TIME AND FLOW AS PARAMETERS IN INTERNATIONAL COMPARISONS: A VIEW FROM AN EIGHTH GRADE ALGEBRA LESSON

Michael N. Fried and Miriam Amit

Ben-Gurion University of the Negev

ABSTRACT: This paper compares the way lessons on systems of linear equations unfold in a classroom in the Negev region of Israel with the way they unfold in a Shanghai and Hong Kong classroom. Lessons are viewed as temporal entities describable not only by the nexus of topics they contain but also by how they flow in time. In this light, the lessons in the classroom studied by the authors contrasts strongly with the Shanghai and Hong Kong classroom, the former having a turbulent flow and latter a smooth directed flow. The result is consistent with previous recognized cultural differences in classroom practice and has implications for the bases of international comparisons.

INTRODUCTION

International studies such as the TIMSS have taught us, among other things, that international comparisons are devilishly difficult to make (e.g. Keitel & Kilpatrick, 1999). Even where curricular complexities may be put aside and a common subject agreed upon, Stigler and Hiebert (1999) and others have shown that lesson structure and presentation can vary greatly from country to country, culture to culture. The present paper adduces further evidence for this fact and underlines a crucial aspect of the presentation of mathematical subjects to be taken into account in international studies, namely, the manner in which lessons unfold *in time*.

How mathematics lessons unfold can be described in two complementary ways. One way is according to their logic or rationale. This rationale is determined partly by mathematics itself and partly by teachers' pedagogical styles. But mathematics lessons also unfold with a certain pace; they have a flow, which one may well describe with musical terms such as rhythm and tempo. The logical unfolding of lessons corresponds, roughly, to what has been called *topogenesis*, and the actual flow of lessons to *chronogenesis* (Chevallard, 1985; Brousseau 1999). The different ways time enters into the teaching and learning of mathematics have been studied broadly by Arzarello, Bartolini Bussi, & Robutti (2002). Some of these ways, such as the 'stream of discussion' (Bertolini Bussi, 1992) and Brousseau's 'didactic memory', are examples of 'external time', that is, they are measurable by an observer's clock, while others are examples of 'internal time', which is "primarily individual and unconscious, although its features may be inferred from external clues" (Arzello *et al.*, 2002, p. 526). In this paper, we shall be concerned only with external time, though we consider internal time no less important. We shall examine, in particular, how algebra lessons on systems of linear equations flow in time and how their pedagogical rationale unfolds. The lessons, which are the focus of this paper, were observed in an eighth grade classroom in the city of Beer Sheva in the southern region of Israel. These are compared with lessons in Shanghai and Hong Kong on the same subject matter, as described by Mok, Leung, Lopez-Real, and Marton (Mok *et al*., 2002).

The comparison of our findings with those of Mok, *et al*. showed that while there were some differences between the lessons in Hong Kong and Shanghai, those lessons were far more comparable with one another than they were with the lessons we observed in Beer Sheva. The latter differed strikingly from either Hong Kong or Shanghai. The most prominent divergence of the lessons we studied from those studied by Mok *et al*. was in the way the very idea of a system of linear equations in two unknowns was developed. And it was here that thinking in terms of flow and time proved useful, as we shall see.

RESEARCH SETTING

The research setting for the results to be presented here, both ours and those of Mok, *et al*., is the *Learners' Perspective Study* (LPS), which is an international effort involving nine countries (Clarke, 1998; Amit & Fried, 2002). The project arose out of the video study connected with the Third International Mathematics and Science Study (TIMSS) in which eighth-grade mathematics classes in Japan, Germany and the USA were videotaped and analyzed to identify national norms for teaching practice, norms that might account for achievement scores attained in each country. The LPS expands on the work done in the TIMSS study (which exclusively examined teachers and only one lesson per teacher) by focusing on *student actions* within the context of whole-class mathematics practice and by adopting a methodology whereby student reconstructions and reflections are considered in a substantial number of videotaped mathematics lessons.

As specified in Clark (1998), classroom sessions were videotaped using an integrated system of three video cameras: one viewing the class as a whole, one on the teacher, and one on a "focus group" of two or three students. In general, every lesson over the course of three weeks was videotaped, that is, a period comprising fifteen consecutive lessons. The extended videotaping period allowed every student at one point of another to be a member of a focus group. Needless to say, video technology with its built-in capacity for measuring time proved an invaluable aid in observing how lessons unfold.

The researchers were present in every lesson, took field notes, collected relevant class material, and conducted interviews with each student focus group. Teachers were interviewed once a week. Although a basic set of questions was constructed beforehand, in practice, the interview protocol was kept flexible so that particular classroom events could be pursued. In this respect, our methodology was along the lines of Ginsburg (1997); this methodology was chosen because the overall goal of

LPS is not so much to test hypothesized student practices as it is to discover them in the first place.

The specific case that formed the basis for this paper was a sequence of 15 lessons on systems of linear equations taught by a dedicated and experienced teacher, whom we shall call Danit. Danit teaches in a comprehensive high school. Her 8th grade class is heterogeneous and comprises 38 students, mostly native-born Israelis, but also new immigrants from the former Soviet Union and one new immigrant from Ethiopia.

DANIT'S LESSONS OVER LONG AND SHORT TIME SCALES

Danit developed the idea of a system of equations and its solution over the course of several lessons. These lessons, to the point at which the algebraic solution of systems was first introduced, unfolded as follows:

Lesson 1: Danit went over the notion of a number line, the coordinate system, and the task of plotting individual points.

Lessons 2-3: Equations in two unknowns were introduced; it was highlighted that such equations generally have an infinite number of solutions.

Lesson 4-5: Danit returned to the coordinate system; the students plotted the solutions of linear equations with her guidance, and the observation was made that solutions of such equations indeed lie along lines; the lines were described by Danit as an equation "in a different language." In the course of this discussion, it is important to remark, another representation was subtly brought into play, a table of values.

Lesson 6: The graphic solution of a system of equations was demonstrated—and it was here that Danit first used the phrase 'system of equations'; here too, she considered the meaning of a solution of a system of equations.

Lesson 7: Still concentrating on the graphic solution, Danit showed that there were cases in which the system can have an infinite number of solutions or no solutions.

Lesson 8: The limits of the graphic solution method were discussed, leading the way to purely algebraic solutions to systems of equations.

In this long sequence of lessons, one observes that the lessons shift from graphical representations to algebraic representations, back to graphical representations, back to algebraic representations. *The back and forth movement is not only characteristic of many lessons taken together: in almost fractal fashion, it is also evident within the details of the lessons themselves*. Consider the following segment from lessons 4-5 (these were taught without a break—in itself a point worth noting). In the preceding lesson, the students had discussed equations in two unknowns and had begun to see that they have an infinite number of solutions. Now, in this lesson, Danit makes the transition, which refers directly at the very start to a shift in the form of representation:

T: [*min. 35*][Writing the equation x+y=6 on the blackboard] Who is willing to tell me what is written here in Hebrew? I want a *translation* into Hebrew, not just "x plus y equals six"!...You've seen this [*i.e.* an equation like this] in your book, and you know to do with them [referring to the exercises given in the last lesson]—now translate it into Hebrew.

S1: Two unknowns you have to find them

T: Ok, but, a little more…[continues to prod the students]

S2: [*min. 36*] One unknown and another unknown equals six

T: Good, but more, even without the word 'unknown'.

S3: Something and something equals six

…

T: [writes: 'Two numbers whose sum is six'] Find me two numbers whose sum is six. In the language of algebra, we say, 'x plus y equals six'. [*min. 37*] Today, we're going to learn to translate this into another language; we're going to sketch this, that is, what is written here, x+y=6, I don't have write in the language of algebra, I don't have to say it in words: I can sketch it.

For the next ten minutes, approximately, Danit guides the class through a point-wise construction of the graph of line given by x+y=6, including the construction of a table of values. Finally, she observes:

T: …[*min. 47*] What we have obtained in fact is a straight line in the coordinate system that represents this equation. Come, see why. The line stands in place of saying x and y equals 6 …

Although Danit says the line stands *in place* of saying 'x and y equals 6', the equation is still very much present in the ensuing dialogue. Indeed, before moving ahead to the graphic translation, she first moved back to a verbal translation of the equation calling to mind the previous lesson. Moreover, as the dialogue continued other ideas from the previous lesson returned, in particular, that equations in two unknowns characteristically have an infinite number of solutions and that that can be shown by choosing an arbitrary value for x and showing that a value of y can be found.:

T: [The line stands in place of saying x and y equals 6] Now, let's see what happens to a point, any point that I happen to pick on the line. Come, I pick at random this point over here. What is the x of this point? [points at the board]

Ss [several students together]: 7

T: What is y?

Ss: -1

T: 7 plus -1 equals 6 [note: this is what she referred to before as the language of algebra] [*min. 48*] How many solutions are there to this equation?...How many points are there on this line?

S1: 6 [there are, in fact, 6 *marked* points on the line drawn on the board]

S2: 5

S3: 6

S4: Infinitely many

T: Infinitely many!

Thus, in this ten minute segment, the lesson has shifted from an algebraic representation of an equation in two unknowns to a verbal representation to a graphic representation back to an algebraic representation, forward again to the graphical representation, and then back to the algebraic ideas of the previous lessons.

SHANGHAI AND HONG KONG

The Shanghai lesson, as described by Mok, *et al*. (2002) develops the idea of a system of equations in one lesson lasting approximately 42 minutes. It began with a real-life problem concerning the purchase of two kinds of stamps, a constraint was then added, and, by doing so, a system of equations was produced. After 10 minutes of exploratory discussion, the teacher presented a definition in terms of the example. The teacher then lead the class into a purely mathematical context and introduces the idea of a *linear* system in a purely algebraic fashion. Several examples were given to reinforce the definition; the lesson returned to the original word problem, and again in terms of the problem what a solution of a system was defined. Finally the students were given exercises designed to apply the definitions.

In the Hong Kong lesson, which was somewhat shorter than the Shanghai lesson (approximately 35 minutes), the teacher began with a whole class discussion arrangement to review the idea of an equation in one unknown. This discussion was the vehicle for reviewing the notions of ‘unknown’, ‘linear’, ‘solution’. Having done this, the teacher could then state the topic of the day, namely ‘simultaneous equations in two unknowns’. From here, the Hong Kong lesson, like the Shanghai lesson, moved on to a motivating word problem—this time, a problem concerning rabbits and chickens. Again, as in the Shanghai lesson, the definition of a system was given in terms of the word problem. The lesson concluded with a shift to a pure mathematics context in the form of ‘worksheet tasks’ asking the students to solve systems of equations.

Although Mok, *et al*. (2002) emphasize the differences between the Hong Kong lesson and Shanghai lesson, we were struck by their similarity. They both have a clear structure: a motivating example, central definitions derived from the example, a return to the motivating example (explicitly, in the Shanghai lesson and hinted in the Hong Kong lesson), and exercises that reinforce the definitions. Moreover, this structure is paced to begin and be completed in exactly one lesson.

DISCUSSION

In the Hong Kong and Shanghai classroom, one moved in a very paced manner, in one lesson, from a motivating example to the definition of system of equations and the solution of a system of equations and then to a summation by means of exercises applying the new definitions; in the classroom we observed, one moved in a slow meandering fashion, over the course of several lessons, through different representations first of equations in two unknowns, then of systems of equations, leading finally to the algebraic solution of a system.

The way Danit's lessons move back and forth between different representations of equations on such a small scale and, simultaneously, on the large scale makes Seeger's (as cited in Arzello *et al*., 2002) comparison of such classroom discussions to *turbulent flow* (as opposed to *laminar flow*) particularly apt. It not only describes the flow of the lesson, it also gives some hint of what the pedagogical effect of such flow is. For in turbulent flow a fluid is constantly being mixed: turbulent lessons are not confused lessons, but ones in which ideas are continually being brought forward and back and compared and contrasted. It is for this reason, we surmise, that Danit, when she finally arrived to the notion of a system of equations, did not see the need to provide an explicit definition: the line representations of the equations and the algebraic equations were continually being mixed, so that the intuitive fact of two lines meeting at a point was immediately being compared to the simultaneous solution of two linear equations.

The turbulent flow of the lessons in the Beer Sheva classroom contrasts strongly with smooth directed flow of both the Shanghai and Hong Kong classrooms. The difference is very likely a cultural one. Stigler and Hiebert (1999) indicated a similar difference between Japanese and American lessons. In Japanese schools, a lesson is considered a perfect whole telling one coherent story. For this reason, a lesson in Japan is not to be disturbed in the middle, and no part of it is to be missed. In American schools, lessons form a series of more or less independent modules: an interruption here or there will not, therefore, ruin the lessons (Stigler & Hiebert, 1999, pp.95-96). Indeed, in our basic set of interview questions, one question asked whether students viewed each mathematics lesson as a single story or as a chapter in an ongoing series, like a 'soap-opera'. In almost every case, the students answered "it is more like a 'soap-opera'."

Accepting the fact of this turbulent flow in the Beer Sheva lessons, one should ask, of course, whether the students benefit from it. Should we, rather, emulate the laminar flow of the Shanghai and Hong Kong lessons? At this point, it is hard to say. We were disturbed to find that when, in the interviews, we asked Danit's students what they understood by 'a system of equations', they had only a vague notion—more than one student identified the system of equation with the coordinate system—even though the same students could often find the solutions to systems without too much trouble. But, to return to musical analogies, it may that in such lessons, such misapprehensions are mere dissonances to be resolved later.

References

Amit, M. & Fried, M. N. (2002). Authority in the Mathematics Classroom and Its Influence on Students' Ability to Reflect. In A. D. Cockburn, E. Nardi, (Eds.), *PME26: Proceedings of the 26th Annual Conference 2*, (pp. 41-48). Norwich, UK: University of East Anglia

Arzarello, F., Bartolini Bussi, M. G., & Robutti, O. (2002). Time(s) in the Didactics of Mathematics: A Methodological Challenge. In L. English (Ed.), *Handbook of International Research in Mathematics Education* (pp.525-552). Mahwah, New Jersey: Lawrence Erlbaum Associates Inc., Publishers.

Bertolini Bussi,M. G. (1992). Mathematics Knowledge as a Collective Enterprise. In F. Seeger. & H. Steinbring (Eds.), *The Dialogue between Theory and Practice in Mathematics Education: Overcoming the Broadcast Metaphor* (pp. 121-151). Bielefeld: Materialien und Studien Band 38.

Brousseau, G. (1999). Education and Didactique of Mathematics. Available at the web site: www.math.washington.edu/~warfield/Ed_and_Didact.html

Chevallard, Y. (1985). *La Transposition Didactique*. Grenoble: La Pensée Sauvage.

Clarke, D. J. (1998) Studying the Classroom Negotiation of Meaning: Complementary Accounts Methodology. In A. Teppo (Ed.) *Qualitative Research Methods in Mathematics Education* (pp. 98-111). Monograph No. 9 of the *Journal for Research in Mathematics Education*. Reston, VA: NCTM.

Ginsburg, H. P. (1997). *Entering the child's mind: The cognitive clinical interview in psychological research and practice*. New York: Cambridge University Press.

Keitel, C. & Kilpatrick, J. (1999). The rationality and irrationality of international comparative studies. In G. Kaiser, E Luna, & I. Huntley (Eds.) *International Comparisons in Mathematics Education* (pp. 241-256). London: Falmer Press.

Mok, I. A. C., Leung, F. K. S., Lopez-Real, F., and Marton, F. (2002). Dimensions of Variation in Mathematics Classrooms: Examples from Hong Kong and Shanghai. Paper presented at the Annual Conference of the *American Educational Research Association*, New Orleans, April 2-5, 2002.

Stigler, J. W. & Hiebert, J. (1999). *The Teaching Gap*. New York: Free Press.