

Democracy and Authority: A Complementarity in Mathematics Education?

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Abstract: There is now a considerable literature exploring the connections between mathematics education and democratic society, much of it theoretical about what could or should occur. The question is what happens when an attempt is made to deliberately realise such a link in a mathematics classroom. It was the investigation of this question, together with a student teacher, who undertook project work in a primary mathematics classroom that gave rise to the theme of democracy and authority. Following a short description of the case, this dual-concept theme is developed, first with reference to data in three domains: whole class, group work and in the interactions between the teacher figures and researcher; and then theoretically, as I explore my proposition that the relation between democracy and authority is best understood and explained with reference to the idea of complementarity.

Kurzreferat: *Können sich Demokratie und Autorität im Mathematikunterricht ergänzen?* Es gibt inzwischen eine beträchtliche Menge an Literatur, in der Zusammenhänge zwischen Mathematikunterricht und demokratischer Gesellschaft untersucht werden. Davon ist vieles theoretischer Natur, was geschehen könnte oder sollte. Die Frage ist, was passiert, wenn ein Versuch unternommen wird, im Mathematikunterricht bewußt solch eine Beziehung herzustellen. Die Untersuchung dieser Frage, zusammen mit einem Lehrerstudenten, der Projektunterricht in der Grundschulmathematik erteilte, führte zu dem Thema Demokratie und Autorität. Nach einer kurzen Beschreibung eines Falles wird dieses Konzept dualer Begriffe entwickelt, zunächst im Hinblick auf Daten aus drei Bereichen: ganze Klasse, Gruppenarbeit sowie Interaktionen zwischen Lehrer, Lehrerstudent und Forscher. Theoretisch wird dann dargelegt, daß die Relation zwischen Demokratie und Autorität am besten unter dem Aspekt der Komplementarität verstanden werden kann.

ZDM-Classification: A40, C60, D40

1. Introduction

It is not uncommon to find current curriculum documents, which outline reforms, making some reference to democracy. Post-apartheid South Africa is not different. The question is what exactly can the relationship between the mathematics curriculum and democracy mean beyond the level of rhetoric, policy or theory?

We could ask: Can mathematics education provide a preparation for democratic life in society – for example, teaching democratic processes such as voting and democratic values such as equality, respect etc.? Can a focus on democracy imply a concern for the fair and equal distribution of mathematical knowledge and other educational opportunities for all members of society? Can democracy and mathematics education have something to do with mathematical content matter questions? Can democracy in mathematics education refer to the very life of a school – that is, pupils learning about democracy by participating in democratic life in a mathematics classroom?¹

There is a rapidly growing literature, especially in the last decade, where the connections between mathematics education and democracy are being explicitly and seriously

explored in a variety of ways (see for example Mellin-Olsen 1987, Adler 1988, D'Ambrosio 1990; Skovsmose 1994, Tate 1996; Woodrow 1997; and the recent special issues 1998/6 and 1999/1 of ZDM). But much of this work is theoretical, about the possibilities of what could or should be done in mathematics education rather than about what does in fact happen when a deliberate attempt is made to realise ideas related to democracy at the chalk-face – in a classroom. It is this challenge that the research reported in this paper rises to, by intervening in a primary mathematics classroom, to shed light on the potentials and pitfalls of the relationship between mathematics education and democracy. It is from this arranged situation that the notion of authority has come to be found to coexist in a special relation with democracy, best captured in the idea of complementarity.

In this paper I will present some data which serves both as illustration and evidence for this dual-concept theme of democracy and authority². After a short description of one particular case, I will show how these two concepts of authority and democracy come to reside in the reality of a mathematics classroom in an antagonistic yet co-operative partnership described by complementarity, in three domains: whole class, group work and teacher-student teacher-researcher interactions. This binary theme is then developed further theoretically when I elaborate the idea of complementarity and its usefulness for understanding mathematics classroom which seek to realise notions of democracy related to a critical mathematics pedagogy.

2. Case description

The theme of authority and democracy emerged from research in which student teachers explored with me, as researcher/teacher educator, what happens in a mathematics classroom, when a serious attempt is made to realise what may be called “a social, cultural, political approach” to the mathematics curriculum, especially if it embeds a critical perspective. This approach was given meaning in different schools through the practice of project work during teaching practice³. For this paper I draw on the work of one student teacher, Sumaiya Desai.

For her teaching practice, Sumaiya chose a school, which is located in what is still largely a middle class “Indian” residential suburb⁴. The composition of pupils is still mainly “Indian”, but there were a number of “African” children who travel to the school from nearby townships or informal settlements. The staff, however, was all “Indian”⁵. Sumaiya worked with a Grade 6 class of 30 pupils who were approximately 11 to 12 years old. Of these, 21 were “Indian”, comprising 8 girls and 13 boys; and 9 were “African” pupils, comprising 5 girls and 4 boys. The teacher who worked with Sumaiya was a qualified and experienced teacher, but teaching mathematics for the first time.

This case⁶ spanned a period of approximately four weeks. Pupils were arranged in 5 groups in the classroom. For their project work, Sumaiya listed five topics for pupils to choose from but almost every project was subverted. Two groups chose “how much money is spent on my education” but used the project to raise questions

about the amount and use of school funds collected by the school. A project “time spent after school” became a project about the problem of being given too much homework. One group chose their own project which was named by the teacher as a “survey about sport” at the school but, which they used to raise questions about the school’s inadequate and non-use of existing sporting facilities. One group chose the “mathematics newsletter” project. A “traffic problem” and a “consumer profile” project were not chosen. Once projects were chosen, pupils brainstormed ideas for their projects and then each group presented these to the class. During the project work, pupils were constantly reminded to demonstrate some mathematics in their projects. Groups discussed, decided, shared and carried out activities for their projects, which had to be completed in time for a formal presentation also used for assessment in the last week.⁷

3. Democracy and authority: an analysis from the case description

The theoretical lens I bring to the data, alluded to earlier as a social, cultural, political approach to the school mathematics curriculum, is informed by a broad theoretical landscape comprising at least four strands: critical mathematics education (see e.g. Skovsmose 1994; Frankenstein 1987); ethnomathematics (see e.g. D’Ambrosio 1990; Vithal and Skovsmose 1994; Powell and Frankenstein 1997); concerns about dimensions of diversity in mathematics education such as race and class and especially gender (see e.g. Willis 1996), and developments in people’s mathematics in South Africa (see e.g. Julie 1993). Within this landscape I emphasise a critical perspective. From this theoretical perspective, an analysis of the data yielded several dual-concept themes⁸ but here I focus on one of these, namely, authority and democracy. To discuss this theme, this section is organised as three interrelated domains: the whole class discussions; the workings of groups; and the teacher-student teacher-researcher interface.

3.1 The whole class domain

A central assertion of an approach that seeks to integrate a critical perspective is the idea that it could be possible for pupils to live a kind of democratic life within the heart of a school, in fact within a mathematics classroom. But any democratic life of pupils exists in relation to the authority of the teacher (and the school). The teacher is at once tied to both the responsibilities of a mathematics teaching-learning contract in the project work situation and to being another member of the classroom democracy. The data below, presented sequentially as it occurred in the classroom and organised in “chunks”, illustrates how democracy and authority clash and support each other. (S=Sumaiya; T=the class teacher)

Pupils from group 3, a race and gender mixed group, are presenting their project ideas to the class during the early stages of their projects, just after they have spent some time brainstorming activities.

Chunk 1: Learning about unequal backgrounds

Devan: We had already written out our school budget. Also getting to know our parents’ salary. We are doing

monthly work out and trying to find out whether our school fees should be higher or brought down. Some people have very little money to pay for food.

Chunk 2: The use of school funds for building a “structure” (a roof supported by metal poles) to serve as a school hall is questioned

Mohan: I don’t think the structure is very important. So much of money is spent on this when our toilet facilities need to be improved. Need money for computers. We shouldn’t worry how our school looks rather on our education.

T: Can I disagree with you immediately. We have very, very hot sun and so much of the time we cannot have all the activities. You know the play you watched “Trouble with Andre”, you paid R1.50 for the 600 people. It will cost more in terms of money, theatre. The disadvantaged students will then not benefit. You need to discuss how often this structure is used ... Do not get side tracked. We are doing it in a graph form.

Chunk 3: Mathematics saves the teacher

S: What graph are you using?

Devan: We are still deciding. We want to use the pizza graph and then make a summary.

S: Are you going to draw one graph for all the pupils or are you going to use different graphs for each individual pupil?

Devan: We are going to take everybody’s points and draw one big graph and explain to the class. Is that ok?

T: Mam, (to S) lets have them (group 5) since they are similar.

Chunk 4: Pupils from group 5 present their project ideas and questioning of the school fund use continues – it’s a strong issue for pupils

Siva: Why the toilets are so filthy? No toilet paper. More sporting activities, better lights. Mam nothing is done with the sporting facilities, they pickle it.

T: Would you be able to find out from a plumber or builder how much it would cost to do the toilet facilities up and then maybe put on a list of priorities. The school is 21 years old. Update the facilities and then get quotations. Then tell the school fund committee, we paid so much school fund, we are recommending you upgrade the toilet facilities.

Vaneel: The school built the monument up there (referring to the “structure”). I think they should have first checked the price before they built it.

Chunk 5: The teacher is forced to respond about the “structure”

T: Can I explain this to you too. It costs R130 000. We got R68 000 from the Ravindra Trust. We raised R30 000 and Mr. P (the principal) made a mistake in that he thought he was going to get a refund of R28 000. Then we discovered, because nobody is a chartered accountant, they assumed that, because we don’t have a vendor’s license, this is a school, we not entitled to that tax. The school fund was used to pay for this. That means the school fund is less R30 000. Next year you will have to buy your own schoolbooks. No money is given from the state.

S: How are you going to represent your findings?

Siva: On a big chart.

T: Thagraj's mother is the secretary, he will find out from her about income and expenditure and how the school has used the money this year.

S: Have you ever complained about the toilet facilities to the principal?

Niren: No.

In the above episode the classroom may be considered a microcosm of society in which pupils represent the democratic citizens and the teacher represents the government (though not elected by them). The pupil-citizens pay school funds to the school as a government who is held accountable and answerable to its people. The teacher as its bureaucratic authority, is responsible for collecting the school funds. Like responsible citizens the pupils question the authority about the use of their funds. In this sense the pupils act out democratic attitude and competence. The teachers assert their authority in quite different ways. At first they use the authority of their positions as teachers and the teaching of mathematics content. But when the questioning persists, the teacher responds according to bureaucratic authority of a democratic government. It is in this second later, almost forced response that the teacher gives some meaning to learning about democratic living in a mathematics classroom by giving pupils information about the decisions made on their behalf. Even as the didactical contract that relies on the authoritarianism of the "normal" mathematics classroom is maintained, the teachers suggest that pupils exercise their responsibilities as pupil-citizens and act on this issue. But, they do not. Democratic living of course requires authority, but schools because of their very nature exercise particular kinds of authorities, which in turn both contradicts that democratic living as pupils are urged to focus on the mathematics, but also supports it as pupils are asked to do something about their concerns.

For the teacher, who had suggested this topic as a pilot project, it offered a new approach to the curriculum, an opportunity to do some mathematics, but also a way of dealing with pupil's resistance to paying school funds. The teacher intended for pupils to see, by working on this project, that the amount was not a great deal in comparison to the many other things parents spent on their children's education and this is borne out in the lists of school expenses and in several graphs pupils drew where the school fund appears as one of the lowest amounts. But their democratic concerns differed. While the teacher wanted to raise more general societal issues of differences and inequalities between the pupils' background such as their parents' different incomes, the pupils focused on the specific issue of their school fees and their needs and concerns at school. The pupils' orientation to the problem was different from the teacher's. Several pupils expressed their surprise at realising how much their parents spend on their education but this served to strengthen their argument that the school fund was too much rather than that it was relatively less than other costs. The pupils took to democratic life quickly and they exploited the smallest opportunity to exercise their right to question but in doing so they also

faced the authority against which this takes place.

Pupils' concern with the amount of school fund is tied to their unhappiness about what it is spent on. The debate about how the school funds are spent point to an urgent problem of a lack of democratic forms available to pupils in the life of schools and the extent to which schools deny and underestimate the capacity of young children to understand and participate in discussions about issues that affect them. What is remarkable is that pupils make their voices heard given the slightest opportunity, despite the deeply authoritarian culture of schools, manifest in the continued use of corporal punishment. In the debate the pupils make telling and reasonable arguments against the teacher who represents the voice of the school authority. The school appears to be bungling, uncaring and incompetent in the face of the very different priorities the pupils set. The pupils' toilets are in a terrible condition (compared to the teachers); sports facilities are inadequate, textbooks are tatty and the school is trying to set up a library. The money could have been better spent. In this respect this project represents a strong parallel to the problems of democracy in civil society.

The possibility for the classroom to serve as the arena for acting out a democratic life is taken further by the pupils. This is evident not only in the numerous activities they undertook such as drawing graphs to show their expenses and interviewing relevant people, but also in the way in which pupils engaged not only the teacher but also each other. In their final presentations, for example, as the all boys group 5 continued to criticize the school about the fees issue in their assessment presentation, they were challenged by other pupils in the class who took up the teacher's arguments, especially group 3. The next day, when group 3 did their final presentation, however, a discrepancy emerged within the group as one pupil held his position against the group and the teacher:

Mohan: Mam its ok, but mam what ... mam its not really a cover mam ... we thought mam when they said they going to build a structure, we thought that it was going to be fully closed and all, and its going to be like a hall.

T: Ok that's two hundred thousand not hundred thousand.

Devan: Mam that wasn't our ... that was like Mohan's question. He agreed to mam, because Harry, Vikesh ... we thought that it was ... school fund is quite ok because the things that we get.

Some pupils agreed with the teacher about the need for the shelter, but the majority of the class did not. Various points of views were argued. This democratic acting, however, is short lived and moreover this opportunity is not fully exploited. The competence to access information that would create a more informed situation and allow more and different kinds of questions to be asked about the funding of their education does not occur. Thagraj did not present any information from his mother who was the school secretary, about what the school fund is spent on. The school's budget or financial statements did not appear in the class. None of the pupils calculated the total amount the school collects in the form of school fund for the year or listed all sources of funding. They did not question

who comprises the school fund committee and how and what decisions they make nor did they ask whether pupils could participate. The focus was on the presentations and its corresponding assessments which marked the end of the projects and the pupils did not act on their findings. Classroom life and organisation seem not to be entirely conducive to democratic living. Ultimately the teacher is the authority and she cannot be voted out of office. There is content to be learned, this is the mathematics class and time is limited.

Democratic life is also curtailed as the authority of the teacher is carried through the assessment requirement, which takes on a competitive character. The competitive nature of school life is carried into the project work situation. Pupils try to guard rather than share project ideas so that they can earn a good mark. Even though I mentioned to Sumaiya the opportunity for the two groups to address different aspects of how education is funded, this does not happen. Neither teachers attempt to contextualise the school fee issue more broadly to what different schools pay across the previous racial divides, nor is it discussed in class. But, one pupil from the sports group, Navin mentions this in his diary: "I think coming to school per year is a hell of a lot of money. Paying R1810,00 and for shoes I pay R250,00. That sure is a lot. But it is not bad because in white schools just for school fund is R3000 to R5000. So it is not bad." The current debate about a more just distribution in the funding of education to address the deep inequalities in the education system inherited from apartheid, were all left unexplored. No one asked who pays for school buildings, for teachers, and so on.

While project work seems to open possibilities for pupils to learn how to act in democratic ways this is mediated through the authority of the school setting in which the teacher gives higher priority to some parts of her role than to others. Learning to become democratic is lower on the list of what pupils need to learn in a situation in which there is so much to teach and learn. Hence it seemed to occur incidentally and on occasion when opportunities did arise the teachers authority takes different forms which may or may not exploit particular learning opportunities. The teacher's suggestion for a debate kept the issue within the realm of a school-like activity where her authority as teacher could probably be preserved and easier to manage. This contrasted with Sumaiya's suggestion to write to the parent-teacher association which is closer to a democratic political activity.

In attempting to live this democratic life, there is also the sense in which pupils demonstrate the limits to which they can act on what is found out. In the absence of having challenged the authority of the school before they do not really believe that it can be done and they lack the skills and competence to do so. Even though the possibility to act on their project findings is mentioned by the teachers, its not taken up seriously as one of the student teachers pointed out "they ... so used to their opinion not being taken into account in school normally". School life is a pseudo real life in which pupils' actions do not translate into real outcomes. This is borne out also by the noise and lack of attention paid by pupils during discussions. Yet the

seriousness of the issue has not escaped them even though this is not so evident in their classroom interactions and behaviour. Siva reflected: "I just love this kind of topic. It is very interesting. You can learn a lot from this project. This is very important to know. You will know how much money your parents spend on you. They work so hard and support you. Money does not come in easily."

Realising some form of democracy in the project work classroom impacted not only on the project work itself but also on the relationships. Giving pupils the freedom to choose and to control their learning forces a different interaction with the teacher figures during project work. The teacher is a facilitator or a supervisor rather than "a bank and dispenser of knowledge". This meant the teachers had to ask different kinds of questions, eliciting pupils' opinions and decisions and giving suggestions rather than telling them what to do. The teacher's authority is not, of course, absent and it still carries through in other ways. After all, the teacher is going to give an assessment. The pupils' relationship with the teacher also changes because of the kind of questioning that the project invites not only in the class but also outside. For the pupils tasks such as interviewing the principal and other teachers in the school fundamentally changed their status in the interaction as they asked questions that potentially criticised the teachers, about too much homework or under-utilisation of sporting equipment. In this respect the pupils spoke to teachers almost as equal. The pupil-citizens wanted answers about the decisions and actions of the "school government". In these situations outside the classroom, pupils confronted not only the authority of the school but also its authoritarianism. Some teachers refused to engage the pupils seriously and others resented questioning of their actions. This raises the question of how any attempt to introduce a democratic classroom life could coexist in a predominantly autocratic school environment.

During project work as democracy comes to have a place in a mathematics classroom authority gets rearranged, and in turn, the kinds of authority influences the forms of democratic life possible within a classroom or school. Typically the teacher's authority is the main form of authority in the class maintained through her position as teacher and the authority of the subject matter, and it is not usually questioned. In project work this authority is questioned and transformed as particular democratic processes are enacted.

3.2 In the groups

We have seen how democracy and authority operate in the whole class interactions but this theme is also woven into the interactions and workings of groups. Instead of the teacher directing the projects, this responsibility was devolved to the groups. This meant that opportunities for pupils to exercise some democratic actions were constantly made available. This weakened the teacher's authority to some extent through its extension to a sort of group authority. In many ways the functioning of the groups as a "micro-society" can be seen to be parallel to the functioning of a democracy. The authority required for democracy in the group to function, however, had to also come from

the group itself. Moreover, this authority and democracy was enacted within the framework made possible by the teacher and school.

The first democratic act undertaken by the groups was to decide on a project and in some groups such as group 3, this required a vote. As a result not all pupils in the group related to the project in the same way or identified with and owned the project to the same extent, which had implications for the nature of the project and their participation in the group. Even though pupils were told they had the possibility to move from the groups they were in, none of them did. In this way this “micro-society” contained the diversity of the society of which it was a part, as they shaped and dealt with their project problem. In choosing the projects, groups chose those projects that they understood, found interesting, relevant and considered important. This also meant that they were free to choose projects that did not require too much work or too difficult mathematics as both groups 3 and 5 admitted in their final presentation.

A second democratic process may be seen in the option groups were given of choosing “group leaders”. Group leaders were required to co-ordinate the group’s activities, and be its representative. Different kinds of leadership emerged in the different groups, though not exclusive of each other. Democratic leadership was evident in groups where the leader tried to include as many members as possible in as many activities as possible and showed a concern for decisions to be shared or as far as possible based on some form of consensus. A second form of leadership was a sort of shared or co-leadership in which there was not one leader but a group leadership. It carried some democratic elements but this leadership form also served to seriously exclude some group members. Third, there was a teacher-leader where the leadership role included a deliberate teaching role and brought with it the teacher’s authority into the group. Fourth, the marginalised leader who may have been elected in the group as the leader but was unable to demonstrate the authority needed to be taken as the leader in the group. Whatever the leadership form, pupils had to act out a democratic process of electing a leader, and with each kind of leadership, pupils also demonstrated different forms of authority. Their authority was related to their position in the group but also derived from their competence in language or in mathematics.

Democratic life within groups involved listening to each other and acting on agreements as well as disagreements as they had to arrive at some consensus on their interpretation of the project topic. The question was both what ideas and whose ideas would come to be taken as representing the project ideas. As groups agreed on project activities, they had to divide the work, which included both “intellectual” and “menial” tasks. In their effort to give everyone in the group a role to play, some pupils got to do things like bringing materials. Others got to produce graphs. Still others did not get to do anything for the group but quietly did some work in the diary-workbooks. The question then is also who gets assigned the less intellectual work and why.

In each group there were a few members who were

active, dominated and took control and responsibility for the group’s work. This had the effect of both ensuring that work needed to be done in the project got done, but also reduced the ownership of the group ideas by some of the other members. A member of newsletter group 2, Sharitha, who is silent in most of the group’s discussion wrote about the rejection of her ideas. “Dear Diary, I like to tell you about my group. What ever I got they said to take it back. They say it is not related to math but I do not have no puzzle. And I do not like my groups people bossing me round.” Participation in the groups varied according to several dimensions such as race, language, and perceived competence to do the agreed tasks and was also affected by absenteeism of group members especially due to the teacher strike. During the presentations the groups relentlessly and sometimes ruthlessly questioned the limited and excessive participation of some members. In this we see authority and control is neither once and for all and nor the preserve of the same people as it shifts and gets challenged in different parts of project work as democratic process are allowed. A democratic concern about fair and equal participation was evident but it was always directed outward to other groups and not inward to the groups’ own functioning.

While the potential to learn how to act and work democratically in a group existed in all groups, the extent to which this opportunity can and is actually taken varied considerably. Some groups operated more democratically than others. The opportunities to act democratically were counterbalanced by the traditional requirements of a mathematics classroom such as assessments and the pressure to do mathematics on the one hand; and the requirements of project work on the other, to involve everyone in the group in the project and the need to do something about a problem deemed important. In managing this daunting and difficult challenge authority sometimes became authoritarianism. These multiple pressures on the group sometimes led democracy in the group to be countered by some pupils importing the teacher’s authority into the group. The difficulty lies in the extent to which pupils are able to act democratically in allowing the capacity and ideas of individual group members to be realised but also at the same to exercising sufficient authority to meet the requirements of the different activities in the project. The struggle with this tension results in many relevant ideas remaining outside the discussion of the project problem which can be seen when comparing the diary-workbooks of individual group members, their participation in the group and the ideas that appear in the final presentations of the group.

Conflicts emerged both within groups but also across groups. The reluctance and resistance of groups to collaborate and co-operate with each other is evident. Despite Sumaiya suggesting to several groups to make contributions to the newsletter, none of the groups took up this offer and nor do the newsletter group solicit articles. The fact that groups are going to be assessed militates against groups working together. It is not in their interest to co-operate, if the aim is to get the highest marks for the group and yourself. Democratic forms of interacting and co-operating contradict in competitive situations and the

competitive aspect of traditional classrooms are preserved in the project work situation through the assessment requirements in the final presentations. Acting on the project findings did not count in getting a good mark.

The authority of the teacher as it related to mathematics teaching and learning itself also involved some democratic concerns. Sumaiya asserted her authority in the groups as a mathematics teacher in introducing and emphasising the need to include mathematical ideas. Some groups were given a text on data handling prior to a conventional lesson on drawing graphs, conducted for the whole class in the middle of the project work experience, using data generated by the groups. The groups were free to decide on the mathematical ideas and representations to include in their projects but they took up the ideas suggested by the teachers. Both Devan and Thandeka chose not to draw pie graphs in their groups because they were considered difficult to draw and because of their perception that the majority of the class were unlikely to understand them during the presentation. Although the teacher's authority forces them to consider mathematical representations in the first place, the choice of the type of representation was a democratic concern. The pupil-citizens made choices so that everyone could participate. Moreover, groups debated about: drawing one graph to reflect a group position or to draw separate individual graphs; what information to show on the graphs; monthly or yearly costs; and whether to include costs incurred by parents for siblings. While the authority of the teacher and its manifestation through the discipline continued, spaces for some democratic forms of acting are created which in turn challenge that authority.

3.3 *The teacher-student teacher-researcher domain*

This theme of democracy and authority is also interwoven in the triad of relationships between the teacher, the student teachers particularly Sumaiya, and myself. Typically student teachers are deemed only learners but in this project work situation the teacher has to acknowledge Sumaiya's knowledge and understanding of the approach. The teacher's authority lies in her knowledge about the learners and about practice in classroom management. Sumaiya's authority is expressed through the way in which she interprets the approach in the classroom and constructs the teaching-learning experience. The potential for conflict is always present and it erupts sometimes in the classroom especially over the amount of teacher direction to be given to pupils. For the most part this is a constructive tension between Sumaiya and the teacher as they mediate both older established forms of authority and new ideas about a democratic classroom.

Both Sumaiya and the teacher demonstrate concerns of a critical mathematics education having to do with making societal inequalities visible, but differ about how it should be handled within classroom practices which points to a contradiction in critical mathematics pedagogy. For Sumaiya, this approach is about giving pupils freedom, a term she uses frequently in her post lesson reflections. She struggles to minimise prescription about what pupils should do in the project and is concerned about imposing her authority on the pupils. But this runs the risk that

pupils may not engage the critical ideas the approach seeks to realise. The teacher argues constantly for giving more guidance so that pupils face these critical ideas and do the necessary work to be evaluated. The concern to offer pupils a democratic environment is in constant opposition to and yet also assists the concern about discipline in the classroom to keep pupils on task.

This theme of democracy and authority, which occurs in the classroom between the teachers and the pupils, is also apparent in the relation between Sumaiya and the teacher on the one hand and myself on the other. I am constructed as an authority by the student teachers and the teacher and school in my role as a teacher educator/researcher. Sumaiya and the teacher defer to me to deal with conflicts and uncertainty in the approach. In my interactions with Sumaiya, I was a resource, but she made her own decisions about what to do in practice in much the same way as she interacted with the groups. I offered reflections about classroom events and she responded to the authority inherent in my position by implementing some ideas, yet also disagreed and chose not to act in ways that I suggested just as the pupils did in response to their teacher's suggestions.

The theme of democracy and authority permeates the domains of mathematics education from the whole class context to the workings in the group and also within situations in mathematics teacher education. But what exactly is this relationship between democracy and authority?

4. **Complementarity, authority and democracy**

In this section I explore the proposition that the concepts of democracy and authority be described and explained as existing in a binary which forms an antagonism and yet also works in co-operation within a mathematics classroom – being both “opposite and complementary”. For this idea, I draw on Brousseau and Otte (1991) who point to “a whole series of pairs of concepts: insight and action, intuition and formalism, and so forth, the character of which (they) have tried to show as both paradoxical and necessary, opposite and complementary. These oppositions are the source of the fragility of the act of knowing and the difficulties in the transmission of knowledge” (p. 35). Here, Brousseau and Otte illustrate implicitly the notion of complementarity, the complexity and the contradictory nature of mathematics teaching and learning when they explain the necessary but paradoxical nature of the didactical contract. Even though I have in mind quite a different mathematics classroom from them, the importance of this idea of complementarity is emphasised because this complexity is increased several times when the goals of mathematics teaching and learning are changed or widened as they are in situations in which a critical democratic concern is brought into the classroom.

In the project work situation described here we see how democracy is brought into the lives of groups of individuals in a classroom and school as it becomes a microcosm of society. What elements of democracy were practised in this mathematics classroom? Pupils voted on project ideas, elected group leaders, acted out that leadership; questioned different kinds of authority, mathematised some aspect of

reality; explained it to their fellow citizens; reacted to a mathematisation from other citizens; distributed work in diverse groups; faced questions about acting with fairness; and tried to get the work done for assessment. They have lived through the many difficulties and dilemmas of democratic life. This democratic life lived in a mathematics classroom is not smooth or easy as it comes into existence against the structures and authority of the institution of the school. The problems are not pseudo problems. They are real and important to the pupil-citizens. But in order to act in the democracy they must already know what it means to be democratic. Learning about being democratic faces the same paradox of the didactical contract elaborated by Brousseau and Otte (1991): "As the person solves a problem inherent in a situation, he obviously has all the knowledge that is necessary for that purpose. The fact that he learned something from the situation is manifest by his failure to solve the problem. The knowledge is the prerequisite as well as the result of the problem solving activity" (p. 34). The failure of the didactical contract represents also its success and following this point, pupils learn about democratic values, attitude and competence precisely when they show a lack of these. This learning however depends on creating the opportunity within the mathematics classroom to bring these into pupils' consciousness or awareness and possibilities for action.

Complementarity seems to offer a way of thinking about what was happening in the classroom that grasps a more complex and deeper understanding of the dual concepts of democracy and authority and of the concepts themselves. With its origins in the work of the physicist Niels Bohr, the idea of complementarity has been invoked in mathematics and mathematics education by several authors (see for example Kuyk 1977; Steiner 1985; Mellin-Olsen 1993; Bartolini Bussi 1994; Sfard 1998). Often, it is not elaborated, but used as a justification for bringing together conflicting but necessary positions or theories in mathematics education. Its most recent and substantive development has been in the work of Otte (1990, 1994)⁹. For Otte, complementarity represents a basic perspective in our coming to understand and provides a way of speaking about our means for understanding as being insufficient. If we take any one perspective, then we exclude another. This does not mean that the other is not present but that when we experience the one the other is excluded. A main example of complementarity explored by Otte (1994) is through the notions of object and tool, and using these he specifies complementarity as constituting two main ideas. First, objects and tools are woven together. They presuppose each other. The one cannot be defined or described without the other. Second, objects and tools are contradictory to each other. They oppose each other. One does not directly show itself in the other. So for Otte the idea of complementarity expresses a basic and essential condition for knowledge production.

In my use and interpretation of this notion of complementarity I do not seek to follow exactly in Otte's footsteps as it were, but rather to use the underlying principle as inspiration for analysing the happenings of a mathematics classroom, particularly one in which we seek to realise

ideas about democracy. The question we might pose is how could the principle of complementarity feature in the way in which we understand and produce knowledge about mathematics classrooms. I am proposing that complementarity offers a theoretical analytical means for exploring a critical mathematics pedagogy.

In the choice of the concepts of democracy and authority, they oppose each other, but they are not in direct opposition. For example, the opposite of democracy could be autocracy or authoritarianism. But this will once again return to an almost unitary conception because it reduces the complementarity of the concepts to a simple negation, which does not serve to explain the complexity of the theme. Complementarity subsumes a duality as its use in the theme points to a special relationship of contradictions and co-operation found in practice, in the classroom, in the attempt to realise the theoretical ideas of this curriculum approach, which integrates a critical perspective. Authority and democracy are both needed to understand teacher' and pupils' actions and reflections within this curriculum. In order to realise any kind of democratic life in the classroom we must assume and expect that there exist particular forms of authority. The authority that exists in a classroom is never absolute. Some expression of democracy is always present in how pupils react to that authority, even if that democratic action refers to resistance. One exists constantly in the context of the other as one form is always present when the other is handled.

The project work situation carries the complementarity of the goals and functioning of the traditional classroom on the one hand and the theoretically or hypothetically realised ideas of a critical mathematics pedagogy on the other. That reality of the classroom that preceded the innovation does not disappear, even as an arranged situation is created and imposed. What occurs in the classroom has fundamentally changed but it remains in a complementarity with the former actual situation. In the arranged situation authority and democracy are placed in antagonism with each other, but are neither inseparable nor reducible to each other. Through complementarity we are able to see not what teachers failed to do or pupils failed to learn but rather how contradictions are lived and held together, especially when a radically different curriculum approach is brought into the classroom. There are good reasons for what participants do in this classroom and complementarity opens for a different perspective in understanding teachers' and pupils' lives in such classrooms that does not build in a deficit view of both learners and teachers.

The complementarity of authority and democracy was made visible in the attempt to give meaning to a critical perspective in mathematics education. But what does it mean to be critical? It means "to draw attention to a critical situation, to identify it, to try to grasp it, to understand and to react to it" (Skovsmose 1994, p. 16) and a critical situation is one that contains crises, which includes conflicts, inequalities, exploitation and so on. If the classroom is to be the place in which pupils are to be critical in this sense, then there seem to be two possibilities. The first, favoured by teachers, is to import a critical situation into the class, and the second, preferred by pupils,

is to constitute the classroom itself as a critical situation. The idea that democracy can refer to the very life of a classroom has to take into account how that democratic life can come to exist against particular forms of existing authority when the crises and conflicts relate not only to society “out there” but to the very classroom or school as a “society”. Both in theory and in practice much of the literature is focused on the former and rather silent on the latter. The assumption is that teachers and pupils are in an unproblematic agreement about what constitutes critical situations in society. But if pupils are free to choose the critical situations to which they are expected to react, their interest is not with a society “out there” but the “society” of school. Critical situations are different for teachers and pupils precisely because they are positioned differently in relation to the situation and have different vested interests. Hence, participants have to negotiate to agree on what constitutes a crisis. It is here that the freedom for pupils to choose and act on a critical situation comes into conflict with the authority of the teacher especially when the critical situation itself implicates the teacher. When this happens, the teacher gets placed in a precarious situation in which she has to exercise at least a dual authority – that inherent in the (mathematical) didactical contract and that of the authority of government in the school.

Whatever the critical situation, it features the complementarity of democracy – the freedom and competence to criticise and act – and authority – that against which the critique and action is directed. The question is both who and how critical situations are defined. We observed that they are developed outwardly, away from the people doing the defining rather than reflexively – teachers direct their attention to society “out there” rather than at their schools or classrooms, and pupils focus on the school or teacher rather than on their own group’s workings.

A main assumption in connecting democracy to mathematics education is that it should take a form that will enable pupils to act in and react critically to those situations in democratic society in which mathematics is integrated. There are two senses in which we can speak of democracy in mathematics classrooms. The first is that mathematics classrooms can teach *about* democracy – about voting and elections, about equality and fairness in the distribution of goods, knowledge and opportunities; and about how mathematics is interwoven into the fabric of society. The second is that the mathematics classroom can teach *through* democratic living by giving learners an opportunity to participate in an actual (or virtual) democracy. The classroom or school itself is seen as a democracy with the social and political responsibility to developing critical pupil-citizens through the communications and relationships found in school. These two ways of approaching democracy are not mutually exclusive. Each of these presuppose the existence of particular forms of authority which could support or thwart democratic action.

Complementarity in this theme is significant for an “education after Apartheid” in which the pupils are effectively speaking out against authoritarian and undemocratic practices of the past both in schools and in society. Not only do young learners demonstrate this combined democratic and

critical competence, they do this in relation to the authority of the teacher and the continued authoritarianism of the school. The task for the pupils is quite difficult as they react to different kinds of authority. There is the authority of the teacher by virtue of the mathematics didactical contract and the authority of the teacher in the context of the “classroom as a democracy”. Equally pupils also have to react to the authority of the school which operates as an autocracy rather than as a democracy. The point to be made here is that authority exists, both as a phenomenon of schools and as a feature of a democracy. Authority and democracy operate in antagonism though not incompatibility with its opposite partner. In fact democracy and authority require each other in a classroom and especially one that is concerned with critique. Each contains elements of the other. In this sense the complementarity between the concepts are also contained within themselves.

If we take authority, it is both positive in that it is required and necessary in a democracy, and negative in that its expression in schools is often as authoritarianism. Giroux (1997) argues: “In its emancipatory model, authority exists as a terrain of struggle and as such reveals the dialectical nature of its interests and possibilities; ... For radical educators ... the dominant meaning of authority must be redefined to include the concept of freedom, equality and democracy” (p. 102). But within a mathematics classroom the teacher has to manage an “emancipatory authority” in a complementarity with a “didactical authority”. The democratic ideal, however important, is still only one goal among several competing goals that schools are expected to achieve. Unless these different expressions of authority become a part of our understanding of a critical pedagogy, we remain unable to take account of the deep contradictions teachers’ face in practice, in realising such pedagogy. The use of the terms authority and authoritarianism correspond to these positive and negative meanings respectively. Moreover, these forms of authority not only exist but are important for learning about democratic life. They play simultaneous contradictory and complementary roles in schools. When pupils face the authority (or even authoritarianism) of the school they learn about the limits of their individual actions but they also learn that such authority exists and against which they have the possibility to raise their voice. The authority of the teacher is an authority that could support their challenge in a democratic situation while at the same time it is also an authority against which they have to react and with which they must interact. The teacher’s authority became an authority against which pupils could practice and understand what it means to question and challenge authority. And in doing so they express a *Mündigkeit* – a capacity to speak for themselves against some authority (Skovsmose 1994).

In what sense can we claim a complementarity in the concept of democracy? Here it is possible to distinguish on the one hand, democracy as a set of theoretical hypothetical ideals that represent a potentiality, possibilities for schools and classrooms and, on the other, the actuality of a lived realised democracy. These may be considered to be in a complementarity. Each is required by the other to give meaning to the relation between mathematics ed-

ucation and democracy since the one provides a source for the development of the other. And yet they also oppose each other as that meaning has to come to reside in schools and a mathematics classroom which were never intended and by and large nor do they operate as places in which to become democratic, learn democratic values, develop democratic competence and a democratic attitude, are given priority. Democratic ideals remain in a complementarity with democratic realities. They need each other, one cannot exist without the other but they also oppose each other, which means we can account for the contradictions they create.

5. Conclusion

As pupil-citizens interact with the authority of the teacher in a mathematics classroom and the school, the school itself, as an institution, has to act and react within larger democratic society and its forms of authority. While the development of a critical and creative pupil-citizenry is encouraged in theory and in the rhetoric and policy of the new curriculum in South Africa, it remains to be seen how the authorities within our democracy interacts with those who react critically toward that authority. As Fasheh (1982) has warned, in many “developing countries” with young and fragile democracies, and histories of some form of autocracies or dictatorships, alternate points of views and critique are often not allowed, let alone tolerated or respected.

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6. Notes

- ¹ These questions are adapted from Skovsmose (1994). They may be taken to refer to the formal, material and ethical conditions as well as the conditions for participating and re-acting within a democracy.
- ² This paper is derived from work in progress for a PhD study with the working title “In search of a pedagogy of conflict and dialogue for mathematics education”. Aalborg University, Denmark.
- ³ For further elaboration of the student teachers’ understanding of this approach see Vithal (1997) and for descriptions of the projects they implemented in school see Vithal et al. (1997).
- ⁴ Despite the demise of Apartheid, many aspects of South African life, including education, remain largely racially segregated in different ways.
- ⁵ Racial categorisations continue to be used in South Africa especially to redress past inequalities. It is also related to language differences. “African” pupils are English second language speakers and “Indian” pupils have English as a first language. The language of instruction is English.
- ⁶ A wide range of data sources were used to develop this case including interviews with the teacher, student teacher, pupils; their journals; classroom videos; all documentation produced by Sumaiya such as lesson plans and project report; and pupils’ written work.
- ⁷ The project work extended well beyond that period of time set aside for mathematics as it spilled into lunch breaks, other lessons when a teacher was absent, after school and continued during a teacher strike that occurred in the middle of the project.
- ⁸ Themes included “authority and democracy”; “structure and freedom”; “mathematics and context”; and “differentiation and

equity”.

- ⁹ In its elaboration here, I rely on the translation and interpretation made in a discussion between myself and Ole Skovsmose of Otte’s (1994) work since this work is not available in English. In both summarising and interpreting the concept of complementarity from a chapter in Otte’s book, I draw on these notes which I have shared with him in personal communications.

7. References

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Vorschau auf Analysethemen der nächsten Hefte

Für die Analysen der Jahrgänge 31 (1999) bis 32 (2000) sind folgende Themen geplant:

- Mathematik und Deutsch
- TIMSS
- Mathematikdidaktische Forschung im Primarbereich
- Mathematik an Hochschulen lehren und lernen
- Analysis an Hochschulen
- Mathematik in der Ingenieurausbildung
- Theoretische Betrachtungen zu Schulbuchanalysen.

Vorschläge für Beiträge zu o.g. Themen erbitten wir an die Schriftleitung.

Outlook on Future Topics

The following subjects are intended for the analysis sections of Vol. 31 (1999) to Vol. 32 (2000):

- TIMSS
- Research in primary mathematics education
- Teaching and learning mathematics at university level
- Calculus at universities
- Mathematics and engineering education
- Concepts and issues in textbook analyses.

Suggestions for contributions to these subjects are welcome and should be addressed to the editor.