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Pseudo Stochastic Dominance. Applications

Cuasi dominancia estocástica. Aplicaciones

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Abstract

The aim of this work is to show that on certain ocasions classic decision rules used in the context of options (Stochastic Dominance criteria and Mean-Variance rules) do not provide a selection of one specific option over the other, therefore, the need of working with other criteria that can help us in our choice. We place special interest in economic and financial applications.

***Key words*:** Mean, Variance, Stochastic dominance.

Resumen

El objetivo de este trabajo es mostrar que en ocasiones las reglas clásicas de decisión sobre inversiones (reglas de Dominancia Estocástica y reglas de Media-Varianza) no siempre conducen a una selección de una inversión sobre otra, surgiendo la necesidad de trabajar con otros criterios que ayudan en dicha elección cuando los clásicos no conducen a ninguna selección concreta. Se pone principal interés en las aplicaciones de carácter económico-financiero.

***Palabras clave*:** dominancia estocástica, media, varianza.

1. Introducción

The use of Mean-Variance rules (MV) or Stochastic Dominance rules (SD) may not be as useful as desired, since it might be the case that these criteria do not lead to selection of an investment over another. For example, suppose that there are two investments X and Y, with the following characteristics:

$$E(X) = 20000, \sigma_X = 20.2$$

$$E(Y) = 1, \sigma_Y = 20$$

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Where $E(X)$ and $E(Y)$ denote the expectations of X and Y, respectively and σ_X and σ_Y their standard deviations. Note that neither is preferred over the other (X is not preferred over Y, and Y is not preferred over X) using MV criteria, this is because $E(X) > E(Y)$ but $\sigma_X > \sigma_Y$. But there is no doubt that almost all investment decision-makers would select X. That is, MV rules have not been capable of choosing one investment over another even though most decision makers would have selected X.

This problem is not new, for example Baumol (1963) noticed this and suggested a different approach to selecting investments known as "Expected Gain-Confidence Limit Criterion" as a replacement for the MV decision rules. Baumol argued that an investment with a high standard deviation σ will be relatively safe if its expected value μ is large enough. He proposed the following index of risk: $RI = \mu - k\sigma$, where k is a positive constant that represents the level of risk aversion of the investor. Another measure to evaluate an investment is known as Sharpe ratio, which measures the profitability of a title independent from the market, that is, it measures the fluctuation of the investment compared to the market.

Let us now propose the following example in which the SD rules will be applied. Let X be the asset which provides 1 euro with probability 0.01 and provides the value 1000000 euros with probability 0.99; and let Y be the asset which provides 2 euros with probability 1. It would not be strange to expect that nearly 100% of investors would prefer asset X over asset Y, but the SD rules are not conclusive in this case. For example, assume utility function:

$$U(x) = \begin{cases} x, & if \quad x \leq 1 \\ 1, & if \quad x > 1 \end{cases}$$

In this case, it is easily verified that, investors who have this utility function will prefer Y over X. From this, it can be deducted that, these investors who have an "extreme utility" do not represent the majority of investors.

For the reasons discussed above, it has been necessary to establish alternative decision rules to help decide in cases where the above rules (SD and MV) do not allow selection of an investment over another. These rules are known as "Almost Stochastic Dominance rules" (ASD). With ASD rules it is possible that, given two assets X and Y, whose distribution functions do not have any preference using SD rules, but with a " minor change" in the expression of the distribution functions, reveal a preference, and it is possible to select one over another. This small change in the distributions removes extreme preferences (profits), considering the profits that are more common among investors. The utility function above example is a case of extreme utility.

The advantages of ASD over SD and MV are:

1. ASD is able to rank investments in cases where SD and MV are inconclusive.
2. ASD remove from the SD efficient set, alternatives which may be worse for most investors.

3. ASD shed light on the efficient portfolio selection problem and the horizon of the investment. It is possible to establish a relationship between the percentage of equity in the efficient portfolio and the investment horizon. That is, ASD can help investors in choosing their investment portfolio.

Let us continue with the previous example with assets X and Y described above. Let F be the distribution function of X defined as:

$$F(x) = \begin{cases} 0, & if \quad x < 1 \\ 1/100, & if \quad 1 \leq x < 1000000 \\ 1, & if \quad x \geq 1000000 \end{cases}$$

and let G be the distribution function of Y defined as:

$$G(y) = \begin{cases} 0, & if \quad y < 2 \\ 1, & if \quad y \geq 2 \end{cases}$$

Their representation is given in the next figure, in that, it is possible to see how the distributions intersect, also it is representing the area between these two distributions:

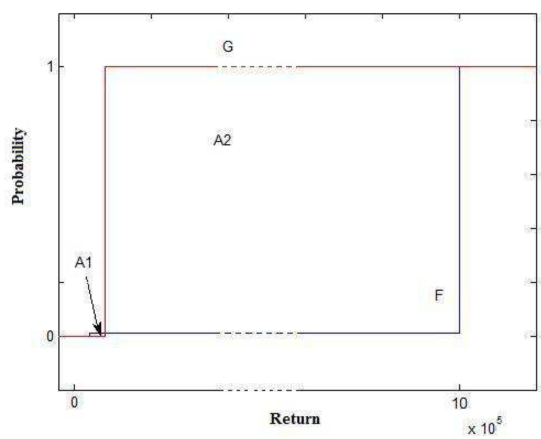

Figure 1: Distributions F and G and area between them.

Although as noted, most investors prefer would F (X) over G (Y), technically, and using the definition of FSD[1], there is no dominance in that sense, because the distributions intersect. Previously, this fact was shown noticing that there are some extreme preferences (profits) which made G preferable (better) to F.

[1]FSD: First order Stochastic Dominance. It is said that random variable X with distribution F dominates random variable Y with distribution G in the first order degree stochastic dominance, if $F(x) \leq G(x)$ for all x and with at least one point in which the inequality is strict.

Moreover, in this example, there is no SSD[2] or MV (for more information about SD or MV see Shaked & Shanthikumar (2007), Almaraz (2009), Almaraz (2010) o Steinbach (2001)). ASD criteria, have come up as an extension of SD criteria to help in these situations. Intuitively, if the area between the two distributions which causes the violation of the FSD criterion (area A_1 in the example) is very small relative to the total area between them (area $A_1 + A_2$ in the figure), then there is dominance of one over another for almost all investors (that is, those with reasonable preference). Hence the name of ASD criteria.

Formally, let S be the range of possible values that both assets can take (or in general two random variables) and S_1 is defined as the range of values in which the FSD rule is violated:

$$S_1(F,G) = \{t : G(t) < F(t)\} \tag{1}$$

where F and G are the distribution functions of the assets (or random variables) under comparison. ε is defined as the quotient between the area in which FSD criterion is violated and the total area between F and G, that is:

$$\varepsilon = \frac{\int_{S_1}(F(t)-G(t))dt}{\int_S |F(t)-G(t)|dt} \tag{2}$$

Another way to write this:

$$\varepsilon = \frac{\int_{S_1}(F(t)-G(t))dt}{\int_{S_1}(F(t)-G(t))dt + \int_{\bar{S}_1}(G(t)-F(t))dt} = \frac{A_1}{A_1+A_2} \tag{3}$$

where $\bar{S}_1$ denotes the complementary set of S_1 and A_i, $i = 1, 2$ are the areas described previously.

For $0 < \varepsilon < 0.5$, it is said that F dominates G by $\varepsilon - AFSD$. The lower the value of ε the higher degree of dominance. Almost First degree Stochastic Dominance criterion (AFSD) is:

Definition 1. Let F and G be two distribution functions with values in the range of S. It is said that F dominates G by AFSD (for a particular ε, or also ε-AFSD) and it is denoted $F \geq_{AFSD} G$, if and only if:

$$\int_{S_1}[F(t)-G(t)]dt \leq \varepsilon \int_S |F(t)-G(t)|dt \tag{4}$$

where $0 < \varepsilon < 0.5$.

And the definition of Almost Second degree Stochastic Dominance criterion (ASSD) is:

[2]SSD: Second order Stochastic Dominance. It is said that the random variable X with distribution F dominates random variable Y with distribution G in the SSD sense if $\int_{-\infty}^{x}(G(t) - F(t)) \geq 0$ for all x and with at least one point in which the inequality is strict.

Definition 2. Let F and G be two distribution functions with values in the range of S. It is said that F dominates G by ASSD (for a particular ε, or also ε-ASSD) and it is denoted $F \geq_{ASSD} G$, if and only if:

$$\int_{S_2} [F(t) - G(t)]dt \leq \varepsilon \int_S |F(t) - G(t)|dt \tag{5}$$

and $E_F(X) \geq E_G(Y)$, where $0 < \varepsilon < 0.5$ y $S_2(F,G) = \{t \in S_1(F,G) : \int_{\inf S}^t G(x)dx < \int_{\inf S}^t F(x)dx\}$.

It can be shown that AFSD implies condition $E_F(X) \geq E_G(Y)$, but in (5) this implication is not true and therefore must appear in the ASSD definition.

The paper is organized as follows: in first Section, the decision problem will be introduced; in Section 2, principal results in the literature about Almost Stochastic Dominance will be shown, in Section 3, examples of ASD criteria applications in the economic context will be explained (laboratory and real examples, which constitute the main practical contribution of the paper). Finally, in Section 4, main conclusions of this work will be presented.

2. Main Results

In this section, the most noteworthy results about ASD will be described.

Proposition 1. *Let X and Y, be two random variables with distributions F and G respectively. Then:*

1. *F dominates G in the AFSD sense, if and only if, there exists a distribution $\widetilde{F}$ such that $\widetilde{F} \geq_{FSD} G$, and it happens that:*

$$\int_S |F(t) - \widetilde{F}(t)|dt \leq \varepsilon \int_S |F(t) - G(t)|dt \tag{6}$$

2. *F dominates G in the ASSD sense, if and only if, there exists a distribution $\widetilde{F}$ such that $\widetilde{F} \geq_{SSD} G$, and it happens that:*

$$\int_S |F(t) - \widetilde{F}(t)|dt \leq \varepsilon \int_S |F(t) - G(t)|dt \tag{7}$$

That is, the difference between F and $\widetilde{F}$ must be relatively small ($0 < \varepsilon < 0.5$). Having condition $\varepsilon < 0.5$ ensures that it is impossible than both distributions F and G to dominate each other according to AFSD, because if F dominates G in AFSD sense, then $E_F(X) > E_G(Y)$ (see proposition 2).

***Proof*.** See Leshno & Levy (2002). □

Proposition 2. *Let X and Y be two random variables with distribution functions F and G, respectively. If F dominates G in the ε-AFSD sense and F and G are not identical, then $E_F(X) > E_G(Y)$. So, it is impossible that F dominates G in the ε-AFSD sense and that G dominates F in the ε-AFSD sense.*

***Proof*.** See Leshno & Levy (2002). □

As in the case of SD, there is also a characterization of the ASD criteria by utility functions. To address this issue, it is necessary to define the following sets:

Definition 3. Let S be the support of the random variables X and Y, the following sets are defined:

- Let U_1 be the set of all non-decreasing and differentiable utility functions, $U_1 = \{u : u' \geq 0\}$.
- Let U_2 be the set of all concave and two time differentiable utility functions, $U_2 = \{u : u' \geq 0, u'' \leq 0\}$.
- $U_1^*(\varepsilon) = \{u \in U_1 : u' \leq \inf\{u'(x)\}[\frac{1}{\varepsilon} - 1], \forall x \in S\}$.
- $U_2^*(\varepsilon) = \{u \in U_2 : -u'' \leq \inf\{-u''(x)\}[\frac{1}{\varepsilon} - 1], \forall x \in S\}$.

Theorem 1. *Let X and Y be two random variables with distribution functions F and G respectively.*

1. *F dominates G in the ε-AFSD sense, if and only if, for all function $u \in U_1^*(\varepsilon)$ it happens that $E_F(u) \geq E_G(u)$.*
2. *F dominates G in the ε-ASSD sense, if and only if, for all function $u \in U_2^*(\varepsilon)$ it happens that $E_F(u) \geq E_G(u)$.*

***Proof*.** See Leshno & Levy (2002). □

Proposition 3. *Let X and Y be two random variables with distribution functions F and G respectively.*

1. *F dominates G in the FSD sense, if and only if, for all $0 < \varepsilon < 0.5$, F dominates G in the ε-AFSD sense.*
2. *F dominates G in the SSD sense, if and only if, for all $0 < \varepsilon < 0.5$, F dominates G in the ε-ASSD sense.*

***Proof*.** The first part of the proposition will be proven.

Let us assume that F dominates G in the FSD sense, then for all t it happens that $S_1(F, G) = \emptyset$, in this way, for all $0 < \varepsilon < 0.5$:

$$\int_{S_1} [F(t) - G(t)]dt = 0 \leq \varepsilon \int_S |F(t) - G(t)|dt,$$

and F dominates G in the $\varepsilon - AFSD$ sense. Let us now assume that for all $0 < \varepsilon < 0.5$, F dominates G in the $\varepsilon - AFSD$ sense. If $\mu(S_1) = 0$, where μ denotes the Lebesgue's measure over $\mathbb{R}$, then as F and G are non-decreasing and continuous on the right functions, for all t, $F(t) \leq G(t)$, that is, F dominates G in the FSD sense. If $\mu(S_1) > 0$ and there is no FSD, it will be proven that there is no AFSD for some $\varepsilon > 0$.

It will be denoted by $\varepsilon_0 = \int_{S_1} [F(t) - G(t)]dt > 0$. For $\varepsilon = \frac{\varepsilon_0}{2 \int_S |F(t) - G(t)| dt}$, we have $\varepsilon_0 = 2\varepsilon \int_S |F(t) - G(t)|dt > \varepsilon \int_S |F(t) - G(t)|dt$. That is, F does not dominate G for any ε, as intended to prove.

Part 2 is analogous. □

3. Financial Applications of Almost Stochastic Dominance

Many authors argue that as the investment horizon increases, an investment portfolio with a higher proportion of assets will dominate, or will be preferred over a portfolio of predominantly government bonds, although this is not in accordance with SD rules, that is, in this case there is some type of dominance, ASD. Therefore, investors prefer long-term assets over bonds, moreover, as the investment horizon increases, the set of "almost all" investors becomes the set of "all" the investors. (See Bernstein (1976), Leshno & Levy (2002) and Bali, Demirtas, Levy & Wolf (2009)).[3]

Examples of this fact will be proposed.

Example 1. Let us consider two simple investments: one bond which has an annual return of 9% with probability 1, and one asset which annual return of −5% with probability 0.5, and 35% with probability 0.5. The target is defining what type of investment is more attractive for investors. The fact mentioned above, will be confirmed, as the horizon of the investment advances, the asset will be more clearly preferred over bonds.

Let X be the random variable which represents the annual return of the asset and let Y be the random variable which represents the annual return of the bond. Let F be the distribution function of X, and G the distribution function of Y. The return of the asset in the first year is $X^{(1)} = 1 + X_0$, being X_0 the initial capital destined to the investment in assets and for the case of the bonds, this will be $Y^{(1)} = 1 + Y_0$ with Y_0 the initial capital destined to the investment in bonds. The return after n periods (years, in this case) will be $X^{(n)} = \prod_{i=1}^{n} [1 + X^{(i)}]$ and $Y^{(n)} = \prod_{i=1}^{n} [1 + Y^{(i)}]$ in assets and bonds, respectively.

For this example, it will be assumed, without loss of generality, that $X_0 = 1 = Y_0$. The procedure that will be followed is to calculate, for each year n, the possible returns of the investment in assets and bonds; this will provide a series of values for random variables with their respective probabilities. After, the

[3] This will be clarified later.

associated distributions will be calculated, they will be denoted as $F^{(n)}$ and $G^{(n)}$ for the assets and bonds, respectively.

For example, for the first year, the returns obtained for the assets are:

$$1 \quad u.m. \begin{cases} 1 - 0.05 * 1 = 0.95 & u.m. \\ 1 + 0.35 * 1 = 1.35 & u.m. \end{cases}$$

where u.m. denotes monetary units, and for the bonds:

$$1 \quad u.m. \longrightarrow 1 + 0.09 * 1 = 1.09 \quad u.m.$$

In this way:

$$F^{(1)}(x) = \begin{cases} 0, & if \quad x < 0.95 \\ 0.5, & if \quad 0.95 \leq x < 1.35 \\ 1, & if \quad x \geq 1.35 \end{cases}$$

and

$$G^{(1)}(x) = \begin{cases} 0, & if \quad x < 1.09 \\ 1, & if \quad x \geq 1.09 \end{cases}$$

These distributions do not verify the FSD criterion because they intercept, as shown in the graphic:

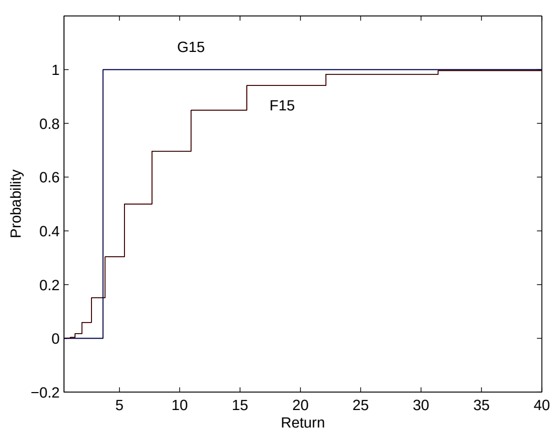

FIGURE 2: Distributions $F^{(1)}$ and $G^{(1)}$.

For the second year, the returns on the investment in assets are:

$$\begin{cases} 0.95 \quad u.m. \begin{cases} 0.9025 & u.m. \\ 1.2825 & u.m. \end{cases} \\ 1.35 \quad u.m. \begin{cases} 1.2825 & u.m. \\ 1.8225 & u.m. \end{cases} \end{cases}$$

and for bonds:

$$1.09 \quad u.m. \longrightarrow 1.1881 \quad u.m.$$

Then:

$$F^{(2)}(x) = \begin{cases} 0, & if \quad x < 0.9025 \\ 1/4, & if \quad 0.9025 \leq x < 1.2825 \\ 3/4, & if \quad 1.2825 \leq x < 1.8225 \\ 1, & if \quad x \geq 1.8225 \end{cases}$$

and

$$G^{(2)}(x) = \begin{cases} 0, & if \quad x < 1.1881 \\ 1, & if \quad x \geq 1.1881 \end{cases}$$

In this case the graphic is:

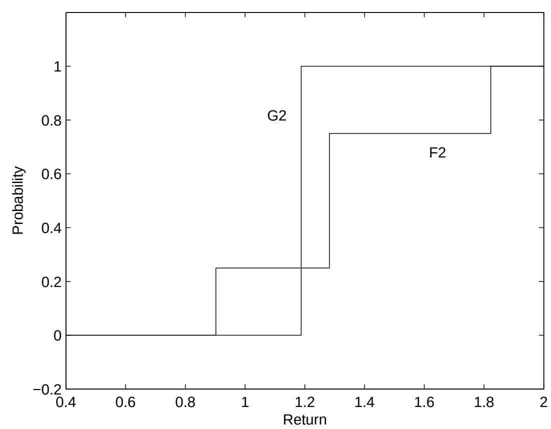

Figure 3: *Distributions* $F^{(2)}$ *and* $G^{(2)}$.

and so on.

Horizons of $1, 2, \ldots, 10, 15$ and 20 years will be considered, and it will be assumed that the investment began in the first of these years. For each year, the value ε will be calculated and it will be proven that this value decreases with the time, reason for which investors will prefer assets to bonds.

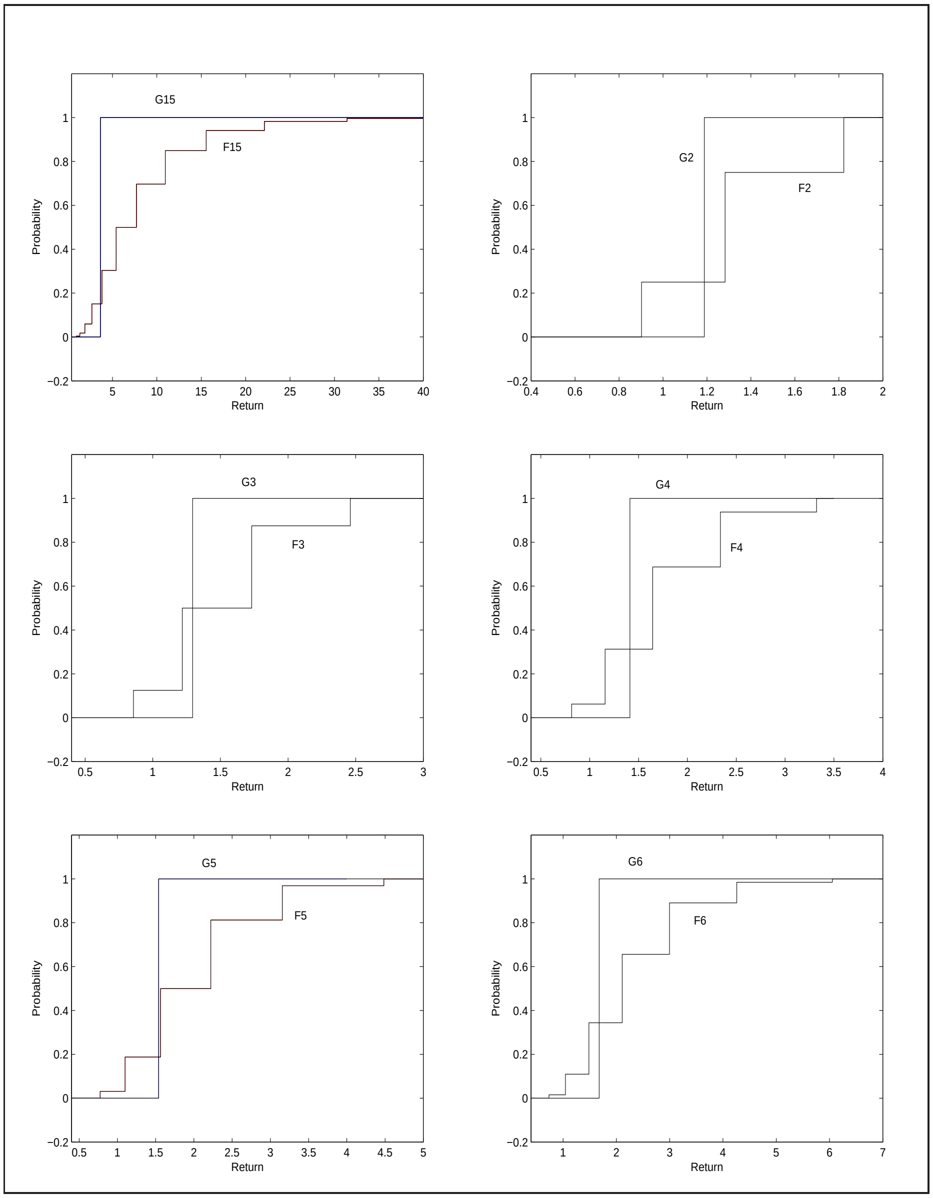

FIGURE 4: Distributions $F^{(n)}$ and $G^{(n)}$, with $n = 1, \ldots, 10, 15$ and 20. As observed, the area of violation of the FSD criterion, namely, the area in which $F^{(n)}$ is above $G^{(n)}$-A_1 of the ε definition-, decreases to the extent that the horizon of the investment increases, the value of ε also decreases, that is, as time increases, investors will prefer assets to bonds.

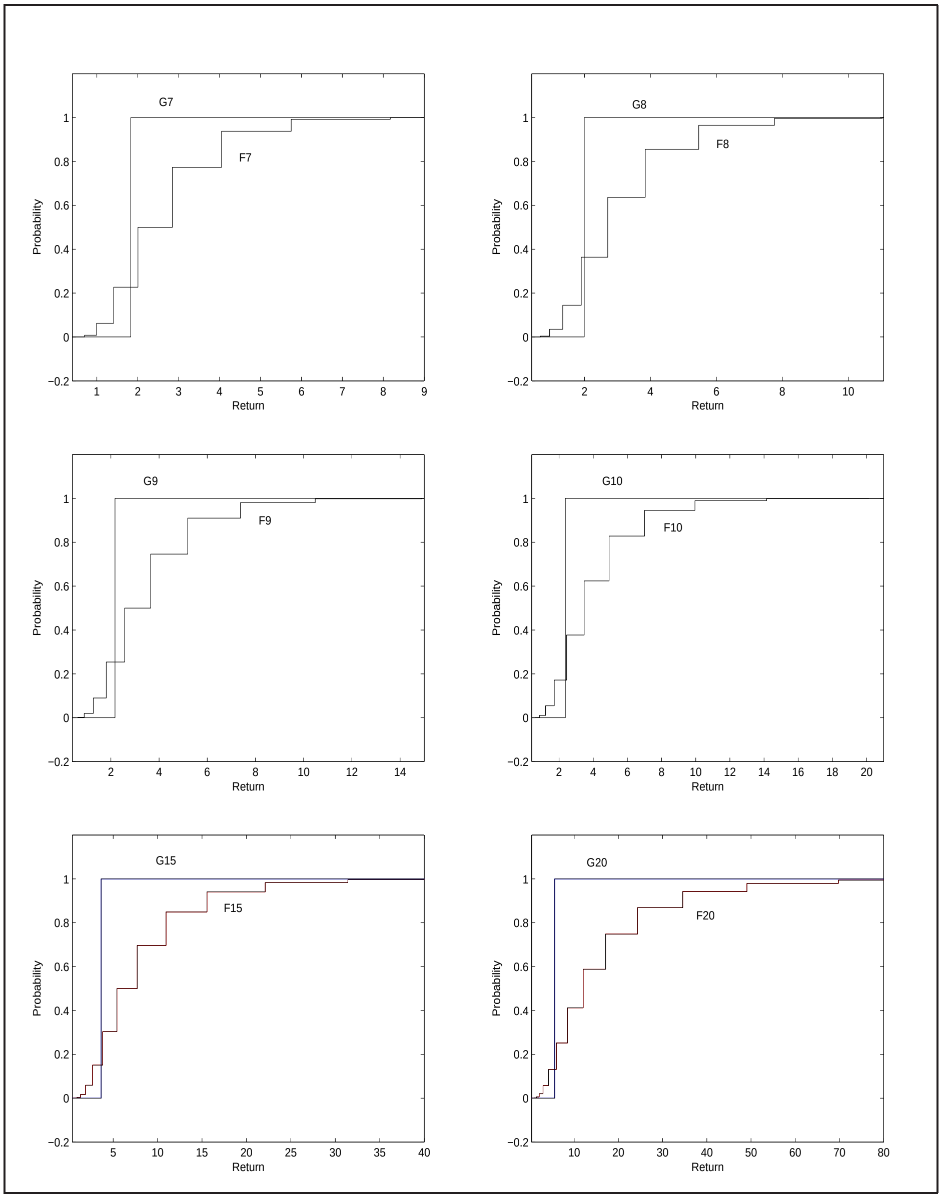

Figure 4: Continuation

Next, ε values will be shown for each horizon of the investment. As shown in the Table 1, these values decrease with time:

Table 1: ε values for each horizon of the investment.

Number of years	ε
1	0.3500
2	0.2576
3	0.2125
4	0.1856
5	0.1406
6	0.1363
7	0.1132
8	0.0972
9	0.0919
10	0.0464
15	0.0414
20	0.0247

Comments at the beginning of this subsection will be explained. As verified, ASD criteria have been used to establish a strong argument in favor of assets over bonds. Let us consider an investor who maximizes expected profits in a period T. Returns are supposed to be independent and identically distributed (i.i.d.) and the investments are supposed to be constant along through time. It is well known that, given different investments with i.i.d. returns and a large enough investment planning horizon, the investment which has higher geometric mean in returns (per period) almost certainly provides a greater benefit than those with lower geometric mean. In the long run, the distribution function of the investment that has a higher geometric mean is almost entirely to the right of the other distributions that represent alternatives, that is, ε decreases with time, as discussed throughout this section. However, there is some controversy in the economic meaning of this fact. Latané (1959), Markowitz (1976) and Leshno & Levy (2004), argue that the decrease in the value of ε is tied to an increase in the range of investor preferences ($U_1^*(\varepsilon)$), that is, they argue that in the long term, all reasonable preferences (profits) are considered. Levy (2009), highlights this fact, saying that, really as time goes by ε decreases (it has been shown in example 1), but the set $U_1^*(\varepsilon)$ does not increase. What happens is that as the periods of the investment increase, the set of all possible values of the random variables also increase, that is, set S is not a fixed set. Of course, if the set S is fixed, the set $U_1^*(\varepsilon)$ increases, but the fact is that S is not fixed. If the last example is observed, set S for the first year is: $[0.95, 1.35]$, for the second year is $[0.9025, 1.8225]$, for the third year is $[0.857375, 2.460375]$, etc.

In summary, there are two facts as time progresses: first ε decreases and this causes an increase of set $U_1^*(\varepsilon)$, and on the other hand, S increases, causing that for a given ε, the set $U_1^*(\varepsilon)$ decreases. The total effect over set $U_1^*(\varepsilon)$ is a mix between these two effects and this depends, on the kind of utility functions that are used.

Definition 4. Given a set S, ε_u is defined as the higher value of ε for which the utility function u still belongs to the set $U_1^*(\varepsilon)$, that is:

$$\varepsilon_u = \left[1 + \frac{\sup\{u'(x), x \in S\}}{\inf\{u'(x), x \in S\}}\right]^{-1} \tag{8}$$

As S increases with time, coefficient $\frac{\sup\{u'(x), x\in S\}}{\inf\{u'(x), x\in S\}}$ increases, and therefore, ε_u decreases. Observe that ε_u shows the higher value of the area alloweb to violate stochastic dominance criteria, for a given utility u such that u still belongs to the set $U_1^*(\varepsilon)$. If $\varepsilon > \varepsilon_u$ then $u \notin U_1^*(\varepsilon)$, otherwise $u \in U_1^*(\varepsilon)$. To be part or not of the set $U_1^*(\varepsilon)$ depends on the speed of decrease of ε and ε_u, that is, the fact that ε decreases is not enough to choose in the long term, it also depends on the utility function.

Let us continue with the last example. Values of ε_u will be calculated for different utility functions u.

Example 2. Let us continue with example 1. Utility functions u will be considered and the associated values of ε_u, will be calculated.

TABLE 2: Values of ε and ε_u for each horizon of the investment.

Number of years	ε	ε_u $u(x) = -\exp^{-x}$	ε_u $u(x) = \ln(x)$	ε_u $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$ $\alpha = 4$	ε_u $u(x) = \frac{(x-0.2)^{1-\alpha}}{1-\alpha}$ $\alpha = 2$
1	0.3500	0.4013	0.4130	0.1969	0.2984
2	0.2576	0.2849	0.3312	0.0567	0.1579
3	0.2125	0.1676	0.2584	0.0145	0.0780
4	0.1856	0.0754	0.1969	$3.6030 * 10^{-3}$	0.0373
5	0.1406	0.02389	0.1472	$8.8595 * 10^{-4}$	0.0176
6	0.1363	$4.8769 * 10^{-3}$	0.1083	$2.1740 * 10^{-4}$	$8.2874 * 10^{-3}$
7	0.1132	$5.6743 * 10^{-4}$	0.0787	$5.3320 * 10^{-5}$	$3.8923 * 10^{-3}$
8	0.0972	$3.1390 * 10^{-5}$	0.0567	$1.3076 * 10^{-5}$	$1.8269 * 10^{-3}$
9	0.0919	$6.4004 * 10^{-7}$	0.0406	$3.2059 * 10^{-6}$	$8.5653 * 10^{-4}$
10	0.0464	$3.3717 * 10^{-9}$	0.0289	$7.8616 * 10^{-7}$	$4.0100 * 10^{-4}$
15	0.0414	$1.1114 * 10^{-39}$	$5.1121 * 10^{-3}$	$3.2858 * 10^{-7}$	$8.5647 * 10^{-6}$
20	0.0247	$3.8185 * 10^{-176}$	$8.8591 * 10^{-4}$	$6.1816 * 10^{-13}$	$1.5380 * 10^{-7}$

For each representative column of values of ε and ε_u, the decrease mentioned above may be observed. Now, if columns 2 and 3 are compared, it can be shown that ε_u decreases faster than ε and for periods of 1 or 2 years, $\varepsilon < \varepsilon_u$, so $u(x) = -\exp(-x) \in U_1^*(\varepsilon)$, whereas periods strictly exceeding 2 years $u(x) = -\exp(-x) \notin U_1^*(\varepsilon)$. In this case, it is evidenced that the set $U_1^*(\varepsilon)$ does not necessarily increase with time. For this type of utility functions, it is not possible to reason as the authors previously mentioned. In case of working with log-utilities the reasoning is analogous, but for horizons of 5 or less than 5 years, and more than 5 years. In the case of columns 5 and 6, it is verified that $\varepsilon > \varepsilon_u$ for the analized periods, in these cases $u(x) \notin U_1^*(\varepsilon)$ for each studied period.

Example 3. In this case, two financial data series will be considered, in particular series of Ibex 35[4] and Nasdaq Composite indexes[5] corresponding to years from 1926 to 2008. A similar construction as that in the previous example will be performed. In this case, ϵ value is 0.3053, concluding that the Nasdaq series dominates Ibex 35 in a AFSD sense. The illustrative graphic is:

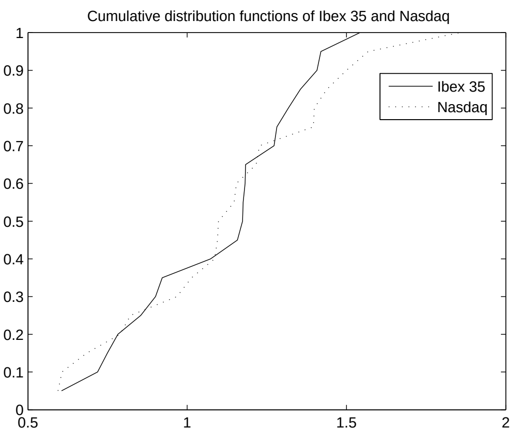

FIGURE 5: Distributions $F^{(1)}$ and $G^{(1)}$ for Ibex and Nasdaq Composite.

4. Conclusions

There are different rules in the literature for comparing investments, for example, Stochastic Dominance rules (SD), Mean-Variance (MV) and Almost Stochastic Dominance (ASD).

SD rules are useful in different areas of knowledge and they arise in a natural way from the need to make comparisons between different choices, using more information available in some situations (distribution functions, density functions, failure rate, etc.) than the mere comparison of averages or other numerical single data.

However, in is situations it may be useful to compare certain functional relationships dependent on means, variances or other measures of uncertainty (for example in the efficient portfolio selection or the scope of the study of the utility). In these cases, MV rules are used.

But sometimes, the use of SD or MV rules is not conducive to a specific selection of an investment over another, consequently, other rules (ASD) arise in response

[4] The official index of the Spanish Continuous Market, which is comprised of the 35 most liquid stocks traded on the market.

[5] A market-capitalization weighted index of the more than 3,000 common equities listed on the Nasdaq stock exchange. The types of securities in the index include American depositary receipts, common stocks, real estate investment trusts (REITs) and tracking stocks. The index includes all Nasdaq listed stocks that are not derivatives, preferred shares, funds, exchange-traded funds (ETFs) or debentures.

to this need for selection. These rules (ASD) are intended to be an extension of SD rules in cases where SD does not respond and they are defined in such manner as to be a useful guide for selection for almost all decision-makers, hence its name.

This paper presents a review of the different classical rules for investment decisions and the importance of ASD concepts selecting some investments over others has been highlighted in cases where there was no clear relationship according to SD and/or MV rules. Likewise, several examples have been proposed, in which applying ASD rules, it has been able to make a clear selection of some investments over others. It is important to note the selection made of Nasdaq Composite Index over the Ibex 35, for an annual series from 1926 to 2008.

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References

Almaraz, E. (2009), Cuestiones notables de ordenación estocástica en optimación financiera, Tesis de Doctorado, Universidad Complutense de Madrid, Facultad de Ciencias Matemáticas. Departamento de Estadística e Investigación Operativa, Madrid.

Almaraz, E. (2010), Reglas de decisión en ambiente de riesgo, Tesis de Master, Universidad Nacional de Eduación a Distancia, Facultad de Ciencias Matemáticas. Departamento de Estadística e Investigación Operativa, Madrid.

Bali, T., Demirtas, K., Levy, H. & Wolf, A. (2009), 'Bond versus Stock: Investors' Age and Risk Taking', *http://ssrn.com/abstract=936648* .

Baumol, W. (1963), 'An expected gain-confidence limit criterion for portfolio selection', *Management Science* **10**, 174–182.

Bernstein, P. (1976), 'The time of your life', *Journal of Portfolio Management* **2**(4), 4–7.

Latané, H. (1959), 'Criteria for choice among risky ventures', *Journal of Political Economy* **67**(2), 144–155.

Leshno, M. & Levy, H. (2002), 'Preferred by all and preferred by most decision makers: almost stochastic dominance', *Management Science* **48**(8), 1074–1085.

Leshno, M. & Levy, H. (2004), 'Stochastic dominance and medical decision making', *Health Care Management Science* **7**, 207–2215.

Levy, M. (2009), 'Almost stochastic dominance and stocks for the long run', *European Journal of Operational Research* **194**, 250–257.

Markowitz, H. (1976), 'Investment for the long run: New evidence for an old rule', *Journal of Finance* **31**, 1273–1286.

Shaked, M. & Shanthikumar, G. (2007), *Stochastic Orders*, Springer Series in Statistics, Springer Series in Statistics.

Steinbach, M. C. (2001), 'Markowitz revisited: Mean-variance models in financial portfolio analysis', *SIAM Review* **43**(1), 31–85.