

ON THE FRAGILE, YET CRUCIAL, RELATIONSHIP BETWEEN MATHEMATICIANS AND RESEARCHERS IN MATHEMATICS EDUCATION

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*The relationship between mathematicians and researchers in mathematics education has often been fragile. Yet it is crucial. We conducted a series of themed Focus Group interviews with mathematicians from six UK universities. Pre-distributed samples of mathematical problems, typical written student responses, observation protocols, interview transcripts and outlines of relevant bibliography were used to trigger an exploration of pedagogical issues. Here we elaborate the theme “the relationship, and its potential, between mathematicians and researchers in mathematics education” that emerged from the data analysis. We do so by presenting the participants’ views on this relationship in terms of: **obstacles, desired characteristics and potential benefits.***

INTRODUCTION

The relationship between theory and practice in mathematics education is often fraught with suspicion (Sierpinska & Kilpatrick 1998), even hostility. This applies across the educational spectrum – for example within the primary and secondary sectors where policy makers often suffer criticism that their decisions are rarely and marginally informed by research in mathematics education [Brown in (Sierpinska & Kilpatrick 1998)]. Nowhere however is this more evident than within the tertiary sector (Ralston 2003). Recent developments in the world of university mathematics, such as the changing enrollment and profile of the student intake (Holton 2001), have resulted in a need for mathematics departments to rethink curricula, e.g. (Kahn and Hoyles 1997), and pedagogical practices, e.g. (McCallum 2003). In doing so a rapprochement between the worlds of mathematics and mathematics education research has become vital [Artigue in (Sierpinska & Kilpatrick 1998)].

Here we elaborate this issue through drawing on the views of university mathematicians participating in a study currently in progress in the UK. For an outline of the methodology of the study see the ENDNOTE. Participants were twenty mathematicians, pure and applied, with teaching experience ranging from a few years to several decades, all but one male and of varying rank. In six out of the eighty *Stories* which formed the analytical units of the study (see ENDNOTE) the participants expressed views on what we present here grouped as: *obstacles* and *desired characteristics* of the relationship between mathematicians and researchers in mathematics education; and, *potential benefits* for mathematicians engaged as educational co-researchers.

OBSTACLES

The participants in the study generally acknowledged that, despite its importance, the relationship between mathematicians and researchers in mathematics education is weaker than it ought to and could be. Across the six *Stories* there was extensive discussion on what these weaknesses are. Issues of *trust*, *access*, *priority*, *communicability*, *applicability* and *subtlety* dominated this discussion.

Participants often stated that we, the team of researchers in mathematics education conducting this study, were the ‘first ones’ they ‘ever talked to’. While appreciating the fact that our mathematical background allowed elaborate examination of learning and teaching issues that are specific to university mathematics (see BENEFITS), they also expressed concern that, if that was not the case, they would have found it difficult and rather unproductive to participate in ‘content-free’ pedagogical discussion.

Asked about where in the mathematics literature a mathematician is likely to find educational articles, the *Notices* of the American Mathematical Society were mentioned as was the *Mathematical Gazette* in the UK. However, participants admitted that they would ‘never come across’ papers such as the ones published in the *PME Proceedings* or in mathematics education journals. At the heart of the problem seems to be the fact that the two worlds do not meet very much: both mathematicians and researchers in mathematics education need to publish in and read journals in their own areas (e.g. for the purpose of research-assessment exercises) and there is precious little time for reading each other’s journals. ‘We are more likely to read pedagogically thoughtful books than journals (e.g. Polyà)’, as one participant put it. ‘In its bulk’ the mathematics community does not ‘look to this type of research as a source of knowledge or ideas about mathematics teaching. It just doesn’t... whether it should or not is a different matter’, he suggested. He then concluded: it is still the case that the image of a mathematics department that pays a lot of attention and contributes to research in mathematics education could suffer as this appears as a digression from its main research agenda.

Issues of access and priority notwithstanding, participants highlighted a deeper difficulty in establishing and maintaining communication between the two worlds. Many mathematics departments are committed to the idea that ‘research driven tertiary education is the only and best way to teach mathematics’, to the ‘terribly arrogant’ idea that ‘we are the holders of the knowledge of how tertiary mathematics should be taught’ and that ‘universities should not be giving [degrees] to people if they don’t have active research faculty’. Under these circumstances rethinking the profile of a mathematics graduate as someone who may also have pedagogical faculty (namely could become a mathematics teacher or a researcher in mathematics education) then becomes extremely difficult – as does an appreciation of what engagement with research in mathematics education may have to offer in terms of university-level pedagogical practice.

On the other hand most participants reserved weary suspicion towards epistemological debates within the educational community which were seen as

moving the target from offering to mathematicians something that is ‘somehow connecting with what I [am] doing in a lecture theatre or in a seminar group’ to ‘egotistical nonsense’ about how, for example, individual educational researchers use particular terminology. As one participant put it: in order to ‘pay attention’, ‘first I would want them to kind of not be arguing about what they are meaning’. This comment reflects a more profound epistemological gap between the two worlds: for these mathematicians concept definitions within mathematics are stable – stability having been achieved through arduous negotiation and evolution over the years – and so should concept definitions be within educational research. Whether this is desirable or even feasible (Schoenfeld 1994) within the community of educational researchers is a matter of debate. It seems however an issue that could be resolved were the two worlds to meet more fruitfully.

Furthermore, according to some participants, the ‘noble cause’ of ‘involving both communities’ is not always best served by a presentation of educational findings that is often seen as ‘almost indecipherable’. Additional evidence to these statements comes from the following fact: across the eleven Cycles of Data Collection, the brief literature reviews on each of the six *Themes* (see ENDNOTE) accompanying the Datasets that participants were invited to consider prior to the interviews *never* became part of the conversation spontaneously – unlike the samples of data (from students’ writing, interviews etc) that proved highly effective triggers for discussion. ‘Indecipherability’ seemed to be the main reason while some participants suggested that some educational ‘jargon’ is inevitable if analysis is to do justice to the subtlety of data. Sophistication in the data analysis was seen as a *sine qua non* for the participants: in the few occasions where, following probing by the interviewees, participants discussed, for example, models of mathematical understanding, they appeared to be variably impressed. On a proactive note, one participant suggested that small glossaries could facilitate the exchange amongst interlocutors from different disciplines. Trying to understand writing in mathematics education is actually a ‘fun’ exercise: ‘in the same way in which you can read a political essay and so on and you can take a certain pleasure just from that’.

Even further, we see the issue of building a common language as not separate from building a ‘mutual agenda’ (Barbara Jaworski, personal communication) of research – all stages of it, from inception through to execution and dissemination. Or, as she put it, for example regarding data analysis: ‘I [wonder] to what extent the mathematicians contribute to the analysis or consider it when it is written’. One way of addressing the issue of communicability of findings is ensuring that the audience these findings aim at addressing has ownership and participation in the process that brought these findings to existence in the first place. Other characteristics of research in mathematics education seen by the participants in our study as desirable are presented in the next section.

DESIRES

Participants often proposed that dissemination of educational research findings should be done more systematically in mathematics departments (for example through seminars, workshops etc.). Tertiary level teacher training, at least in the UK currently in the form of courses on higher education practice for newly appointed lecturers (<http://www.ilt.ac.uk>), was proposed as ‘the kind of places in which this kind of research should be disseminated’. However some participants suggested that often these courses are bogged down to ‘epitomising the worst aspects of professional education’ by being ‘content-less’. ‘If I had gone to one of those things’, observed one experienced lecturer, ‘and someone had said something about the ε - δ definition of continuity or the definition of a group or a specific [...] thing in chemistry or had talked in these terms about it, I would have been fully attentive and I would have got something from it. So... it is not that the audience is not there’, because ‘the problem [of effective teaching, of recruitment etc.] is there’. While not underestimating the value of interdisciplinary discussion of pedagogy, a younger lecturer amongst the participants observed that, in the face of the challenge of a new lecturing job, where the pressures to prove one’s worth in the substantive field (see also OBSTACLES), ‘content-less’ training courses can engender impatience and be seen as a waste of time.

Beyond ways in which to improve the dissemination of educational research findings within the mathematical community, discussions amongst the participants focused primarily on characteristics of research in mathematics education which would enhance for them the appeal of *collaboration and engagement*. In these discussions issues of methodology (e.g. selecting focus of the research, methods for collecting and analysing data, language) dominated.

In terms of selecting foci for educational research, participants expressed a strong interest in studies which focus on the teaching and learning of specific concepts or topics. Cross-topical studies of mathematical learning were also mentioned. One participant outlined an example: ‘a longitudinal study of a single student’, one that would focus on ‘what in fact are the conceptual hurdles one must overcome’. While acknowledging interviewers’ comments that student-centred studies are already in existence, the participants stressed that mathematicians are not always aware of these studies (returning to the issues raised in OBSTACLES) and, furthermore, would hugely appreciate a consideration in these studies of their own experiences and views. Or as one participant put it: we ‘all have an intuitive feeling on what goes or what doesn’t go. You look at these students, you look at their faces, you know that they are lost’.

In terms of building a common language to discuss the teaching and learning of mathematics at university level, the participants often emphasised that a sufficiently strong mathematical background on the part of researchers in mathematics education helps alleviate some deeply rooted suspicion on the side of mathematicians. Or as one participant strongly put it: ‘...to be honest if people want to find open doors in a mathematics department they need to be able to talk to mathematicians about mathematics, and, if they can’t, maybe they are in the wrong business.’ Another

participant followed this comment with a recommendation for ‘friendliness’ towards the community of mathematics educators. Significantly he also raised the issue of the epistemological gap (see OBSTACLES) between the two communities and made a plea to the mathematical community for a less ‘absolutist’ spirit when engaging in pedagogical discussion.

Despite reservations towards educational research findings that do not always directly prescribe effective practice (see OBSTACLES), participants in this study expressed a preference for a type of research that ‘brings questions, not answers to the table’. The latter, one participant observed, is ‘far less interesting’. Drawing on specific examples from the Datasets used in this study, the participants often expressed a preference for a methodology that allows naturalistic data to be examined in an open forum where the researchers in mathematics education, given that they are the apparent educational experts in the discussion, do not assume a position of authority but engage in a more equitable and collaborative exploration of pedagogical issues – a characteristic of this study that was emphatically appreciated by the participants (see BENEFITS).

In addition to a preference for open, naturalistic methodologies, some participants made statements about how this openness needs to be combined with attention to detail and sensitivity to the complexity of pedagogical issues. In particular some participants expressed reservation towards studies which attempt to explain mathematical behaviour in terms of broad variables, thus being exposed to ‘the danger of smudging ... different things together’. And ‘smudging together the effect of certain conditioning, perceivable conceptual things, cognitive things, just smudging all together under the same banner’ would be worrying, one participant concluded and stated a preference for a highly focused, ‘clinical’ approach.

But, we wondered, behind some of these broad variables, e.g. gender, do not often lie controversial issues (such as the issue of gender representation in mathematics faculties in the UK)? For example, in the context of this study one could claim that, by apparently focusing in the Datasets on mathematically-specific issues on learning, by maintaining ‘that there is something relatively safe in the kind of questions that we are asking’ and by ‘not asking for acting upon the problems’ (our words during the last group interview) we have stayed clear of controversy in the fear of risking the mathematicians’ participation. As a ‘safer route’ into ‘how people involved in a mathematics course are thinking’ this adoption of a minimally interfering observer’s role, prior to attempting to effect change, was seen by the participants as an appropriately subtle method. One pinned this idea as follows: ‘... we have been talking about things covered in several different years with as little as possible change on how we view them. [...]. If we were sort of on the fly during the process, changing how we presented certain concepts what we would be saying would change very rapidly and we would be somehow observing something, we would be participating in it and [...] not to the benefit of understanding what is going on. [...] it would be like trying to hit a moving target.’ This insight into pedagogical issues was described by participants as the major benefit from collaborative engagement with educational research. In the following section we elaborate their comments on this issue.

BENEFITS

Participants evaluated their experience of the study and often these evaluations led to more general statements about potential benefits from engagement as educational co-researchers. 'There are things I will teach differently. There are things that I feel like I understand better of mathematics students than I did', said one. In particular the research process helped the realisation to emerge how 'one should be liaising with the other lecturers' in order to discuss 'what things we are doing that confuse [students]'. The questioning, 'all guards down', exploratory and non-prescriptive spirit of these discussions coupled with the 'refreshing discipline' in the specificity of questions being asked in the Datasets were also appreciated (see also DESIRES). As one participant put it, 'this is a platform by which we start to rationalise'.

The naturalistic character of the data ('these are real people struggling with a real difficulty, in real time') used as trigger for discussion was often praised. Its character almost steered the conversation away from the hectic frame of mind within which the lecturers' encounter with the masses of student writing usually takes place: there was no need to determine marks or distinguish between 'right' and 'wrong' answers. 'It is really a confirmation that [...] the quantitative story about learning is devoid of meaning' suggested one participant. '... a tick box of the average understanding of the concept of group among the students is a figure that has absolutely no meaning at all in it, in comparison with the individual detailed discourse of this student'. And while this may vary across students 'there are similar hurdles and I think that defining the hurdles is important'.

The challenge within the 'rationalisation' mentioned above was also seen as 'a perfect role reversal': 'this idea of having *lecturers* [as opposed to students] thinking hard about what they are understanding when we write [...] on the board'. 'I think now I don't have any more answers than when I started [the study] but certainly I don't take things for granted anymore, from colleagues or from students. I think I am much more open-minded on what might be going on inside other people's brains', one participant said of the 'fascinating' increase in pedagogical awareness that participating in the study resulted in. 'The intellectual challenge for me', he continued referring to his teaching 'is [...] converting what I know into communicating that to students' and this is 'no routine task'. 'I think we should take an interest in this because obviously we are spending a lot of effort on things that aren't working. The routine milling that we do since the middle ages probably won't produce students who have a broad view of mathematics as an exciting intellectual challenge. I think we can do this much better and I think we can start by mathematicians talking to each other about their own prejudices [...] before having a go at innocent students!'. This is 'starting at the right end' if we are to consider alterations of practice, he concludes. The participants often suggested ways in which their teaching practice was affected by the opportunity for a closer look at students' thinking and ways in which students and mathematics teachers across the educational levels can benefit from such reflective processes (we elaborate these elsewhere, for update information: <http://www.uea.ac.uk/~m011>).

CONCLUSION

The participants in the study acknowledged that, despite its importance, the relationship between mathematicians and researchers in mathematics education is weaker than it ought to and could be. *Trust, access, priority, communicability, applicability* and *subtlety* were the dominant **obstacles** they identified. Beyond ways in which to improve the dissemination of educational research findings within the mathematical community (for example through seminars, workshops, tertiary level teacher training etc.) participants often focused on **desired characteristics** of research in mathematics education that would enhance the likelihood of their engagement. These primarily included issues of *methodology* (e.g. selecting focus of the research, methods for collecting and analysing data, language). Finally, with regard to potential **benefits**, the participants valued the opportunity to appreciate the *qualitative paradigm* exemplified by the study, expressed a preference for *in-depth* studies of teaching and learning and elaborated the *gains in pedagogical insight*, both in terms of awareness and in terms of specific modifications in their practice. With regard to the latter, and once the current phase of data analysis is complete, we envisage an Action Research study, where a selection of John Mason's 'tactics' (2002) will be employed and evaluated by practitioners in the context of lectures, seminar groups and assessment of students' written work.

ENDNOTE

This 15-month, LTSN-funded (<http://www.ltsn.ac.uk>) study engages groups of mathematicians from six institutions in the UK as educational co-researchers (Wagner 1997). There were 11 Cycles of data collection, six with five mathematicians from the University of East Anglia (Cycles 1-6), where the authors work, and five from elsewhere (Cycles 1X-5X). Six Data Sets were produced for each of Cycles 1-6 on the themes *Formal Mathematical Reasoning I: Students' Perceptions of **Proof** and Its Necessity; Mathematical Objects I: the Concept of **Limit** Across Mathematical Contexts; Mediating Mathematical Meaning: **Symbols and Graphs**; Mathematical Objects II: the Concept of **Function** Across Mathematical Topics; Formal Mathematical Reasoning II: Students' Enactment of **Proving Techniques** and Construction of Mathematical Arguments*; and, *A **Meta-Cycle**: Collaborative Generation of Research Findings in Mathematics Education*. The Datasets for Cycles 1-5 were used also for Cycles 1X-5X. Each Dataset consisted of: a short literature review and bibliography; samples of student data (e.g.: students' written work, interview transcripts, observation protocols) collected in the course of the authors' previous studies (<http://www.uea.ac.uk/~m011>); and, a short list of issues to consider. Participants were asked to study the Dataset in preparation for a Focus Group Interview - see Madriz (2001) and Nardi & Iannone (2003) for a rationale for using this tool for data collection. Interviews were digitally recorded. The interviews from Cycles 1-6 were fully transcribed (the data from Cycles 1X-5X were used as supportive material in the analytical process). Each interview was about 200 minutes long and generated a *Verbatim Transcript* of about 30,000 words. In the

spirit of Data Grounded Theory (Glaser & Strauss 1967) eighty *Episodes*, self-contained extracts of the conversation with a particular focus, emerged from a preliminary scrutiny of the transcripts and were transformed into *Stories*. These are narrative accounts in which we summarise content, occasionally quoting the interviewees verbatim, and highlight conceptual significance. The eighty *Stories* were grouped in terms of the following five *Categories*: *students' attempts to adopt the 'genre speech' of university mathematics* (Bakhtin 1986); *pedagogical insight: tutors as initiators in 'genre speech'*; *the impact of school mathematics on students' perceptions and attitudes*; *one's own mathematical thinking and the culture of professional mathematics*; and, ***the relationship, and its potential, between mathematicians and mathematics educators*** (25, 25, 4, 20 and 6 *Stories* respectively). Here we focus on the last *Category*.

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