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ALMOST SYMPLECTIC N-LINEAR CONNECTIONS IN THE BUNDLE OF ACCELERATIONS

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Abstract

In the present paper we treat an almost symplectic d-structure in the bundle of accelerations $E = Osc^2M$, defined as an alternate, nondegenerate d-tensor field of type $(0, 2)$, we determine all almost symplectic N-linear connections in the bundle of accelerations, we study the group of transformations of these connections and its invariants.

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1 Introduction

Motivated by concrete problems in variational calculus, the differential geometry of higher order Lagrange spaces is introduced and studied by R.Miron and Gh.Atanasiu in [6] – [9].

The various applications of the Lagrange geometry of order k in physics and mechanics are considerable, [2], [12].

The study of higher order Lagrange spaces is grounded on the k- osculator bundle notion. The bundle space of accelerations correspond in this study to $k = 2$, [1], [5].

The almost symplectic geometry of Finsler spaces was studied by many geometers. (See e.g.[11], [2], [3]).

In the present paper we shall treat an almost symplectic d-structure, defined as an alternate, nondegenerate d-tensor field of type $(0, 2)$.

We first introduce the notion of almost symplectic d-structure, define the notion of almost symplectic N-linear connection and study the properties of these notions (§2). And, the structure of the set of all almost symplectic N-linear connections preserving a nonlinear connection is discussed (§3), and finally the group of their transformations gives us the various important invariants (§4).

As to the terminology and notations we use those from [10], which are essentially based on M.Matsumoto [4].

2 The notion of almost symplectic d-structure in the bundle of accelerations

Let M be a real $n = 2n'$-dimensional C^∞-manifold and (Osc^2M, π, M) its 2-osculator bundle, or the bundle of accelerations. The local coordinates on the total space $E = Osc^2M$ are denoted by $(x^i, y^{(1)i}, y^{(2)i})$. If N is a nonlinear connection on E with the coefficients $N_{(1)}{}^i{}_j$, $N_{(2)}{}^i{}_j$, then let be $D\Gamma(N) = (L^i_{jk}, C_{(1)}{}^i_{jk}, C_{(2)}{}^i_{jk})$ an N- linear connection D on $E = Osc^2M$.

Definition 2.1 *A d-tensor field $a_{ij}(x, y^{(1)}, y^{(2)})$ of type $(0,2)$ on a differentiable manifold $E = Osc^2M$ is called an almost symplectic d-structure on E, if it is alternate and nondegenerate:*

$$(2.1)\quad a_{ij}(x, y^{(1)}, y^{(2)}) = -a_{ji}(x, y^{(1)}, y^{(2)}),$$

$$(2.2)\quad det(a_{ij}(x, y^{(1)}, y^{(2)})) \neq 0,\ \forall y^{(1)} \neq 0,\ \forall y^{(2)} \neq 0,$$

with the restriction on $TM = Osc^1M$, also nondegenerate.

Given an almost symplectic d-structure a_{ij}, we associate Obata's operators:

$$(2.3)\quad \Phi^{ir}_{sj} = \frac{1}{2}(\delta^i_s\delta^r_j - a_{sj}a^{ir}), \Phi^{*ir}_{sj} = \frac{1}{2}(\delta^i_s\delta^r_j + a_{sj}a^{ir}),$$

where (a^{ij}) is the inverse matrix of (a_{ij}):

$$(2.4)\quad a_{ij}a^{jk} = \delta^k_i.$$

a^{ij} is also alternate. Obata's operators have the same properties as ones associated with the Finsler space [10].

Definition 2.2 *An N-linear connection* $D\Gamma(N) = (L^i_{jk}, C_{(\alpha)}{}^i_{jk}), (\alpha = 1,2)$, *with the property:*

$$(2.5) \quad a_{ij|k} = 0, \; a_{ij} \overset{(\alpha)}{|}_k = 0, \; (\alpha = 1,2),$$

is said to be an almost symplectic N-linear connection, or compatible with the almost symplectic d-structure (2.1), (2.2).

Proposition 2.1 *Let* $D\Gamma(N)$ *be an almost symplectic N-linear connection. Then Obata's operators are h-and* v_α*-covariant constants* $(\alpha = 1,2)$:

$$(2.6) \quad \Phi^{ir}_{sj|l} = 0, \; \Phi^{ir}_{sj} \overset{(\alpha)}{|}_l = 0, \; \Phi^{*ir}_{sj|l} = 0, \; \Phi^{*ir}_{sj} \overset{(\alpha)}{|}_l = 0, \; (\alpha = 1,2),$$

and the Ricci identities, applied to a d-tensor field a_{ij}, *give:*

$$(2.7) \quad a_{im}R_j{}^m{}_{kh} + a_{mj}R_i{}^m{}_{kh} = 0,$$

$$(2.8) \quad a_{im}P_{(\alpha)j}{}^m{}_{kh} + a_{mj}P_{(\alpha)i}{}^m{}_{kh} = 0, \; (\alpha = 1,2)$$

$$(2.9) \quad a_{im}S_{(21)j}{}^m{}_{kh} + a_{mj}S_{(21)i}{}^m{}_{kh} = 0,$$

$$(2.10) \quad a_{im}S_{(\alpha\alpha)j}{}^m{}_{kh} + a_{mj}S_{(\alpha\alpha)i}{}^m{}_{kh} = 0, \; (\alpha = 1,2),$$

where $R_h{}^i{}_{jk}$, $P_{(\alpha)h}{}^i{}_{jk}$, $S_{(21)h}{}^i{}_{jk}$, $S_{(\alpha\alpha)h}{}^i{}_{jk}$ *are curvature d-tensor fields.*

From (2.6), (2.7), ..., (2.10) we easily get

Theorem 2.1 *The curvature d-tensor fields* $R_m{}^r{}_{pq}$, $P_{(\alpha)m}{}^r{}_{pq}$, $S_{(21)m}{}^r{}_{pq}$, $S_{(\alpha\alpha)m}{}^r{}_{pq}$, $(\alpha = 1,2)$ *of an almost symplectic N -linear connection have the following properties:*

$$(2.11) \quad \Phi^{*ir}_{sj}R_r{}^s{}_{kl} = 0, \; \Phi^{*ir}_{sj}R_r{}^s{}_{kl|l_1 \dots l_p} = 0,$$
$$\Phi^{*ir}_{sj}R_r{}^s{}_{kl} \overset{(\alpha)}{|}_{l_1 \dots l_p} = 0, \; (\alpha = 1,2; \; p = 1,2 \dots),$$

$$(2.12) \quad \Phi^{*ir}_{sj}P_{(\alpha)r}{}^s{}_{kl} = 0, \; \Phi^{*ir}_{sj}P_{(\alpha)r}{}^s{}_{kl|l_1 \dots l_p} = 0,$$
$$\Phi^{*ir}_{sj}P_{(\alpha)r}{}^s{}_{kl} \overset{(\beta)}{|}_{l_1 \dots l_p} = 0, \; (\alpha = 1,2; \; p = 1,2 \dots; \; \beta = 1,2),$$

$$(2.13) \quad \Phi^{*ir}_{sj}S_{(21)r}{}^s{}_{kl} = 0, \; \Phi^{*ir}_{sj}S_{(21)r}{}^s{}_{kl|l_1 \dots l_p} = 0,$$

$$\Phi^{*ir}_{sj} S_{(21)r}{}^{s}{}_{kl} \overset{(\alpha)}{|}_{l_1 \ldots l_p} = 0, \ (\alpha = 1,2;\ p = 1,2\ldots),$$

(2.14) $$\Phi^{*ir}_{sj} S_{(\alpha\alpha)r}{}^{s}{}_{kl} = 0, \ \Phi^{*ir}_{sj} S_{(\alpha\alpha)r}{}^{s}{}_{kl|l_1 \ldots l_p} = 0,$$
$$\Phi^{*ir}_{sj} S_{(\alpha\alpha)r}{}^{s}{}_{kl} \overset{(\beta)}{|}_{l_1 \ldots l_p} = 0, \ (\alpha = 1,2;\ p = 1,2\ldots,\ \beta = 1,2).$$

3 The set of almost symplectic N-linear connections

Let $D\overset{0}{\Gamma}(N) = (\overset{0}{L}{}^{i}_{jk}, \overset{0}{C}{}_{(\alpha)}{}^{i}_{jk})$, $(\alpha = 1,2)$ be a fixed N-linear connection on E. Then any N-linear connection on $E : D\Gamma(N) = (L^i_{jk}, C_{(\alpha)}{}^{i}_{jk})$, $(\alpha = 1,2)$ can be expressed in the form

(3.1) $$L^i_{jk} = \overset{0}{L}{}^{i}_{jk} - B^i_{jk},\ C_{(\alpha)}{}^{i}_{jk} = \overset{0}{C}{}_{(\alpha)}{}^{i}_{jk} - D_{(\alpha)}{}^{i}_{jk},\ (\alpha = 1,2),$$

where B^i_{jk}, $D_{(\alpha)}{}^{i}_{jk}$, $(\alpha = 1,2)$ are components of the difference d-tensor fields of $D\Gamma(N)$ from $D\overset{0}{\Gamma}(N)$ [4].
In order that $D\Gamma(N)$ is almost symplectic, that is (2.5) holds for $D\Gamma(N)$, it is necessary and sufficient that B^i_{jk}, $D_{(\alpha)}{}^{i}_{jk}$, $(\alpha = 1,2)$ satisfy

(3.2) $$a_{\underset{ij|k}{0}} + a_{sj} B^s_{ik} + a_{is} B^s_{jk} = 0,\ a_{ij} \overset{(\alpha)}{\underset{0}{|}}{}_{k} + a_{sj} D_{(\alpha)}{}^{s}_{ik} + a_{is} D_{(\alpha)}{}^{s}_{jk} = 0,\ (\alpha = 1,2),$$

which is eguivalent to

(3.3) $$\Phi^{*ir}_{sj} B^s_{rk} = -\frac{1}{2} a^{im} a_{\underset{mj|k}{0}},\ \Phi^{*ir}_{sj} D_{(\alpha)}{}^{s}_{rk} = -\frac{1}{2} a^{im} a_{mj} \overset{(\alpha)}{\underset{0}{|}}{}_{k},\ (\alpha = 1,2),$$

where $\overset{0}{|}$ and $\overset{(\alpha)}{\overset{0}{|}}$, $(\alpha = 1,2)$ denote the h-and v_α-covariant derivatives $(\alpha = 1,2)$, with rapport to $D\overset{0}{\Gamma}(N)$.

Proposition 3.1 *Let $D\overset{0}{\Gamma}(N)$ be a fixed N-linear connection on E. Then the set of all almost symplectic N-linear connections $D\Gamma(N)$ is given by (3.1), where B^i_{jk}, $D_{(\alpha)}{}^{i}_{jk}$, $(\alpha = 1,2)$ are arbitrary d-tensor fields satisfying*

(3.3).
Especially, if $D \overset{0}{\Gamma}(N)$ *is almost symplectic, then* (3.3) *becomes:*

(3.4) $\Phi^{*ir}_{sj} B^{s}_{rk} = 0,\ \Phi^{*ir}_{sj} D_{(\alpha)}{}^{s}_{rk} = 0,\ (\alpha = 1, 2).$

From Theorem 5.4.3[12], however, the system (3.3) has solutions in B^{i}_{jk}, $D_{(\alpha)}{}^{i}_{jk}$, $(\alpha = 1, 2)$. Substituting in (3.1) from the general solution we have:

Theorem 3.1 *Let* $D \overset{0}{\Gamma}(N)$ *be a fixed N-linear connection. The set of all almost symplectic N-linear connections* $D\Gamma(N)$ *is given by*

$$(3.5)\quad \begin{cases} L^{i}_{jk} = \overset{0}{L}{}^{i}_{jk} + \frac{1}{2} a^{im} a_{\substack{mj \overset{0}{|} k}} + \Phi^{ir}_{sj} X^{s}_{rk}, \\ C_{(\alpha)}{}^{i}_{jk} = \overset{0}{C}_{(\alpha)}{}^{i}_{jk} + \frac{1}{2} a^{im} a_{mj} \overset{(\alpha)}{\overset{0}{|}}{}_{k} + \Phi^{ir}_{sj} Y_{(\alpha)}{}^{s}_{rk},\ (\alpha = 1, 2), \end{cases}$$

where X^{i}_{jk}, $Y_{(\alpha)}{}^{i}_{jk}$, $(\alpha = 1, 2)$ *are arbitrary d-tensor fields, and* $\overset{0}{|}$, $\overset{(\alpha)}{\overset{0}{|}}$, *denote the h-and* v_α*-covariant derivatives* $(\alpha = 1, 2)$ *with respect to* $D \overset{0}{\Gamma}(N)$.

As the particular case $X^{i}_{jk} = Y_{(\alpha)}{}^{i}_{jk} = 0$, $(\alpha = 1, 2)$ in Theorem 3.1 we have

Theorem 3.2 [15] *Let* $D \overset{0}{\Gamma}(N)$ *be a given N-linear connection. Then the following N-linear connection* $D\tilde{\Gamma}(N)$ *is almost symplectic:*

$$(3.6)\quad \begin{cases} \tilde{L}^{i}_{jk} = \overset{0}{L}{}^{i}_{jk} + \frac{1}{2} a^{im} a_{\substack{mj \overset{0}{|} k}}, \\ \tilde{C}_{(\alpha)}{}^{i}_{jk} = \overset{0}{C}_{(\alpha)}{}^{i}_{jk} + \frac{1}{2} a^{im} a_{mj} \overset{(\alpha)}{\overset{0}{|}}{}_{k},\ (\alpha = 1, 2), \end{cases}$$

where $\overset{0}{|}$ *and* $\overset{(\alpha)}{\overset{0}{|}}$ *denote the h-and* v_α*-covariant derivatives* $(\alpha = 1, 2)$ *with respect to* $D \overset{0}{\Gamma}(N)$.

If we take an almost symplectic N-linear connection (*e.g.*$D\tilde{\Gamma}(N)$) as $D \overset{0}{\Gamma}(N)$ in Theorem 3.1, we have:

Theorem 3.3 [15] *The set of all almost symplectic N-linear connections* $D\Gamma(N)$ *is given by:*

$$(3.7)\quad \begin{cases} L^i_{jk} = \tilde{L}^i_{jk} + \Phi^{ir}_{sj} X^s_{rk}, \\ C_{(\alpha)}{}^i_{jk} = \tilde{C}_{(\alpha)}{}^i_{jk} + \Phi^{ir}_{sj} Y_{(\alpha)}{}^s_{rk}, \ (\alpha = 1, 2), \end{cases}$$

where X^i_{jk}, $Y_{(\alpha)}{}^i_{jk}$, $(\alpha = 1, 2)$ *are arbitrary d-tensor fields.*

4 The group of transformations of almost symplectic N-linear connections.

Let us consider the transformations $D\Gamma(N) \longrightarrow D\bar{\Gamma}(N)$ of almost symplectic N-linear connections [13], which preserve the nonlinear connection N. Owing to Theorem 3.3 they are given by:

$$(4.1)\quad \begin{cases} \bar{L}^i_{jk} = L^i_{jk} + \Phi^{ir}_{sj} X^s_{rk}, \\ \bar{C}_{(\alpha)}{}^i_{jk} = C_{(\alpha)}{}^i_{jk} + \Phi^{ir}_{sj} Y_{(\alpha)}{}^s_{rk}, \ (\alpha = 1, 2), \end{cases}$$

where X^i_{jk}, $Y_{(\alpha)}{}^i_{jk}$, $(\alpha = 1, 2)$ are arbitrary given d-tensor fields.

Evidently we have:

Theorem 4.1 *The set of all transformations* (4.1) *and the mapping product form an abelian group* $\mathcal{G}_{as}$, *which is isomorphic to the additive group of the triads of d-tensor fields* $(\Phi^{ir}_{sj} X^s_{rk},\ \Phi^{ir}_{sj} Y_{(1)}{}^s_{rk},\ \Phi^{ir}_{sj} Y_{(2)}{}^s_{rk})$.

We determine the invariants of the group $\mathcal{G}_{as}$. The torsion d-tensor fields $T_{(0)}{}^i{}_{jk}$, $S_{(\alpha)}{}^i{}_{jk}$, $R_{(0\alpha)}{}^i{}_{jk}$, $P_{(\alpha\beta)}{}^i{}_{jk}$, $Q_{(\alpha\beta)}{}^i{}_{jk}$, $(\alpha = 1, 2;\ \beta = 1, 2)$ are given in §3.4 [12]. We denote with:

$$(4.2)\quad t_{(\alpha)}{}^i{}_{jk} = \mathcal{A}_{jk}\{\frac{\delta N_{(\alpha)}{}^i{}_j}{\delta y^{(\alpha)k}}\},\ (\alpha = 1, 2),$$

where $\mathcal{A}_{jk}\{\ldots\}$ denotes the alternate summation: $\mathcal{A}_{jk}\{A_{jk}\} = A_{jk} - A_{kj}$.

Since $R_{(0\alpha)}{}^i{}_{jk}$, $t_{(\alpha)}{}^i{}_{jk}$, $(\alpha = 1, 2)$, $Q_{(21)}{}^i{}_{jk}$, $P_{(12)}{}^i{}_{jk}$, $P_{(21)}{}^i{}_{jk}$ depend on N only, they are invariants of $\mathcal{G}_{as}$.

We make some notations:

$$(4.3)\quad \begin{cases} t_{(\alpha)}{}^*{}_{ijk} = \mathcal{S}_{ijk}\{a_{im} t_{(\alpha)}{}^m{}_{jk}\},\ T_{(0)}{}^*{}_{ijk} = \mathcal{S}_{ijk}\{a_{im} T_{(0)}{}^m{}_{jk}\}, \\ R_{(0\alpha)}{}^*{}_{ijk} = \mathcal{S}_{ijk}\{a_{im} R_{(0\alpha)}{}^m{}_{jk}\},\ S_{(\alpha)}{}^*{}_{ijk} = \mathcal{S}_{ijk}\{a_{im} S_{(\alpha)}{}^m{}_{jk}\},\ (\alpha = 1, 2 \end{cases}$$

where $\mathcal{S}_{ijk}\{\ldots\}$ denotes the cyclic summation: $\mathcal{S}_{ijk}\{A_{ijk}\} = A_{ijk} + A_{jki} +$

A_{kij}, and

$$(4.4)\quad \begin{cases} \overset{1}{k}_{(\alpha\alpha)ijk} = a_{km}T_{(0)}{}^{m}{}_{ij} + \mathcal{A}_{ij}\{a_{im}P_{(\alpha\alpha)}{}^{m}{}_{jk}\}, \\ \overset{2}{k}_{(\alpha)ijk} = a_{im}S_{(\alpha)}{}^{m}{}_{jk} + \mathcal{A}_{jk}\{a_{km}C_{(\alpha)}{}^{m}_{ij}\}, \\ \overset{3}{k}_{(\alpha\beta)ijk} = \mathcal{A}_{jk}\{a_{km}P_{(\alpha\beta)}{}^{m}{}_{ij}\},\ \overset{4}{k}_{(\alpha)ijk} = \mathcal{A}_{ij}\{a_{im}C_{(\alpha)}{}^{m}_{jk}\}, \\ \alpha = 1,2;\ \beta = 1,2. \end{cases}$$

It is noted that $t_{(\alpha)}{}^{*}{}_{ijk}$, $R_{(0\alpha)}{}^{*}{}_{ijk}$, $T_{(0)}{}^{*}{}_{ijk}$, $S_{(\alpha)}{}^{*}{}_{ijk}$ are alternate, and $\overset{1}{k}_{(\alpha\alpha)ijk}$, $\overset{4}{k}_{(\alpha)ijk}$ are alternate with respect to i, j and $\overset{2}{k}_{(\alpha)ijk}$, $\overset{3}{k}_{(\alpha\beta)ijk}$ are alternate with respect to j, k.

Theorem 4.2 *The d-tensor fields* $t_{(\alpha)}{}^{i}{}_{jk}$, $R_{(0\alpha)}{}^{i}{}_{jk}$, $t_{(\alpha)}{}^{*}{}_{ijk}$, $R_{(0\alpha)}{}^{*}{}_{ijk}$, $T_{(0)}{}^{*}{}_{ijk}$, $S_{(\alpha)}{}^{*}{}_{ijk}$, $\overset{1}{k}_{(\alpha\alpha)ijk}$, $\overset{2}{k}_{(\alpha)ijk}$, $\overset{3}{k}_{(\alpha\beta)ijk}$, $\overset{4}{k}_{(\alpha)ijk}$, $(\alpha = 1,2;\ \beta = 1,2)$, *are invariants of the group* $\mathcal{G}_{as}$.

Proof. By a transformation of almost symplectic N-linear connections (4.1) we find $\bar{T}_{(0)}{}^{i}{}_{jk} = T_{(0)}{}^{i}{}_{jk} + \frac{1}{2}\mathcal{A}_{jk}\{X^{i}_{jk} - a_{sj}a^{ir}X^{s}_{rk}\}$.
By direct calculations we have $\bar{T}_{(0)}{}^{*}{}_{ijk} = T_{(0)}{}^{*}{}_{ijk}$ etc.

Proposition 4.1 *Between the invariants in Theorem* 4.2 *there exist the following relations:*

$$(4.5)\quad \begin{cases} \mathcal{S}_{ijk}\{\overset{1}{k}_{(11)ijk}\} = 2T_{(0)}{}^{*}{}_{ijk} + t_{(1)}{}^{*}{}_{ijk},\ \mathcal{S}_{ijk}\{\overset{2}{k}_{(\alpha)ijk}\} = 2S_{(\alpha)}{}^{*}{}_{ijk}, \\ \mathcal{S}_{ijk}\{\overset{3}{k}_{(11)ijk}\} = T_{(0)}{}^{*}{}_{ijk} + t_{(1)}{}^{*}{}_{ijk},\ \mathcal{S}_{ijk}\{\overset{4}{k}_{(\alpha)ijk}\} = S_{(\alpha)}{}^{*}{}_{ijk}, \\ \overset{2}{k}_{(\alpha)ijk} + \overset{4}{k}_{(\alpha)jki} = S_{(\alpha)}{}^{*}{}_{ijk},\ (\alpha = 1,2). \end{cases}$$

Theorem 4.3 *Let N be a nonlinear connection on $E = Osc^2M$.*
(1) *The invariant* $T_{(0)}{}^{*}{}_{ijk}$ *(resp.* $S_{(\alpha)}{}^{*}{}_{ijk}$, $(\alpha = 1,2)$*) vanishes if and only if there exists an almost symplectic N-linear connection $D\Gamma(N)$ with* $T_{(0)}{}^{i}{}_{jk} =$

0 *(resp.* $S_{(\alpha)}{}^{i}{}_{jk} = 0$, $(\alpha = 1, 2)$).
(2) *The invariants* $T_{(0)}{}^{*}{}_{ijk}$ *and* $S_{(\alpha)}{}^{*}{}_{ijk}$ *vanish if and only if there exists an almost symplectic N-linear connection* $D\Gamma(N)$ *with* $T_{(0)}{}^{i}{}_{jk} = S_{(\alpha)}{}^{i}{}_{jk} = 0$, $(\alpha = 1, 2)$).

Proof. If we put $X^{i}_{jk} = \lambda T_{(0)}{}^{i}{}_{jk}$ in (4.1), where λ is a real number, we have $\bar{T}_{(0)}{}^{i}{}_{jk} = T_{(0)}{}^{i}{}_{jk} + \mathcal{A}_{jk}\{\Phi^{ir}_{sj}X^{s}_{rk}\} = (1 + \frac{3}{2}\lambda)T_{(0)}{}^{i}{}_{jk} - \frac{\lambda}{2}a^{ir}T_{(0)}{}^{*}{}_{rjk}$.

Taking $\lambda = -\frac{2}{3}$, $T_{(0)}{}^{*}{}_{ijk} = 0$ implies $\bar{T}_{(0)}{}^{i}{}_{jk}$. The converse is evident. The statement about $S_{(\alpha)}{}^{*}{}_{ijk}$, $(\alpha = 1, 2)$ is proved in the same way. (2) follows from the independence of two procedures in (1).

Paying attention to $a_{ij} \overset{(\alpha)}{|}_{k} = \frac{\delta a_{ij}}{\delta y^{(\alpha)k}} - C_{(\alpha)}{}^{h}_{ik} a_{hj} - C_{(\alpha)}{}^{h}_{jk} a_{ih} = \frac{\delta a_{ij}}{\delta y^{(\alpha)k}} - \overset{4}{k}_{(\alpha)ijk} = 0$, $(\alpha = 1, 2)$, Proposition 4.1 tells us the condition that a_{ij} be a usual almost symplectic d-structure:

Theorem 4.4 *An almost symplectic d-structure* a_{ij} *does not depend on* $y^{(\alpha)}$, $(\alpha = 1, 2)$, *if an only* $\overset{4}{k}_{(\alpha)ijk} = 0$, $(\alpha = 1, 2)$, *which is equivalent to* $\overset{2}{k}_{(\alpha)ijk} = 0$, $(\alpha = 1, 2)$. *In this case it holds* $S_{(\alpha)}{}^{*}{}_{ijk}$, $(\alpha = 1, 2)$.

Proposition 4.2 (1) *If* $\overset{1}{k}_{(11)ijk} = 0$, *then* $2T_{(0)}{}^{*}{}_{ijk} = -t_{(1)}{}^{*}{}_{ijk}$.
(2) *Assume that* $\overset{1}{k}_{(11)ijk} + a_{km}R_{(01)}{}^{m}{}_{ij} = 0$, $\overset{2}{k}_{(\alpha)ijk} + \overset{3}{k}_{(11)ijk} = 0$, $(\alpha = 1, 2)$ *and* $S_{(\alpha)}{}^{*}{}_{ijk}$, $(\alpha = 1, 2)$. *Then,* $R_{(01)}{}^{*}{}_{ijk} = 0$ *is equivalent to* $T_{(0)}{}^{*}{}_{ijk} = 0$.
(3) *Assume that* $\overset{4}{k}_{(\alpha)ijk} + \lambda\,\overset{1}{k}_{(11)ijk} + a_{km}R_{(01)}{}^{m}{}_{ij} = 0$, $\lambda\,\overset{2}{k}_{(\alpha)ijk} + \overset{3}{k}_{(11)ijk} = 0$ *and* $S_{(\alpha)}{}^{*}{}_{ijk} = 0$, $(\alpha = 1, 2)$, *where* $\lambda \neq \pm 1$ *is a real number. Then,* $T_{(0)}{}^{*}{}_{ijk} + \lambda R_{(01)}{}^{*}{}_{ijk} = 0$ *is equivalent to* $T_{(0)}{}^{*}{}_{ijk} = 0$.

Proof. Forming the cyclic summations of each of the assumed formulas, the proof follows from Proposition 4.1.

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