for teachers is to take into consideration the students' preferences for the method/s they use in MPS, in order to encourage them to develop the strategies they are not sufficiently competent at. Moreover, findings concerning the function of decorative pictures in MPS underline that the use of decorative pictures in MPS should be handled with great attention in regard with the development of mathematics textbooks, materials, resources and instruction.

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