FLAT AFFINE AND SYMPLECTIC GEOMETRIES ON LIE GROUPS

ANDRÉS VILLABÓN

Communicated by Robert Low

Abstract. In this paper we exhibit a family of flat left invariant affine structures on the double Lie group of the oscillator Lie group of dimension 4, associated to each solution of classical Yang-Baxter equation given by Boucetta and Medina. On the other hand, using Koszul’s method, we prove the existence of an immersion of Lie groups between the group of affine transformations of a flat affine and simply connected manifold and the classical group of affine transformations of $\mathbb{R}^n$. In the last section, for each flat left invariant affine symplectic connection on the group of affine transformations of the real line, describe by Medina-Saldarriaga-Giraldo, we determine the affine symplectomorphisms. Finally we exhibit the Hess connection, associated to a Lagrangian bi-foliation, which is flat left invariant affine.

MSC: 53C07, 53B05, 53D05, 17B62, 17D25
Keywords: Development of flat affine manifold, flat affine Lie groups, oscillator Lie group

Contents

1 Introduction 96

2 Flat Affine Geometry on Four-Dimensional Oscillator Lie Group 99
   2.1 Flat Affine Structures on $O$ ................................. 100
   2.2 Left Symmetric Algebra Cohomology ......................... 102
   2.3 Double Lie Groups of the Oscillator Group $O$ Relative to a Solution of Classical Yang-Baxter Equation ......................... 105

3 Development of Flat Affine Manifold 109

4 Symplectic Invariant Geometry on the Lie Group $\operatorname{Aff}(\mathbb{R})$ 117

References 120

doi: 10.7546/jgsp-46-2017-95-121 95