ON THE QUESTION OF SOLVABILITY OF THE PERIODIC BOUNDARY VALUE PROBLEM FOR A SYSTEM OF LINEAR GENERALIZED ORDINARY DIFFERENTIAL EQUATIONS

Let $\omega$ be a positive number. $A = (a_{ik})_{i,k=1}^n : \mathbb{R} \to \mathbb{R}^{m \times n}$ and $g = (g_i)_{i=1}^n : \mathbb{R} \to \mathbb{R}^n$ be a matrix function and a vector function from $BV_{\omega}^{m \times n}$ and $BV_{\omega}$, respectively.

We consider the $\omega$-periodic boundary value problem
\[ dx(t) = dA(t) \cdot x(t) + dg(t), \quad x(0) = x(\omega). \tag{1} \]

The use will be made of the following notation and definitions: $\mathbb{R} = [-\infty, \infty]$; $\mathbb{R}^{m \times n}$ is the set of all real $m \times n$-matrices; $I$ is the identity $n \times n$-matrix; $\mathbb{R}^n = \mathbb{R}^{1 \times n}$, $BV_{\omega}^{n}$ is the set of all matrix functions $X : \mathbb{R} \to \mathbb{R}^{m \times n}$ such that $X(t + \omega) = X(t) + X(\omega)$ for $t \in \mathbb{R}$, and the restriction on $[0, \omega]$ of every its components has bounded total variation; $X(t-)$ and $X(t+)$ are the left and the right limits of $X$ at the point $t \in \mathbb{R}$; $d_1 X(t) = X(t) - X(t-)$, $d_2 X(t) = X(t+) - X(t)$.

If $g : \mathbb{R} \to \mathbb{R}$ is nondecreasing, $x : \mathbb{R} \to \mathbb{R}$ and $s < t$, then
\[ \int_s^t x(t)d\tau = \int_s^t \int_{[s,t]} x(\tau)d\mu \, d\tau + \int_{[s,t]} x(t)d_1g(\tau) + x(s)d_2g(s), \]
where $\int_{[s,t]} x(\tau)d\mu = \text{Lebesgue-Stieltjes integral over the open interval } [s,t] \text{ with respect to the measure } \mu_g \text{ corresponding to } g$. (If $s = t$, then $\int_s^t x(\tau)d\mu = 0$).

Let natural numbers $m$ and $n_1, \ldots, n_m$ ($0 = n_0 < n_1 < \cdots < n_m = n$), nondecreasing functions $c_{ij} : [0, \omega] \to \mathbb{R}$ ($l = 1, 2; j = 1, \ldots, m$), functions $\alpha_{lj} \in L_\omega(c_{lj})$ ($l = 1, 2; j = 1, \ldots, m$) and matrix functions $P_{ij} = (p_{ijk})_{k,l=1}^{1, \omega}$ ($l = 1, 2; j = 1, \ldots, m$).
Then the problem where differential equations.

Let there exist natural numbers \( m_1, \ldots, m_m \) (0 = \( n_0 < n_1 < \cdots < n_m = n \)), functions \( c_j \) and \( \alpha_j \) (\( l = 1, 2; j = 1, \ldots, m \)) and matrix functions \( P_j = \{ p_{ijk}(t, \mathbf{x}) \}_{i,k \in \mathbb{N}} \) such that (2) holds. Let, moreover,

\[
\det(t + (-1)^k t^k A(t)) \neq 0, \quad (1 + \sigma_j) d_1 c_j(t) + (1 - \sigma_j) d_2 c_j(t) < 2,
\]

\[
(1 - \sigma_j) d_1 c_j(t) + (1 + \sigma_j) d_2 c_j(t) \neq -2
\]

and

\[
\exp(c_j(t)) - \sum_{0 < \tau \leq t} d_1 c_j(\tau) - \sum_{0 < \tau \leq t} d_2 c_j(\tau) > \frac{1}{2} \left[ (1 + \sigma_j) \prod_{0 < \tau \leq t} \left( 1 - d_1 c_j(\tau) \right) \prod_{0 \leq \tau \leq t} \left( 1 + d_2 c_j(\tau) \right)^{-1} + (1 - \sigma_j) \prod_{0 < \tau \leq t} \left( 1 + d_1 c_j(\tau) \right)^{-1} \prod_{0 \leq \tau \leq t} \left( 1 - d_2 c_j(\tau) \right) \right]
\]

for every \( t \in [0, \omega] \) and \( j \in \{1, \ldots, m\} \), where

\[
c_j(t) = \sum_{l=1}^{m} \int_{0}^{t} \alpha_{lj}(\tau) d c_j(\tau).
\]

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