MODELING OF MINIMAL SURFACE BASED ON AN ISOTROPIC BEZIER CURVE OF FIFTH ORDER

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Abstract. A method is proposed for constructing minimal surfaces based on fifth-order Bezier isotropic curves specified in a vector-parametric form, allowing control of the guide curve and the surface in user mode. The coefficients of the basic quadratic forms were calculated and it was shown that the surfaces would be minimal. An example of a surface constructed by the proposed method is given.

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1. Introduction

The problem of building a smooth surface containing specified points or curves is particularly relevant due to the intensive development of mechanical engineering, the construction industry and computer technologies [12, 19]. Previously used for such problems shells of zero Gaussian curvature [22, 23], although simple in design and manufacture, do not always give an optimal result when covering complex structures, and the carrying capacity of such shells significantly depends on small deviations of their overall contour from ideal shape [23]. The elimination of these drawbacks in the most natural way is possible by using minimal surfaces [16], whose theory has been successfully developed for a long time.

The minimal surface is a surface whose average curvature $H$ is zero at all its points. The minimal surface is thus a surface of negative Gaussian curvature. The first studies of minimal surfaces were performed by Lagrange, who investigated the variation problem of finding the surface of the smallest area stretched over a given contour [7, 29]. Later, Monge established that the minimalism of the area leads to the condition that the mean curvature is zero. The classical problem of Plato [10] is also known, which consists in finding the surface of the smallest area passing through a given curve. The physical implementation of the task is achieved by immersing a rigid wire frame of some given shape into soapy water and then removing it – the shape of the resulting soap film is a solution to Plato’s problem.