

## Didactics of mathematics as a scientific discipline: A sketch of its development from a personal (autobiographic) point of view<sup>1</sup>

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*Dedicated to the memory of my "two fathers", both born a hundred years ago: Alfred Steiner (2.2.1898–29.3.1979) and Heinrich Behnke (9.10.1898–10.10.1979)*

**Abstract:** At the 8th International Congress on Mathematical Education (ICME-8) held in Sevilla, Spain, July 1996, Working Group 25 was devoted to the theme "Didactics of Mathematics as a Scientific Discipline". The proceedings of this activity have been published by Nicolina A. Malara (s. footnote 1). The plenary talk given by the author on "Basic Characteristics and Recent Trends in the Development of Didactics of Mathematics as a Scientific Discipline" as well as the round-table discussion under his chairmanship are documented in the proceedings. Furthermore the author has contributed to the proceedings an autobiographically oriented paper which goes back to his own time as a schoolboy and refers to his professional development in connections with various national and international institutions and movements. The present slightly extended reproduction of that contribution is mediating aspects of developmental dynamics which can hardly be grasped by highly objectified presentations.

As for a similar paper see H. G. Steiner: "Some notable anniversaries and some personal German-Italian memories in didactics of mathematics. ZDM 26 (1994), 211–214; also in Italian: L' insegnamento della matematica e delle scienze integrate (Numero Speciale), vol. 5 N.17A–17B, Sept.–Oct. 1994, 425–437.

**Kurzreferat:** *Didaktik der Mathematik als wissenschaftliche Disziplin: Ein Abriß ihrer Entwicklung von einem persönlichen (autobiographischen) Gesichtspunkt.* Auf dem 8. Internationalen Kongreß für Mathematikdidaktik 1996 in Sevilla war die Arbeitsgruppe 25 dem Thema "Didaktik der Mathematik als wissenschaftliche Disziplin" gewidmet, wozu Nicolina A. Malara einen Berichtsband herausgegeben hat (s. Fußnote 1). Der vom Autor gehaltene Hauptvortrag zum Thema "Basic Characteristics and Recent Trends in the Development of Didactics of Mathematics as a Scientific Discipline" und die von ihm geleitete Podiumsdiskussion sind im Berichtsband dokumentiert. Für den Berichtsband hat der Autor sodann einen stark autobiographisch geprägten Beitrag abgefaßt, der bis auf die Schulzeit des Autors zurückgeht und einige Etappen seiner eigenen Entwicklung in Verbindung mit der nationalen und internationalen Entwicklung umreißt. Die hier punktuell präzisiertere und erweiterte Wiedergabe des Beitrags vermittelt Aspekte von Entwicklungsdynamiken, die in stark objektivierten Darstellungen schwerlich zu erfassen sind.

**ZDM-Classification:** A30

### 1. Kamen in Westphalia: At secondary school 1939–1949

When in 1949 I matriculated at the 1780 founded *University of Münster* (Westphalia, Germany), 40 km north of my home town Kamen (at the eastern border of the steel- and

coal-based industrial area known as the "Ruhr-district"), this university which presently has an enrolment of about 45000 did not comprise more than 3000 students. During the Second World War the centre of the city of Münster had been destroyed up to 80 % by repeated air-raids and so had been the buildings of the university. In 1949 an intense re-building process was under way and the faculties had already been differentiated – though with a thin coverage of staff – into institutes, seminars, special research and teaching areas, related chairs and professorships etc., thus representing from my personal point of view quite well the universe of scientific work of that time.

I had graduated from high-school by the "Abitur"-exam taken at the *Gymnasium (grammar school) in Kamen* in January 1949 and began my university studies in the fall intending to become a theoretical physicist. In between I had established relations to the laboratory and the development and research department of the big *iron and steel factory Hoesch (Westfalenhütte)* at Dortmund, a large industrial city west of Kamen, and I worked there for half a year in 1949. By continuation during semester-vacations in the following years, I worked at Hoesch in the domains of physics, metallography and stability testing as well as deformability testing altogether for more than 3 years time and particularly learnt a lot about applications of physics, chemistry and mathematics and did my *first cooperative and individual research*.

My *school time at the Gymnasium* in Kamen that I had entered at the age of 10 in the spring of 1939, which in Nazi-Germany was also the beginning of a compulsory membership of political youth organizations (first 4 years "Jungvolk" (Young Folks); second 4 years "Hitler Jugend" (Hitler Youth) in which I had chosen the division for building and flying airplanes, beginning with gliders) was over-shadowed by the war, particularly because of an increasing number and growing dimension of air raids. From 1942 onward much schooling time was spent in air-raid-shelters. In January 1944 – I had my 15th birthday in November 1943 – the boys in my class like equally aged boys from other grammar schools in Germany were drafted to become *anti-aircraft cannoneers* at guns of calibers 2 cm, 3.7 cm, 8.8 cm, 10.5 cm, 12.8 cm as had already been practiced with corresponding age groups of gymnasium pupils in the previous year. The status of anti-aircraft artillery (flak)-aids was partly independent from the membership in Hitler Youth and partly defined by the air-force: the official name was "air-force-aids". The boys of my class were appointed to a battery with half-automatic guns of caliber 3.7 cm first for normal military training and then for special flak-training. We then belonged to a unit of 3 guns, each provided with a corporal and 6 flak-aids plus some Russian prisoners of war belonging to a special group with anti-sowjet goals. Units of batteries of the various calibers were concentrically surrounding the *hydro-generation (synthetic oil) plant*, operating in our neighbour-town *Bergkamen*. We stayed in barracks and after the end of the flak training a kind of *school program* for 3 afternoons per week was established in which we partly met with flak-aids from schools of other towns in the area and which was taught

<sup>1</sup>Corrected and slightly extended version of a paper, published in Nicolina A. Malara (Ed.): Proc. ICME-8, WG25 – Didactics of Mathematics as a Scientific Discipline. Univ. of Modena (It.), Dept. of Math. 1997, 159–167.

by teachers coming from various schools involved. One of the math and physics teachers from another town (Ahlen in Westphalia) talked much about his special hobby field machine-building and by his enthusiasm made me try to read in related books he gave me, which then actually resulted in a life long hobby field of mine supported by a christmas-present from my parents who had found in a second-hand bookshop a 12 volume encyclopedia on machine-building. Another teacher for German literature who had been deeply involved in the war and as an injured soldier been released from the army turned out to be a thoughtful and inspiring philosopher who tried to open us a way to Kant's epistemology and ethics.

In September 1944 the heavy caliber batteries of the anti-aircraft protection-belt to the Bergkamen synthetic oil plant were moved to the German east front in the Sowjet-Union. Some days later a fleet of 300 Fortress II-bombers of the US-airforce flying at an altitude of about 3000 m, unreachable for the low caliber guns left, totally destroyed the plant and a great part of the town of Bergkamen, including several of the 2 cm flak emplacements on the inner circle. Several hundred people of Bergkamen and several of the school boys serving as flak aids were killed.

In December 1944 a part of my battery including my unit was moved to a position near the *city of Werl*, 25 km south-east of Kamen, for the protection of a night-fighters' airport, lodging some of the new jet driven airplanes. In February and March of 1945, without a reasonable number of German counterparts, the air in this area was full of low flying Allies' air-fighters (of types Spitfire, Thunderbolt etc.) attacking not only the airport but also trains, cars and waggons on the roads, as well as our own emplacements from which a very busy shooting took place.

After the invasion of British and US troops at the French coast on June 6, 1944 the Allies had reached river Rhein in September 1944 and crossed it in March 1945 with further pursuing the plan to conquer the Ruhr-district by encircling it in pushing forward east along the northern and the southern border of the district and then concentrically penetrating it. In the beginning of April when American troops were already close to Werl a classmate and myself were ordered to the headquarters of our battery, placed near *the town of Hemer*, 35 km south of my hometown in a mountainous area, for the protection of a lime works. The two of us had to do our services in an observation position above a strategically important street along a valley (the "Hönnetal"). According to the name of the nearby small village it was called "Tanks outpost Volkringhausen", and we were equipped with a tent, 2 Czech booty rifles plus 100 pieces of cartridge, 4 bazookas (anti-tank bombs: Panzerfäuste), a telephone and a field-glass.

Other classmates at the Werl-unit, some months older than us, had already been drafted in February to do full services in the German army at the Western frontlines. We were informed that Werl had been occupied by US troops on April 8 and that two days before one of the guns in my former unit had been destroyed by a bomb and that no classmate was among those who were killed or injured.

The area of Hemer was reached by American soldiers April 13. They did not come to our area from the north

through the valley and therefore could not be observed from our outpost. We were called back to the head-quarters and saw them the next day how they had distributed several tanks in the background and how they came through the woods and over the hills individually and in groups moving softly like leopards in their crepe-rubber shoes, their camouflage suits and helmets, how they came negligently hanging in their jeeps, their machine-guns on their knees, surrounded by the sound of their wireless telecommunication. They looked like beings from another star.

At noon of April 14 all members of our battery had gathered in a grove. Our commander had decided that we surrender, but he made mistakes in communicating this to the American soldiers. So they withdrew and attacked the grove by trench-mortars which caused several injuries on our side. During a fire-pause two members of our battery came out of the grove holding a big white sheet. From this moment on I became a *prisoner of war* at a time when the Allies made millions of German prisoners within only a few weeks. And this meant being outdoors in camps, even without a tent, for a long time at the border of starving, surrounded by many co-prisoners who were suffering from and often dying of dysentery or typhus, until I was released June 14, 1945: first having been together with 5000 near Hemer, then with 30000 in Brilon, and from April 25 on with 300000 in the notorious PW-camp on the Rhein-meadows near Remagen.

When I *came home*, Kamen had become a town in the British Zone. The buildings of my school had been destroyed by bombs shortly before Kamen was occupied on April 8. I was obliged to do repairing work in town, e.g. filling bombing craters. Information said that my school would most likely not be re-opened before spring 1946. So I tried to find a job at one of those beautiful looking farms I had learned to love on my several bicycle rides between Kamen and Werl when I had weekends free from my battery. Also I had enjoyed summer vacations spent with my family at the big farm my father came from in East-Prussia near Insterburg, 1200 km north-east of Kamen, which I had visited for the last time as a flak-aid on vacation together with my mother in August 1944, shortly before the Russian occupation of East-Prussia began.

I was lucky to be welcome by the owner, a lady who liked to ride on horse-back and play the organ in church services, and by the supervisor of the historically well known *Böckelmann's farm at Kump*, a village north of Werl. It turned out that they actually had no other full time farm laborer, and some days later, they accepted another school boy of my age from Stadthagen (near Hannover), by the name of Herbert Bürstenbinder, who had also been with the flak and a prisoner of war in a British camp. Until spring of 1946 the two of us then ran the farm of 40 ha (100 acres) mainly on our own, partly – particularly in harvest time – supported by temporary helpers. And this meant taking care of the animals (the bull, 16 cows, cattle, 25 pigs, 8 horses plus 2 stallions belonging to the Kump cover-station which had been destroyed by bombs, cocks and hens, geese and ducks), feeding, maintaining, milking; going behind and guiding the horses when pulling the plough, the harrow, the cultivator; using the scythe,

the reaper and binder, the harvester thresher; bringing in the barley, which was ripe when we came, the rape, the rye, the oat, and the wheat as well as the beets and the sweet turnips; plugging the fruits according to the seasons, particularly the pears from high trees by binding ladders together, especially dangerous when the bull was on the meadow down there waiting for you to get you on his horns; in winter time crushing oats, sawing and cleaving wood, maintaining machines, stables, the fences around the pasture-grounds etc., etc., all the beautiful and sometimes hard work, which also gives you time for dreaming and thinking, throughout the wonderful cycle of the year in the country. On Sundays, Herbert and I tried to play our favorite instruments with our tormented hands and fingers, he the flute and me the piano, admired by our lady whose most attractive bookcase was another opportunity in her saloon for us to keep up with some non-agricultural culture.

In March 1946 my *Gymnasium at Kamen* was re-opened. Many of the former class-mates (boys and girls) did not return and new ones showed up. All of our former teachers had undergone a political evaluation and none of them had been dismissed which corresponded to our opinion about them; only the director was replaced in this position by another teacher. The former school system had comprised 8 years after 4 years of elementary school. Instead of 8 there were now 9 years and our school became a "Neusprachliches (Modern-Language) Gymnasium", in its curriculum particularly built on Latin, English and French, of which we had had English from the first and Latin from the third year. In spite of the fact that those who had been flak aids, did not have much schooling in 1944 and almost none in 1945, we were assigned to gymnasium grade 7 (Obersekunda) and in addition to this had to pass only two more grades (Unterprima and Oberprima). Our last school record (called "airforce-aids record") was of July 1944, signed by the director of our Gymnasium, a "teacher of confidence" (the one mentioned, from Ahlen) and the chief-commander of our battery. It included for all of us the promotion to grade 6.

Since there had been a rather strong anti-intellectual attitude and spirit in Nazi-Germany and an extreme isolation from the rest of the world except from being the occupier, which was another kind of isolation, there was now an overwhelming interest in reading and learning more about what had happened in other countries, what had been hidden or forbidden like Jewish literature, the (by the Nazis) so called degenerated art, scientific and technological progress etc. and especially what would be a humanistic philosophy on which to build our future, particularly after more and more became known about what Germany had done to Jewish people in Germany and in many other countries. Such questions I had already discussed with older, mature friends I had found in the camp of Remagen. They were also on my mind when being a farmer.

At school, two of my classmates with whom I was closely connected already from the first elementary school grade on and particularly during the flak time, and who as frontsoldiers had been prisoners of war in camps in France

and Belgium until the end of 1945, formed together with me a group of three which according to the famous group in Ancient Rome we called the "*Triumviri*", and we addressed each other, besides by our nicknames, especially in funny or quasi-ceremonious situations, by "Caesare", "Crasse", "Pompeje" (the last one applying on me), using well the Latin vocative. Caesar was the primus in class, excellent in all school-subjects, including sports, who more and more developed a deep interest in philosophy and linguistics. Crassus' favorite fields were German literature, especially lyrics, modern history, and classical languages. My strongest fields were mathematics, physics, astronomy, and music, the latter both in theory and practice, playing the piano and singing "Lieder" by Beethoven, Schubert, Brahms, Hugo Wolf, Max Reger, accompanying myself at the piano.

We became members of a *movie-club* and went there with our girl-friends to watch and discuss many of the outstanding international films one couldn't see in Germany during the war, and we attended evening lectures and discussions held in the aula (great hall) of our Gymnasium, about the Caux (Switzerland) based *movement of moral re-armament* and about humanistic education programs built on *anthroposophic ideas* and related interpretations of the works of our greatest poet and writer Johann Wolfgang von Goethe, who also played a central role in German literature lessons at school. We got interested in religious and theological questions and attended services held in the 700-years old protestant Paulus church at Kamen, often discussed the lecture with the pastor, and we even became members of the church's brass-band which on New Year's Eve used to climb the inside ladders of the high church tower up to the level of the bells and play chorals which could be heard in a wide region around the church.

We regularly met in our homes and discussed school matters, assisted each other in different subjects, and had additional programs on our own such as learning the basics of Ancient Greek (which did not belong to the subjects taught in our school) in order to be able to at least partly read and interpret the original texts parallel to the German translations of Aristotle, Plato, and the pre-socratics who had become important sources for philosophical orientations in Germany. We were especially inspired by the novels, dramas, and the related philosophical texts on *existentialism* developed by Jean Paul Sartre and Albert Camus which now were appearing in German translations, and we tried to get access to the sources of existence-philosophy with Jaspers and Heidegger and behind them in Husserl's phenomenology, matters we wanted to study more profoundly later at the university of Münster which we envisaged as our intellectual stronghold hopefully being reconstructed soon from the ruins.

The new *curriculum in mathematics* for the upper 2 grades (8 and 9) at our Gymnasium comprised analytic geometry and conics, spherical trigonometry, the number system from the natural to the complex numbers, algebraic equations, basics in differential and integral calculus related to algebraic and trigonometric functions and their inverses as well as logarithms and exponential functions.

For us grade 7 was mainly used to fill the enormous gaps most of us had in mathematical pre-knowledge and experience according to the extremely bad schooling situation in 1944 and 1945. Our math teacher who also taught us physics did an excellent job by his own teaching in coping with this situation. He knew me since school year 1943 and had from there on supported my particular interest in mathematics and physics by providing me with books and giving me special tasks. In grade 7 he asked me to meet twice a week in the afternoon in our classroom with those classmates who wanted special support in following and comprehending the lessons and in doing the problems. This kind of working with a group of classmates which was quite different from coaching individual students whose classroom situation one doesn't know showed me a lot about how classmates in very different individual ways had transformed the matter taught into their matter learnt influenced by their conceptions or misconceptions but also often by the indifference or vagueness of the matter taught which made me discuss this with my teacher. 42 years later, within the activities of the international working group on Theory of Mathematics Education (TME) I came back to those observations by organizing (jointly with Alfred Vermandel) an international conference on "Investigating and Bridging the Teaching-Learning Gap" held in Antwerp, July 11–15, 1988.

With respect to the basics taught in mathematics in grades 8 and 9, I tried, for my own understanding and use, to elaborate the details, the foundations, the systematic interrelations, the applications in physics, geography and astronomy, and in these connections I also studied several university level books on foundational positions, the role of logics, extensions of calculus into complex analysis and differential equations, the use of non-commutative algebra in quantum theory and of non-euclidean geometry in cosmology. I also was concerned with the question in which way some of these things could be incorporated into the teaching at school and into school text-books. When we had passed the Abitur exam in January 1949 and because of formal regulations had to stay at school for another 2 weeks to be used for free arrangements between teachers and students, I got a chance to give a sequence of lectures with discussions to my classmates, following my own approach, about non-commutative algebra, especially quaternions and their applications, non-euclidean geometry and special relativity theory, which was rather well received by my classmates and is still being remembered by many of them till today and talked about when having our annual meetings in our traditional favorite pub at Kamen.

## **2. Münster in Westphalia: The university; the Seminar for Didactics of Mathematics; regional, national, and international activities: 1949–1967**

When I began my studies at Münster in the fall of 1949, the university had *outstanding professors in various fields* who were of particular importance for me. In mathematics there was Heinrich Behnke on the chair for analysis (special research domain: function-theory of several complex variables), F. K. Schmidt for algebra (algebraic analy-

sis), in experimental physics E. Kappler (atomic physics), in theoretical physics A. Kratzer (relativity theory and quantum mechanics), in the field of research on foundations of mathematics and mathematical logic (the only chair in Germany at this time) Heinrich Scholz (and his disciples G. Hasenjaeger and H. Hermes). In philosophy there were Joachim Ritter, a disciple of Heidegger, (existence philosophy, aesthetics and ethics, social and political philosophy, history of philosophical terminology) and F. Kaulbach (epistemology, philosophical foundations of the natural sciences), in sociology (especially sociology of education) H. Schelsky, in pedagogy H. Döpp-Vorwald and E. Lichtenstein, in psychology W. Metzger, and in German literature Benno von Wiese and Jost Trier.

At that time in Germany after the war, it was recommended to students who were prospective university researchers, for the sake of their potential professional security, to fulfill in their registered studies and examinations, if probable, the conditions of the first state exam for becoming a gymnasium teacher in 2 school-subjects, including the execution of a component called "philosophicum", consisting in some courses and exams in philosophy and pedagogy. In my case this combination, built on a broad and deeply rooted study of its components and their interrelations, including actually being involved in practical teaching, led to a very fruitful synthesis, namely to a model of an *exemplary career for a professor in didactics of mathematics*.

Important conditions for this possibility had basically been established by Professor Behnke. After his appointment as a mathematician at Münster in 1927, he had become internationally known in his research field and before the war had intensely cooperated with Henry Cartan in Strasbourg and later in Paris. Also he had found close connections to Otto Toeplitz who in 1928 became Professor of mathematics at Bonn – not too far away from Münster. At his former university in Kiel, Toeplitz had already been very much interested in supporting his students, who were mainly prospective Gymnasium teachers of mathematics, in better linking what they had learned in mathematics courses with what they would have to teach in school. He had therefore founded already in 1920 a "*mathematical didactical colloquium*" in which students, university mathematicians, and Gymnasium teachers of mathematics met weekly to present and discuss problems of mathematics education and their possible improvement. At Bonn Toeplitz continued this practice and Behnke participated with groups of students and teachers from Münster in some of Toeplitz' meetings, whereas Behnke himself organized similar meetings once a year at Münster out of which the well known *Münster Conferences for the Furtherance of the Connection Between University and School* arose. Also Behnke and Toeplitz founded a new Journal "*Mathematisch-physikalische Semesterberichte*", which was continued by Behnke after the war and became one of the pioneering journals in promoting the scientifically based modern math reform in Germany. Toeplitz had left Germany for Palestine in 1938. After the war, Behnke re-established his research group in function theory of several complex variables (complex analysis), and it

became one of the *most successful mathematical schools* in Germany as well as with respect to worldwide measures, with disciples like Karl Stein, Friedrich Hirzebruch, Hans Grauert, Reinhold Remmert and the outstanding mathematicians in the following generations. This went together with building up a worldwide cooperation especially with mathematicians in France and the USA. Meanwhile French mathematicians like Henry Cartan, André Weil, and Jean Dieudonné under the pseudonym “*Nicolas Bourbaki*” had begun to reorganize large domains of mathematics from a *set-theoretical and structural point of view*, the sources of which could be found already in many developments of the 19th century particularly in Germany, and this deeply affected the conceptual means of research, especially in complex analysis, but also in the whole understanding of even elementary and school mathematics. When I came to Münster, Bourbaki’s ideas were already present and being used in research and teaching mathematics at the university, and this was fundamentally supported by the Münster school in the foundations of mathematics and mathematical logic going back to Heinrich Scholz which now also started an explosive development.

Behnke had continued his annual conferences on the relations between university and school in 1947, after an interruption of 7 years, and in 1951 he founded his *Seminar for the Didactics of Mathematics* as an institution related to his chair and as regular meetings with lectures and discussions, attended by university staff-members, mathematics teachers from Münster and the region, and students. In the beginning of my study of mathematics I had been closer related with Professor F. K. Schmidt and had started to work at problems of algebraic analysis. However, Schmidt left Münster soon and so I moved over to the “Behnke school” and became a member of his Advanced Seminar for Complex Analysis. When we became more personally acquainted, Behnke discovered my special interest in philosophical and educational aspects of mathematics and asked me to join his didactical seminar and give talks about ideas and problems which were on my mind, and especially about some experiments I had made during visits of schools, such as elaborating with Gymnasium 8th graders a proof for the fundamental theorem of algebra saying that in the complex number field each algebraic equation has a (complex) solution, a theorem which thus far was considered by teachers and in textbooks as in its provability being located above school level. In the seminar I was well received and got motivated to work in a more systematic way on didactics of mathematics, especially its *historical development* and its *new trends* which were particularly started in the United States, France and Belgium, a typical sign being the foundation in 1950 of the *International Group for the Study and the Improvement of Mathematics Teaching* by the logician E. W. Beth (Amsterdam), the psychologist Jean Piaget (Geneva), the pedagogue C. Gattegno (London) and the French mathematicians G. Choquet, J. Dieudonné, A. Lichnerowicz. I got more involved in the program for the didactical seminar and suggested to invite speakers from the new innovative groups and projects.

It was Behnke’s idea that I should go *a double way*:

on one hand become qualified as a researcher in mathematics and physics, also in domains like the foundations of mathematics and mathematical logic, and on the other hand get the qualifications for a math- and physics-teacher at a Gymnasium by doing the first state-exam, which included philosophy and pedagogy, then participating in the second phase of qualification with one year related to a school and another one to a special teacher training seminar called “Studienseminar”. On that basis I could return to the university and take a long range position at the didactical seminar with the possibility of also doing experimental and innovative teaching at school and combine theoretical and empirical research and development.

I graduated in *mathematics and physics* in the fall of 1955 and simultaneously also passed the first state-exam. I took a tutored position (called “*Studienreferendar*”) for a year as a math and physics teacher at a *Gymnasium in Hamm*, 20 km east of Kamen, and I extremely enjoyed the being embedded in the life of a school. A special theoretical and experimental study I did was concerned with the problems of introducing the *concepts of mass and force* in mechanics respecting the fact that both concepts are theoretical concepts which cannot be introduced by simple assignments, but need reference to Newton’s equation  $f = m \cdot a$ . Much later I came back to this in a talk titled “The nature of theoretical concepts in physics and mathematics-implications for the fragility of knowledge in the educational context”. The talk was given in 1989 at the Second Jerusalem Convention on Education, organized by Shlomo Vinner.

During a 3 weeks stay, spent with a class of 8th graders at a *school-boarding place* owned by the city of Hamm on the East-Frisian isle of Wangerooge, I used the time given for free activities and the wonderful location at the shore belt, to do observations and studies with the students in *astronomy* and furthermore with respect to *phenomena and theoretical explanations of the tides*. At school in Hamm, I also taught a class of 7th graders in *music* for some weeks when their music teacher was ill. Besides practicing, hearing, and explaining music, we spent a great deal of time, partly exchanging the music hall with the physics laboratory, in analysing the *laws of harmony* and investigating how they were studied and used from Pythagorean beginnings to the cycle of quintes and Bach’s well-tempered piano and more generally in building and playing music instruments of all kinds, including the beginnings of modern electronic music. I also tried to give the students an idea of *twelve-tone-music* and *techniques* (dodecaphonia) which had been condemned as “degenerated music” in Nazi-Germany with the consequence that Arnold Schoenberg, one of its creators (cf. his book “Style and Idea”, 1950) was expelled from Germany in 1933. He first went to France and then to the USA. I had pursued the revival of twelve-tone-music in Germany which began with the first Vacation Course on New Music held at Castle Kranichstein near Darmstadt in 1946, followed annually by other courses of this kind and by 2 congresses especially devoted to theoretical and practical problems of “dodecaphonia”, attended by composers, musicians, theoreticians, journalists and held at Munich in 1950 and at Darmstadt

in 1951. Also the *Donaueschingen Music-Festivals*, which already took place before the war from 1921 to 1926 and then were continued at Baden-Baden, were started again in 1950, in some separation from the Kranichstein courses in basic directions and participating people. I was particularly motivated by the early compositions of *Karlheinz Stockhausen* (born 1928), *Hans Werner Henze* (1926) and the Italian *Luigi Nono* (1924) who were strongly influenced in theory and practice by the Schoenberg specialist *René Leibowitz* of Paris, known by his books “Schoenberg et son école” (1947) and “Introduction à la musique de douze sons” (1949). Leibowitz played a most inspiring role at Kranichstein 3 in 1948, where Henze experienced him as “the great event” and saw in him “a representative of modern existentialist France”. At school in my music class experimentation and analysis, hearing and discussing broadcast transmissions and audio tapes of performances of related compositions together with further developing and using conceptual and theoretical background information which I had provided, gave many students an open-minded and active attitude towards both the traditional forms and the modern developments of what at that time in Germany was still called “serious music”.

In *mathematics*, I taught Gymnasium 1st graders the *elements of number theory*: the set  $N$  of natural numbers, positional notation, divisibility, prime-numbers, unique prime-factor decomposition, proof for the infinity of the set of prime-numbers, least common multiple and greatest common divisor as (algebraic) operations on  $N$  and on special finite subsets such as the set of all divisors of a given element of  $N$ , trying out the *idea of playing operational checker games on these systems*, which I had invented and later further developed in US-projects and finally published at McGraw-Hill.

Also I was given the chance to teach in a class of 8th graders an *approach to infinite sequences and series* as fundamental concepts in real analysis and calculus, based on modern conceptual clarity with respect to *limits* and *clusterpoints* à la Cauchy and Weierstraß but also on analysing with the students historical examples of misconceptions and obstacles, for which I was particularly inspired by M. Reiff’s excellent book on “Geschichte der unendlichen Reihen” (1889).

The second year of qualification as a teacher I spent at the “*Studienseminar*” in Münster with only a reduced relation to a particular school, much abstract debate about education and general didactics, which I found very disappointing, and test-lessons given by the candidates which I often found artificial. I was a very critical and provocative member of the *Studienseminar* which comprised about 50 second year “*Studienreferendare*” of almost all school subjects, and – being in opposition to the rather conservative directorate of the seminar – I often insisted and got involved in plenary discussions about controversial positions related to the goals of education, to the role of school subjects in reaching these goals, to the relations between general didactics and the didactics of school subjects or between theory and practice in education and didactics, matters which we already had dealt with in the Seminar for the Didactics of Mathematics at the university and

which would gain great importance in the coming years in my work at the different institutions and projects in Germany and abroad.

I was given a class of 3rd graders at the Schlaun-Gymnasium in Münster for doing an *experimental teaching of congruence geometry in the plane* on the basis of *congruence mappings* (motions) and I gave a test lesson in that class on the group of symmetries of the square and the use of its subgroups for a classification of quadrilaterals. In *physics* I taught a 9th grade in *electrodynamics* and gave a test lesson on the principles and use of the *ballistic galvanometer*. A more extended activity was the teaching in an 8th grade class of the *construction of complex numbers* from the reals and an *explorative approach to an elementary proof of the fundamental theorem of (classical) algebra*, which I had already tried out as a student when visiting a Gymnasium in Unna, the county town related to Kamen, and about which I had already reported in Behnke’s didactical seminar, but now became a matter of detailed heuristic and inductive procedures with strong participation of the students, e. g. by their introducing the very fruitful concept of closure of degree  $n$  of an algebraic field. The *written report* on this experiment was a component of my obligations for passing the final examination at the end of the second year. In its full elaboration and based on further repeated experiments in coming years it grew up to one of my most influential publications, which was particularly appreciated as a model in explaining the *concept of elementarity* and *elementarization* in the context of mathematics education.

The final examination was in the fall of 1957. I was then formally assigned as a *Studienassessor* to a new math-science oriented Gymnasium (the outstanding Hittorf-Gymnasium in Münster) for possible experimental teaching and returned as a staff member to Behnke’s seminar for didactics, to which I had continuously been in contact by participating in most meetings and in planning the program as well as by giving several talks. Actually, during the two years of being a *Studienreferendar* I had continued, intensified and broadened my own *studies on the foundations, historical developments, new trends and future perspectives of didactics of mathematics*. This included the *further pursuing and reconstructing the development of modern mathematics*: from its beginnings in the 19th century, its explosive growth before 1933 to the actual state of Bourbaki’s elaboration of his program “*Les Structures Fondamentales de l’Analyse*”. It included the further study of the work on *logical, epistemological and philosophical foundations* of modern mathematics, which also goes far back into the 19th century, as well as the new way of viewing and practicing the *relation between pure and applied mathematics*, particularly based on the *concept of model*, related to a mathematical theory, as was clearly exemplified e. g. by J. von Neuman and O. Morgenstern in their “*Theory of Games and Economic Behavior*” (1944).

In winter-semester 1956/57 I had given a talk in the didactical seminar in which I presented a kind of scenario for fundamental changes of school mathematics to come during the next years. The talk was received with enthusiasm and Heinrich Behnke invited me to give along the

lines of my talk a main paper at the 23rd Conference for the Furtherance of the Connection Between University and School, which was to be held at Münster in June 1957, and I spoke there about “The penetration of modern mathematical concepts into the teaching of mathematics at school”. The paper which in a much extended version was published under the title “Modern mathematical thinking and school mathematics” in the new reform-oriented Journal “Der Mathematikunterricht”, has since been considered a decisive impulse and factor for getting a strong reform-movement started at the Gymnasium in Germany.

In 1954, at the International Congress of Mathematicians in Amsterdam Heinrich Behnke had been elected for 4 years as president of the *International Commission on Mathematical Instruction (ICMI)*. For the next congress to be held at Edinburgh in 1958, ICMI had planned to produce national reports on 3 themes which were presented under: (a) Mathematical Instruction up to the Age of Fifteen (H. F. Fehr), (b) The Scientific Foundation of Secondary School Mathematics (H. Behnke), (c) Comparative Studies of Teaching Beginning Geometry (H. Freudenthal). For the West-German contribution to (a) and (b) Behnke had organized 2 big groups of reporters and writers who regularly met in Münster and actually were given the goal to write books on these topics which were published on time and represented an excellent achievement by the authors and by the headquarters in Münster at the Seminar for Didactics. I was involved in both projects and with respect to (b) particularly cooperated with Hans Freudenthal who was invited as a guest writer from the Netherlands. I participated in the congress and met with a large group of mathematicians and people involved in mathematics education from all over the world. I also attended the congresses 1962 at Stockholm and 1966 at Moscow where I gave an invited talk on the role of the axiomatic method in school mathematics.

Before the Edinburgh congress Howard F. Fehr of Teachers College, Columbia University, New York, spent a sabbatical at Münster and we became very closely related. 1960 he invited me to New York to give a 3 weeks course to math-teachers participating in a summer institute to improve their mathematical and didactical background in order to be better prepared for the reforms of school mathematics on the lines of the new math. I did my first trip to the US by M.S. Mauretania of Cunard Line from Le Havre via Cork. My second invitation to the US was extended in 1964 by the American Association for the Advancement of Science (AAAS) on recommendation by Edward G. Begle, the director of the US-School Mathematics Study Group (SMSG), with whom I had been cooperating at the 1963 OECD-Conference on “mathematics to-day”, held in Athens, Greece. My AAAS task was to go on a 6-weeks lecture tour to visit 8 colleges and universities all over the country which were running summer institutes or academic year institutes for teachers to improve their abilities for better understanding and teaching the new math curricula.

Fehr had discussed with me the plans he had concerning the start of a curriculum project at Columbia under the title “Secondary School Mathematics Curriculum Im-

provement Study” (SSMCIS). In 1965 he invited me for a week-end action meeting to New York on the basis of positive reactions he had from the US Office of Education and asked me for being the European co-director of this project, which I accepted. The project work which lasted for several years actually began in the summer of 1966 with a group of mathematicians from the US (Marshall Stone, A. W. Tucker, H. Levi, et al.) and from Europe (G. Choquet, Lennart Råde, André Revuz, et al.) and a group of teacher-writers. The first product was the design of a curriculum using modern mathematical concepts and ideas in a fascinating way. Many mathematics educators who came as visitors from all over the world to Columbia and got a copy of the program reported even after years how much they were stimulated and inspired by the material. Some of Fehr’s and Myron Rosskopf’s Ph. D.-students as Jeremy Kilpatrick and Jim Fey, now internationally renowned educators and researchers, belonged to the project team.

Some days before the beginning of the project Fehr held a conference to which he had invited many people who could be possible consultants. Among them was Burt Kaufman who had just moved from Ft. Lauderdale to Carbondale Ill. to transform his Florida project for gifted students into a new conception of a highly individualized program which he called “Comprehensive School Mathematics Program” (CSMP). He had received a positive response from the Central-Midwestern Regional Educational Laboratory in St. Louis for substantial support. He convinced me that it would be a fascinating experience working in his project, so that I accepted to be his chief mathematics consultant. We closely cooperated for the next 8 years. Among the CSMP consultants and staff-associates were mathematicians and math-educators such as Robert B. Davis, Robert Exner, Hans Zassenhaus, Peter Braunfeld, Peter Hilton, Lennart Råde, Vincent Haag, W. E. Deskins, Frédérique Papy.

During the 60ies there was a strong reform movement in mathematics teaching in Germany, particularly for the Gymnasium, and the Münster seminar for didactics was strongly involved, supported especially by the seminar which Günter Pickert, assisted by Arnold Kirsch, had founded at the University of Giessen. I had done a lot of explorative teaching at the Hittorf-Gymnasium, which became rather well known among educators, also from neighbour-countries, so that I often had visitors in my classroom. Before I left for Karlsruhe at the end of 1967 there were about publications of mine reporting about the various experiments and experiences plus a huge number of talks I gave at conferences, at meetings of teachers, teacher trainers, administrative people etc. Besides the cooperation with the US-projects, there were many contacts to and often intense cooperations with ICMI, OECD and UNESCO in running conferences, writing reports and publishing results.

At Münster University a kind of interdisciplinary cooperation was established with mathematicians, logicians, philosophers, pedagogues, psychologists, especially with young researchers and assistants, to clarify conceptual and theoretical problems related to different approaches, al-

ternative interpretations etc. Actually modern mathematics and the investigations related to the foundations had an enormous impact on trying to really understand even among experts what all that means especially with respect to so-called elementary mathematics: number systems, geometry, algebra, analysis, variables, relational thinking etc.

Especially fruitful for me and my work was the cooperation with Professor Ritter in building up a new edition of the Historical Dictionary of Philosophy, which has extended to more than 10 volumes, where I wrote rather detailed contributions to concepts like “function”, “relation”, “order”, “symmetry”, “isomorphy”, “morphism”, “structure”, “categories”, “set-theory”. Then there was a working group with Professors Hermes (logician), Kaulbach, Kambartel (philosophers) and myself about the edition of professor Scholz’s inheritances where we found most interesting sources about a controversy between the mathematicians Hilbert and Frege concerning the concepts of variable, axiom, implicit definitions etc. which I analysed with an enormous effect on understanding obstacles students have in comprehending the axiomatic method. A special experience was the writing of a volume on mathematics in the encyclopedia “Fischer-Lexikon” jointly with professors Behnke, Remmert, and Tietz (translated into Italian by Professor Lombardo Radice), where I wrote the articles on “cardinal and ordinal numbers”, “logic and methodology”, “sets – mappings – structures”, “research on the foundations of mathematics” which were extremely well received not only by mathematicians, teachers, philosophers who were not familiar with those topics, but also by artists, writers, musicians (among them e.g. Hans-Magnus Enzensberger) who were fascinated and in their letters asked for more and had many further questions.

### **3. Karlsruhe: The Center for the Didactics of Mathematics at the University of Karlsruhe and the Zentralblatt für Didaktik der Mathematik ZDM (International Reviews on Mathematical Education): 1967 – 1970 – today**

Münster had given me a very rich platform to live didactics of mathematics as a very fertile field of acting, searching, thinking, communicating, doing interdisciplinary research and seeing that in the center there is actually a *scientific discipline*. I should mention that this was particularly reflected in courses I had developed for students being prospective mathematics teachers or researchers themselves. The organization was a cycle of courses, some of which were printed in the text-series of the mathematics institute and there were also sets of problems and tests to find out whether the students had well comprehended the matter. Much of this could be used and applied by my successors.

So I used the opportunity to establish a seminar for didactics with Professor Heinz Kunle in Karlsruhe and develop new directions of work. There was a chance in 1967 to broaden the cooperation with CSMP by establishing what we called a *center for the didactics of mathematics*, an *institute at* and not within the university, supported by an association to be found or founded.

The establishment of the center at the university was supported in its status by the university and also in terms of rooms and facilities provided by the university. The Association for the Furtherance of Didactics of Mathematics with 3 American and 4 German members was founded and financial means brought in from potential users of outcomes from the center. One outcome of the center was expected to be a *new journal for documentation and information in didactics of mathematics*, called “*Zentralblatt für Didaktik der Mathematik*” (ZDM). This all was realized in 1968 and especially the ZDM still exists in its 30th year, internationally appreciated and used. First co-workers in the center were A. Engel, I. Weidig, H. Stever, H. Wäsche. A lot of material was produced for CSMP and conceptional output created for general use but also for particular projects run by the center. Two conferences were held, one about the role of set theory and logic in future school curricula analysed particularly from constructivist points of view, the second one about the development of a series of reading and working books for pupils in which particular aspects of mathematics such as geometrical, algebraic etc. are emphasized. Some of the ideas were used for CSMP. The center presently is mainly a component of the administration of the ZDM which is as a whole integrated into the Fachinformationszentrum Karlsruhe (information center for mathematics, physics, energy and various other fields).

Though I left Karlsruhe in 1970 for a professorship at the College for Teacher Training in Bayreuth and as a member of the founding committee for the University of Bayreuth which now exists for more than 20 years, I’m still director of the Center and chairman of the Furtherance Association and could particularly make use of these connections, when the *3rd International Congress of Mathematical Education (ICME-3)* was held in Karlsruhe in 1976 and I was chairman of the international program committee. Built on many substantial activities in terms of well prepared survey-trend-reports brought into the congress, discussed there and published (in joint editorship with Bent Christiansen) in 4 languages by UNESCO in its series “*New Trends in Mathematics Teaching*”, ICME-3 has definitely contributed to an improved professionalization of our discipline.

### **4. Bayreuth: College for teacher education and university: 1970–1973 (1982, 1988)**

In *Bayreuth*, I became very much involved in the *didactics of mathematics for elementary schools* and cooperated in this area with CSMP and the Centre Belge de Pédagogie de la Mathématique (Frédérique and Georges Papy at Brussels). Together with teachers and my assistants I ran an interesting and resultful experiment and empirical study with first graders, using a. o. Cuisenaire rods, arrow diagrams for the representation of relational and operational connections between natural numbers, Papy’s minicomputer for number representation and computing with its particular operational properties of a Piagetian Grouping.

In 1971 we organized a strongly internationally oriented annual conference of our Community for Didactics of Mathematics with particular emphasis on analyses of

and discussions about research and graduation programs in mathematics education (which Professor Iwasaki (Hiroshima) referred to in his Sevilla talk (s. footnote 1)).

Jointly with the mathematician Friedrich Kasch of Munich University who has also been like me a member of the Bayreuth university founding committee for 10 years, I initiated a new Journal "Didaktik der Mathematik" (1973) which in 1982 was the forum of a decisive controversy about the role of research in mathematics education (Steiner contra Zeitler!). As members of the university founding committee Kasch and I designed a concept for the prospective mathematical faculty at Bayreuth with strong components in various actual mathematical research areas and with 4 chairs in didactics of mathematics plus one in the domain of history of mathematics.

Early consequences drawn by the political and cultural administration in Munich from a predicted surplus of mathematics teachers in Bavaria prevented the realization of the intended strong didactico-historical component which could have been a dynamic partner for the IDM established at Bielefeld in 1973.

One of my main contributions to the profile of Bayreuth University has been the initiative and design for the founding of an *Institute for Music-Theatre* in a broad sense, jointly brought forward with Dr. Dietrich Mack – the then scientific and artistic consultant of Wolfgang Wagner, the director of the Bayreuth Festspielhaus. I was asked by the university founding committee to chair and to jointly organize with Dietrich Mack a widely ranged interdisciplinary, theory and practice connecting conference with about 40 highly competent national and international participants which by its outstanding results and perspectives laid the ground for the actual realization in 1976 of the idea for the first and till now unique institute in Germany with its – in some similarity to the IDM – characteristic orientation towards theory, practice, development, information, and consultation. Till 1982 I belonged to the advisory board of this institute.

Another field of cultural activities at Bayreuth lasting till 1988 has been my role as vice-president of the *Bayreuth International Center for Youth Culture*. One of my special contributions has been the planning and negotiation of the integration of a German component of the *German French Youth Work* into the Bayreuth Centre parallel to the simultaneous establishment of a French component at Bourges (France).

### 5. Bielefeld: The Institute for the Didactics of Mathematics (IDM): 1973–today

The IDM was founded in 1973 as a central and supra-regional institute of the University of Bielefeld which had successfully applied for receiving a start-funding of 9 million DM from VW-foundation for the establishment of such a center. After a period of 3 years' time there were 3 full professors (Bauersfeld, Otte, Steiner, all of them being in their position by the end of 1973), 1 associate professor (Stowasser), 14 researchers and 11 members of non-scientific personnel at the institute which made it one of the biggest places of this kind in the world. The leading ideas were to critically investigate the reforms of mathe-

matical curricula and mathematics teaching which took place in German schools especially in the late 50-ies and the 60-ies and to develop a standard of research and development which would make future trends more stable and controllable. It was also expected that the research would make more use of interdisciplinary approaches and international cooperation in order to better cope with the complexity of the conditions and processes.

To make a long and complex story about the dynamic development of IDM short and simple: An excellent *evaluation of IDM's work* was given by many colleagues, institutes, and organizations from all over the world, when the IDM got under pressure in 1991 to be closed down in order to give room at Bielefeld University for new faculties such as technology or health sciences. Since it helped the institute to survive I'm closing with some quotations (to be found in IDM-Occasional Paper 129–June 1993). The *Executive-Committee of ICMI* wrote: "In a unique way, IDM has substantially contributed to the theoretical and empirical foundation, development and furtherance of all essential aspects of mathematics education as a field of study and practice ... IDM is engaged in giving its extraordinarily important contribution to the shaping of a new academic field ...". Professor Richard Skemp, Coventry explained: "I have a high regard for the work of this Institute in all its aspects. This includes interdisciplinary work at a theoretical level, applications to teacher education etc. . . . What I personally regard as particularly important is the development of mathematical curricula on sound theoretical basis . . .". Professor Kilpatrick stated: "All of us in mathematics education who are aware of what is going on internationally recognize the IDM as the single most important center in our field. At no other place in the world can one find such a productive mixture of theoretical work, empirical research, curriculum development and inservice education . . .". I think these quotations are somehow also applicable on various other groups and activities in didactics of mathematics and allow the generalization that Didactics of Mathematics is a scientific discipline. Another evidence is given by the publication in 1994 of a book (vol. 13 within Kluwer's Mathematics Education Library) also titled "Didactics of Mathematics as a Scientific Discipline" which was planned and edited by 4 researchers (Rolf Biehler, Roland W. Scholz, Rudolf Strässer, Bernard Winkelmann) who have been cooperating with me in a continuous IDM-working group for more than 15 years and who brought together 37 international authors to contribute in an affirmative way to the theme of the book and the intentions of this paper.

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#### Editor's Note

*In addition to this article it should be mentioned that Hans-Georg Steiner celebrated his 70th birthday on 21 November, 1998. He was the father of this journal and founded it together with Klett publishers in 1968. He was always very keen to develop ZDM and to establish it in the*

*mathematics education community. All colleagues working at ZDM say again thank you to Hans-Georg Steiner and wish him all the best.*

## **IASE Round Table Conference**

### **Training of Researchers in the Use of Statistics**

Tokyo, Japan, August 2000

The conference is organized by the International Association for Statistical Education (IASE) and will be held at the Meiji University, Tokyo.

It has been usual for these Round Table Conferences to be held as satellite meetings to each meeting of ICME (International Congress on Mathematics Education), which is held every four years. Goal of the Round Table Conferences is to bring together a small number of experts, representing as many different countries as possible, to provide opportunities for developing better mutual understanding of common problems, and for making recommendations concerning the topic area under discussion.

The following are possible topics to be discussed:

1. Statistical competencies that researchers in different disciplines should acquire in their postgraduate training
2. Needs and problems in the statistical training of researchers in specific fields
3. Main learning problems, misconceptions and errors concerning advanced statistical concepts and procedures
4. Design/evaluation of courses for training researchers in particular statistical topics
5. Effects of technology on the statistical training of researchers
6. Assessing/identifying frequent errors in the use of statistics by researchers
7. Researchers' attitudes towards statistics and its effect on the role of data analysis in experimental research
8. Consultation as a teaching/learning process
9. Informal statistical learning from reading research literature.

A book containing a set of refereed conference papers and summaries of discussions, which presents a global overview of the conference subject, will be published.

The scientific committee consists of: Carmen Batanero, Spain, Chair; Theodore Chadjipadelis, Greece; Joan B. Garfield, USA; Anne Hawkins, UK; Yuki Miura, Japan; David Ospina, Colombia.

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