CONTINUOUS DEPENDENCE OF THE SOLUTION OF A CLASS OF NEUTRAL DIFFERENTIAL EQUATIONS ON THE INITIAL DATA AND ON THE RIGHT-HAND SIDE

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Let \( J = [a, b] \) be a finite interval, \( \mathbb{R}^n \) be an \( n \)-dimensional Euclidean space, \( O \subset \mathbb{R}^n \) an open set, \( \eta : J \to \mathbb{R}^1 \) and \( \tau : J \to \mathbb{R}^1 \) be continuously differentiable functions satisfying respectively the conditions: \( \eta(t) < t, \eta(t) > 0; \tau(t) \leq t, \tau(t) > 0 \); moreover, let \( L_1(J, \mathbb{R}^1) \) be the space of summable functions \( m : J \to \mathbb{R}^1 \), \( \mathbb{R}^1 = [0, +\infty), \Delta(J, \mathbb{R}^{\times n}) \) be the space of piecewise continuous \( n \times n \) matrix functions \( C : J \to \mathbb{R}^{\times n^n} \) with a finite number of points of discontinuity of the first kind, \( \| C \| = \sup_{t \in J} |C(t)| \), \( C^1(J, O) \) be the space of continuously differentiable functions \( \varphi : J_1 \to O, J_1 = [\tau, b], \tau = \min\{\eta(a), \tau(a)\}, \) for which \( \| \varphi \| = \max_{t \in J_1} |\varphi(t)| \), and let \( E_f \) be the space of the functions \( f : J \times O^2 \to \mathbb{R}^n \) satisfying the following conditions:

1. the function \( f(t, x, y) : J \to \mathbb{R}^n \) is measurable for every \((x, y) \in O^2;\)
2. for any compactum \( K \subset O \) and any function \( f \in E_f \) there exist \( m_{f,K}(\cdot), L_{f,K}(\cdot) \in L_1(\mathbb{R}^1, \mathbb{R}^1) \) such that

\[
\left| f(t, x, y) - f(t, x', y') \right| \leq L_{f,K}(t) \left( |x - x'| + |y - y'| \right), \forall (t, x, x', y, y') \in J \times K^4.
\]

Introduce the sets:

\[
V_1(K, \delta) = \left\{ f \in E_f : \max_{(t', x, y) \in J \times K^2} \left| \int_{t'}^t f(t, x, y) \, dt \right| \leq \delta \right\},
\]
\[
V_2(K, \alpha) = \left\{ f \in E_f : \int J \left[ m_{f,K}(t) + L_{f,K}(t) \right] \, dt \leq \alpha \right\},
\]
\[
W(K, \delta, \alpha) = V_1(K, \delta) \cap V_2(K, \alpha),
\]
where \( K \subset O \) is a compact set, \( \delta > 0 \) and \( \alpha > 0 \) are arbitrary numbers.

To every element \( \sigma = (\tau_0, x_0, \varphi, C, f) \in \Sigma = J \times O \times C^1(J_1, O) \times \Delta(J, \mathbb{R}^{\times n}) \times E_f \) we assign the neutral differential equation

\[
x(t) - C(t) \dot{x}(x(t)) + f(t, x(t), x(\tau(t)))
\]
with the initial condition

\[
x(t) = \varphi(t), \quad t \in [\tau_0, t_0), \quad x(t_0) = x_0,
\]
where \( \tau_0 = \min\{\eta(t_0), \tau(t_0)\}. \)

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Definition. Every function $x(t; \sigma) \in O$ defined on the interval $[\tau_0, t_1] \subset (\tau, \delta]$ will be called a solution corresponding to the element $\sigma \in \Sigma$ if it satisfies on $[\tau_0, t_0]$ the condition (2), is absolutely continuous on $[\tau_0, t_1]$ and almost everywhere satisfies the equation (1).

Theorem. Let $\tilde{x}(t) - x(t; \sigma), t \in [t_0, t_1]$, be a solution corresponding to the element $\tilde{\sigma} = (\tilde{t}_0, \tilde{x}_0, \tilde{\varphi}, \tilde{C}, \tilde{f}) \in \Sigma$ and let the compactum $K_1 \subset O$ contain a neighborhood of the set $K_0 = \{x(t) : t \in [\tau_0, t_1]\}$, where $\tau_0 = \min[\eta(t_0), \tau(t_0)]$. Then for any $\varepsilon > 0$ there exists a number $\delta = \delta(\varepsilon) > 0$ such that to every element

$$\sigma \in V(\tilde{\sigma}, K_1, \delta, \alpha_0) - V(\tilde{t}_0, \delta) \times V(\tilde{x}_0, \delta) \times V(\tilde{\varphi}, \delta) \times V(\tilde{C}, \delta) \times W(K_1, \delta, \alpha_0)$$

there corresponds the solution $x(t; \sigma)$ defined on the interval $[\tau_0, \tilde{t}_1 + \delta]$. Moreover, if $\sigma_i = (t_{0i}, x_{0i}, \varphi_i, C_i, f_i) \in V(\tilde{\sigma}, K_1, \delta, \alpha_0), i = 1, 2$, then

$$|x(t; \sigma_1) - x(t; \sigma_2)| \leq \varepsilon, \quad t \in [\tilde{t}_0, \tilde{t}_1 + \delta],$$

where $\tilde{t}_0 = \max(t_{01}, t_{02}), \alpha_0 > 0$ is a fixed number.

Here $V(\tilde{t}_0, \delta), V(\tilde{x}_0, \delta), V(\tilde{\varphi}, \delta), V(\tilde{C}, \delta)$ are $\delta$-neighborhoods of the points $\tilde{t}_0, \tilde{x}_0, \tilde{\varphi}, \tilde{C}$ in the spaces $\mathbb{R}^1, \mathbb{R}^n, C^1(J_1, O), C^1(J, \mathbb{R}^{n \times n})$, respectively.

The above formulated theorem is an analogue of a theorem stated in [1], [2] (see also [3] and [4]). This theorem can be proved by the method described in [2].

In conclusion it should be noted that if the right-hand side of the equation (1) depends non-linearly on $\dot{x}(\eta(t))$, then the theorem is, generally speaking, invalid. The appropriate example is given in [3].

References


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