

Research Article

Probabilistic and Fuzzy Arithmetic Approaches for the Treatment of Uncertainties in the Installation of Torpedo Piles

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The “torpedo” pile is a foundation system that has been recently considered to anchor mooring lines and risers of floating production systems for offshore oil exploitation. The pile is installed in a free fall operation from a vessel. However, the soil parameters involved in the penetration model of the torpedo pile contain uncertainties that can affect the precision of analysis methods to evaluate its final penetration depth. Therefore, this paper deals with methodologies for the assessment of the sensitivity of the response to the variation of the uncertain parameters and mainly to incorporate into the analysis method techniques for the formal treatment of the uncertainties. Probabilistic and “possibilistic” approaches are considered, involving, respectively, the Monte Carlo method (MC) and concepts of fuzzy arithmetic (FA). The results and performance of both approaches are compared, stressing the ability of the latter approach to efficiently deal with the uncertainties of the model, with outstanding computational efficiency, and therefore, to comprise an effective design tool.

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1. Introduction

1.1. Context: offshore platforms, mooring systems, and anchors

Petroleum companies around the world have been faced with the challenge of developing offshore oil production activities in deep and ultradeep waters. In shallow water, the traditional solution consists in employing platforms supported by fixed framed structures, such as the

jackets where the foundation system consists of driven piles [1]. Presently, as oil fields have been identified in deeper water such as in the Campos Basin (southeastern Brazil), offshore platforms have included several types of floating units, such as the *semisubmersible* platforms, the *tension-leg platforms* (TLPs), and *floating production, storage, and offloading* (FPSOs) units based on ships.

Floating platforms can be maintained in position by different types of mooring systems, which in turn may employ anchors based on different types of foundation elements. Semisubmersible platforms and FPSO units, for instance, may be kept in position by mooring lines in *catenary* or *taut-leg* configurations. Mooring lines in a free-hanging catenary configuration transmit essentially horizontal loads to the foundation system. This fact introduces a greater flexibility in the selection of the appropriate anchor type. However, the mooring radius (the horizontal distance, measured at the sea bottom, from the center of the platform) is relatively large; typically, about two to three times the water depth. Therefore, the application of catenary configurations may not be feasible in deep or ultradeep waters, due to the increased weight of the mooring lines, and also due to installation problems that may arise in congested scenarios with several platforms close together (as is the case of some oil fields in the Campos Basin).

The taut-leg configuration has been proposed to tackle these constraints. This configuration, where the lines are not slack, allows the use of smaller line lengths. When associated with the use of new materials (such as polyester fiber ropes) [2], this leads to considerable reduction in the weight of the mooring system. Moreover, since at the anchor point the lines are not in contact with the seabed, and may reach inclinations around 45° , the mooring radius is typically equal to the water depth, therefore, considerably shorter than in catenary configurations.

However, taut-leg mooring systems transmit vertical loads to the foundation system. This is also the case with the tension leg platforms, which are moored by vertical tendons. Therefore, care should be taken in the selection of anchor types that can withstand vertical loads.

Amongst the foundation elements that have been applied in deep water systems, two types of anchors can be mentioned: the *suction anchor* and the *vertically loaded anchor* (VLA) [3]. However, some installation difficulties have been reported for suction anchors, due to added mass effects and the resonant period for the lifting system at the installation depth that may approach the dominant wave period at the site [4]. Vertically loaded anchors are easier to install, but require drag procedures that may hinder their correct positioning, mainly in congested areas with many others nearby platforms.

1.2. The torpedo pile

The *torpedo pile* (illustrated in Figure 1) was proposed [5] as a solution to withstand vertical loads while circumventing the problems associated with other types of anchors. It consists simply in a metallic pipe, with closed tip, filled with scrap chain, and concrete [6].

The installation does not require drag procedures such as employed in VLAs; the procedure is quite simple, and is illustrated in Figure 2. First, the installation vessel hangs the pile (connected to the mooring line) at a specified drop height, above the target point on the seabed. The design embedment is then reached by simply releasing the pile, letting it accelerate, fall freely, and then penetrate into the soil.



Figure 1: Torpedo pile.

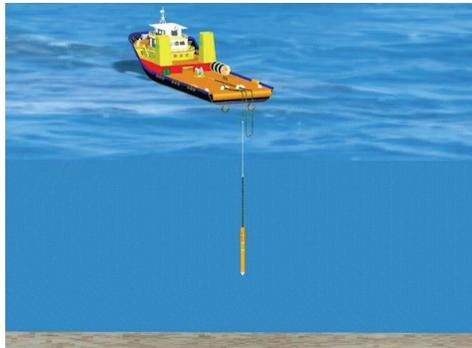


Figure 2: Installation of torpedo pile [5].

More than one hanging configuration has been conceived, for instance, one of the alternatives (considered for the installation of torpedo piles to anchor flexible risers or *mobile drilling units* (MODUs)) has a chain loop at the top of the installation line, as shown in Figure 3. As the pile falls, this loop is pulled and unfolded. Therefore, the torpedo pile presents not only low cost of manufacture, but also low cost of installation, since the same vessel can transport and install the pile.

There is another configuration, for permanent mooring of production units, which does not present the chain loop, but requires two vessels to hang, respectively, the installation line and the mooring line (to which the torpedo pile is connected). In this configuration, the bottom end of the installation line is connected to an intermediate point of the mooring line, therefore, maintaining the pile suspended at the desired drop height. Above this connection point, there is a trigger that allows the mooring line and the pile to be released, causing the pile to fall (dragging with it the mooring line), and penetrate the soil.

Another advantage of the torpedo pile concept is that, since it can withstand horizontal and vertical loads, it can be used with mooring lines in a taut-leg configuration that, as mentioned before, is the preferred alternative for semisubmersible platforms and FPSO units in deeper waters and congested scenarios.

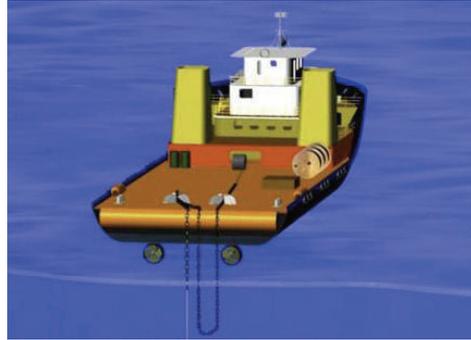


Figure 3: Chain loop [6].

1.3. Objective of the paper

The design of a torpedo pile should employ theoretical models to predict pile penetration depth, such as the dynamic penetration model proposed by True [7]. This model relies on soil parameters whose values are assumed as known, fixed, and deterministic.

However, it is well known that the soil properties present a significant degree of variability that, associated to imprecisions in the determination of their design values, can affect the accuracy of the response given by the simulation method. The objective of this paper, therefore, is to study techniques to deal with the uncertainty of the soil parameters, and to associate these techniques to an analytical/numerical penetration model for the torpedo pile.

Two different approaches are considered for the treatment of uncertainties of the penetration model. The first is a probabilistic approach, based on the classical Monte Carlo method. The second is a “possibilistic” approach, derived using concepts from *fuzzy arithmetic* and fuzzy sets.

The following sections of the paper begin by describing the theoretical model and solution procedure considered for the simulation of the pile penetration. Firstly, the analytical formulation originally presented by True [7] is described; then a numerical solution procedure in the time domain is described, followed by an application where a pile dropped from a height of 200 m above the seabed is analyzed for deterministic, fixed values of the soil parameters.

The paper then proceeds by describing the soil parameters that are considered uncertain. Methodologies to assess the sensitivity of the response to the variation of these uncertain parameters are then presented, based on the Monte Carlo method (MC) and fuzzy arithmetic (FA). More important, such methodologies allow the designer to incorporate, into the analysis method, techniques for the formal treatment of these uncertainties.

Finally, results of applications of these concepts for the treatment of uncertainties are presented for an actual case study, beginning with results of deterministic parametric studies in order to assess the sensitivity of the response to the variation of the uncertain parameters. Results for the “probabilistic” MC approach are then presented, followed by the novel implementation and application of the approach based on FA. The results and performance of both approaches are compared, stressing the ability of the latter approach to efficiently deal with the uncertainties of the model, with outstanding computational efficiency, and therefore, to comprise an effective design tool.

2. The Penetration Model

2.1. Original formulation: penetration of projectiles

Studies on the behavior of penetration of projectiles were initially intended for military applications [8] and were followed by studies on the prediction of final embedment depth of projectiles into soils [9, 10], and estimation of undrained shear strength [11, 12].

The development of a dynamic penetration model by the US navy was required to represent the penetration of propellant-embedded plate anchors into seafloor soils [13]. This kind of anchor is directly positioned on the mud line and an explosion, caused by the propeller system, pushes the anchor fluke down. In order to fulfill this objective, True [7] took into account recommendations given by authors of empirical models (such as Young [9]) and modified traditional bearing capacity formulations (for deep foundations in cohesive soils) to consider variations in penetration resistance with velocity and penetrator shape.

The analytical model developed in [7] to simulate the dynamic penetration of plate anchors is based on Newton's second law. Considering that the penetrator velocity v can be expressed as $v = dz/dt$ (where z stands for the soil depth), and therefore, its acceleration dv/dt can be expressed as $(dv/dz)(dz/dt)$, the governing equation can be written as follows:

$$M' \frac{dv}{dz} v = W_s - F_D - F_T - F_S + F_E, \quad (2.1)$$

where M' is the effective mass of the penetrator, given by

$$M' = M + 2\rho \cdot V. \quad (2.2)$$

In this latter equation, M and V are, respectively, the structural mass and the volume of the penetrator, and ρ is the mass density of the soil. It can be seen that the term $2\rho V$ is similar to the "added mass" term of the Morison equation [14], which has been traditionally employed to calculate hydrodynamic drag and inertia loads on cylinders immersed in fluid. In the present case, when multiplied by the acceleration at the left-hand side of (2.1), the term $2\rho V$ introduces an additional inertia force that corresponds to the contribution of the soil in which the penetrator is immersed.

The forces in the right-hand side of (2.1) are W_s (the submerged weight of the penetrator); F_D , F_T , and F_S (which are, resp., the drag force, the tip resistance, and the side resistance); and F_E (the external driving force applied by the propeller system).

The submerged weight W_s is defined in terms of the weight in air W , volume V , and the unit weight of soil γ by the following expression:

$$W_s = W - V \cdot \gamma. \quad (2.3)$$

The drag force F_D is similar to the longitudinal drag component given by Morison's equation [14], which is expressed as:

$$F_D = \frac{1}{2} \cdot v \cdot |v| \cdot A_f \cdot C_D \cdot \rho, \quad (2.4)$$

where A_f is the frontal or cross-sectional area of the penetrator and C_D is the empirical drag coefficient, that can have the value of 0.7 as proposed by True according to [13].

The classic formulation for *static* bearing capacity of deep pile foundation states that, for undrained conditions, the tip resistance Q_T and side resistance Q_S [15] are defined by

$$\begin{aligned} Q_T &= Su \cdot N_c \cdot A_f, \\ Q_S &= Su \cdot \alpha \cdot A_s, \end{aligned} \quad (2.5)$$

where Su is the undrained soil shear strength; N_c is the bearing capacity factor (assumed equals to 9 for homogeneous clay); α is the dimensionless side adhesion factor; and A_f and A_s are, respectively, the frontal and lateral areas of the pile.

The dynamic tip and side resistance F_T and F_S are now considered by the inclusion, in this classic static formulation, of a *side adhesion reduction factor* δ , a *soil strain rate factor* $S_{\dot{\epsilon}}$, and the *soil sensitivity value* Sti . The latter represents the loss of shear strength that clays suffer when remolded, and is defined as the ratio of undisturbed and remolded strengths [16]. Thus, the tip resistance F_T and side resistance F_S are defined by the following expressions:

$$\begin{aligned} F_T &= Su \cdot N_c \cdot A_f \cdot S_{\dot{\epsilon}}, \\ F_S &= \frac{Su \cdot A_s \cdot \delta}{Sti} \cdot S_{\dot{\epsilon}}. \end{aligned} \quad (2.6)$$

Values for the side adhesion reduction factor δ were determined in [17] based on results of model tests. An expression for the strain rate factor $S_{\dot{\epsilon}}$ was also defined in [17], as a function of the velocity v and the diameter (or thickness) of the penetrator d , the undrained soil shear strength Su , and other empirical parameters. This expression can be written as

$$S_{\dot{\epsilon}} = \left(\frac{S_e}{1 + \left(1/\sqrt{(C_e \cdot v/Su \cdot d) + C_0}\right)} \right). \quad (2.7)$$

Values for the empirical parameters S_e (maximum soil strain rate at high velocity values), C_e (strain rate velocity factor), and C_0 (strain rate constant) were also determined in [17] based on results of model tests.

In the penetration model considered for offshore applications in the Campos Basin [5], the undrained shear strength Su of the soil is assumed to vary linearly with depth z , according to the following expression:

$$Su(z) = Su_0 + Su_k \cdot z, \quad (2.8)$$

where Su_0 is the undrained shear strength at the mudline; and Su_k is the rate of increase with depth.

Original solution procedure

To solve (2.1), True [7] developed an incremental finite-difference algorithm and considered that the penetrator is a point object at the i th depth increment, thus some simplifications could be made:

$$M' \cdot v_i \cdot \frac{v_{i+1} - v_{i-1}}{2\Delta z} = W_s - F_D - F_T - F_S. \quad (2.9)$$

Substituting the expressions for M' , F_D , F_T , F_S , and W_s in (2.9), ignoring the external driving force F_E , and assuming $C_0 = 0.06$ (according to [7]), the following expression is obtained, which can be applied repeatedly to obtain the velocities of the penetrator at each depth increment Δz :

$$v_{i+1} = v_{i-1} + \frac{2\Delta z}{v_i(M + 2\rho_i V)} \cdot \left((W - V\gamma_i) - \left(\frac{1}{2}v_i^2 A_f C_D \rho_i \right) - \text{Su}_i \left(A_f N_c + \frac{A_s \delta}{\text{St}_i} \right) \right) \cdot \left(\frac{S_e}{1 + \left(1/\sqrt{(C_e v_i / \text{Su}_i d) + 0.06} \right)} \right). \quad (2.10)$$

The final penetration depth can then be seen as the product of the depth increment Δz by the number of the increment for which the penetrator velocity drops to zero. It should be recalled that, as in any numerical solution procedure, the accuracy of the results also depends on the careful selection of the depth increment. This will depend on the particular example that is analyzed, and may also involve the use of different increment values to assess the convergence to an accurate solution.

2.2. Formulation for free-falling pile

A free-falling cylindrical penetrometer, dropped from a given height above the mudline, is studied in [12] for the prediction of penetration depth and undrained shear strength. Equation (2.1), with some modifications, is also applied to describe the movement of this penetrometer. Firstly, since it is free falling and there is no external driving force, the term F_E is omitted. Also, the added mass in (2.2) is considered negligible for slender penetrometers moving along their long axis, thus, the effective mass is equal to the structural mass ($M' = M$). Therefore, also replacing the velocity v by dz/dt and considering that the acceleration a is equal to $(dv/dz)(dz/dt)$, (2.1) becomes

$$Ma = W_s - F_D - F_T - F_S. \quad (2.11)$$

Obviously, torpedo piles behave similarly to the free-falling penetrometers, therefore, their motions can also be described by the True formulation [7], with the same considerations as employed in [12], resulting in (2.11). Moreover, the traditional Morison hydrodynamic formulation can also be applied to describe the forces acting in the pile while it is still in the water, before reaching the seabed, and the same submerged weight can be assumed for both media.

It should be emphasized that such semiempirical formulations incorporate some assumptions, which in turn leads to uncertainties in the model. However, as mentioned in the introduction, this work is focused on the influence of the uncertainties in selected soil parameters. Studies regarding uncertainties associated with the penetration model itself will be dealt with in future works.

Solution procedure in the time domain

It can be observed that the procedure originally proposed by True [7], as described in (2.9) and (2.10), involved the spatial integration of (2.1) to obtain velocities as functions of depth z . This indeed would be the more natural solution procedure if one is concerned only with the representation of the isolated pile and the natural random variability of the soil parameters with depth. However, as will be commented later, there are other sources of uncertainty to be concerned as well.

Moreover, as will be commented in the final section of this paper, the final goal of the developments presented here is to incorporate the penetration model (including the techniques that will be described later, based on fuzzy arithmetic, for the formal treatment of uncertainties) in a finite element (FE) spatial discretization scheme, associated to a time domain nonlinear dynamic solver. The idea is to model not only the isolated pile, but also all the other components of the system being installed (i.e., the mooring line itself and the other lines and chains involved in the installation procedure), in a complete 3D model submitted to other loadings such as marine current.

With this scenario in mind, it is more convenient to integrate (2.11) in the time domain. At the current stage, where the focus is in modeling an isolated pile and evaluating the uncertainties in the soil parameters, the added mass can still be disregarded as assumed in [12] and in (2.11). In a posterior implementation of the penetration model in a time domain solver associated to the full FE model, the dynamic equations will also incorporate the added mass effects of the complete configuration of the pile with the installation and mooring lines.

The solution in the time domain, in terms of the acceleration a_{n+1} , velocity v_{n+1} , and displacement d_{n+1} , at a given time t_{n+1} , can be accomplished by applying a time-integration algorithm such as the Chung and Lee explicit method [18] that can be stated as:

$$\begin{aligned} M \cdot a_{n+1} &= f_n; & f_n &= W_s - F_D - F_T - F_S, \\ d_{n+1} &= d_n + \Delta t v_n + \Delta t^2 \left((1/2 - \beta) a_n + \beta a_{n+1} \right), \\ v_{n+1} &= v_n + \Delta t \left((1 - \gamma) a_n + \gamma a_{n+1} \right), \end{aligned} \quad (2.12)$$

where β and γ are algorithmic parameters defined as $1 \leq \beta \leq 28/27$, $\gamma = 3/2$; and Δt is the time step, which should not exceed a critical time step (Δt_c) in order to maintain the stability of the numerical solution [18]. The full time domain solution procedure is shown in Algorithm 1. It should also be recalled that the displacements, velocities, and accelerations are positive in the downward direction.

Application

The application of the penetration model described above is now illustrated for a problem also studied in [19], corresponding to a pile dropped from a height of 200 m above the seabed. The pile and soil data are presented in Tables 1 and 2. It should be emphasized that the soil parameters and the sensitivity value of 3 are related to a specific deposit and may not necessarily be representative of general applications.

This application is not intended to represent an actual installation procedure for a torpedo pile (such as the depicted in Figure 2), since, as mentioned before, the current implementation of the penetration model represents only the pile. Therefore, in order to take

(a) Initial calculations	
1.	Calculate the torpedo mass M .
2.	Initialize d_0 , v_0 , f_0 , and a_0 , where $a_0 = f_0/M$.
3.	Select the appropriate algorithmic parameter β and time step Δt , $\Delta t \leq \Delta t_c$ and calculate the integration constants, $\beta_1 = \Delta t^2 \left(\frac{1}{2} - \beta \right)$, $\beta_2 = \Delta t^2 \beta$, $\gamma_1 = -\frac{1}{2} \Delta t$, $\gamma_2 = \frac{3}{2} \Delta t$.
(b) For each time step ($n = 0, 1, \dots, N - 1$).	
1.	Calculate the forces, $f_n = W_s - F_D - F_T - F_S$.
2.	Calculate the acceleration at time $t_{n+1} = t_n + \Delta t$, $a_{n+1} = f_n/M$.
3.	Calculate the displacement at time $t_{n+1} = t_n + \Delta t$, $d_{n+1} = d_n + \Delta t v_n + \beta_1 a_n + \beta_2 a_{n+1}$.
4.	Calculate the velocity at time $t_{n+1} = t_n + \Delta t$, $v_{n+1} = v_n + \gamma_1 a_n + \gamma_2 a_{n+1}$.
5.	$n \leftarrow n + 1$, go to (1) of step (b).

Algorithm 1: Solution procedure in the time domain.**Table 1:** Problem definition: pile data.

W : weight in air	396 kN
W_b : submerged weight	340 kN
d : diameter	0.762 m

Table 2: Problem definition: soil data.

Su_0 : undrained shear strength at the soil surface	5.0 kPa
Su_k : rate of increase with depth	2.0 kPa/m
St_i : sensitivity	3.0
N_c : bearing capacity factor	9.0
δ : side adhesion reduction factor	0.9
S_e : empirical maximum strain rate factor	5.0
C_e : empirical strain rate velocity factor	0.02 kN · s/m ²

into account the increase on drag effects due to the mooring line and chain loop that are not explicitly represented, the model employs a value for the drag coefficient C_D equal to 2.7, larger than the value of 0.7 as proposed by True according to [13]. In future works, which will consider the implementation of a coupled finite element-based, time domain simulation program, there will be no need to perform this “fudging” of the drag coefficient C_D , since the coupled 3D model will explicitly include the complete installation configuration (e.g., the mooring line and chain loop for the application in MODUs described earlier).

The time domain solution considered a total time of 15 seconds (enough, as will be seen, for the pile to fully penetrate the soil). The analysis is performed with a time increment of 0.002 seconds. The value considered for the algorithmic parameter β of the time-integration algorithm is $\beta = 28/27$.

The results are presented in Figure 4, in terms of a graph relating the vertical position of the pile to its velocity, and in Figure 5 in terms of time histories of depth and velocity. The origin

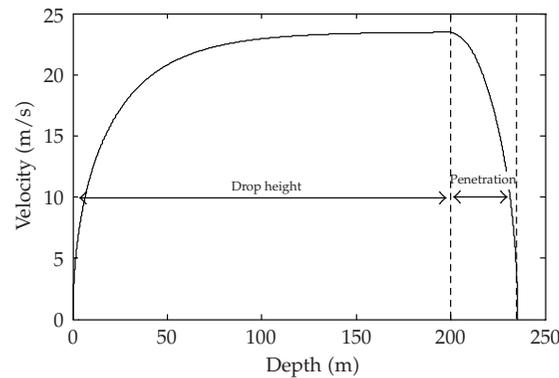


Figure 4: Deterministic pile penetration analysis: depth \times velocity.

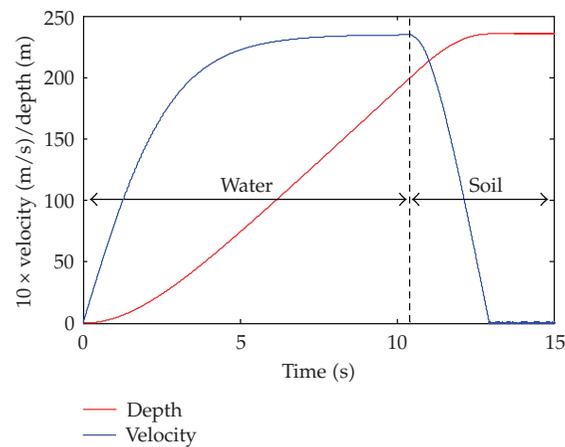


Figure 5: Deterministic pile penetration analysis: time histories.

of the graph of Figure 4 corresponds to the pile in its initial position, before being dropped (therefore, with velocity and displacement equal to zero). It is seen that, as the pile drops in the water, its velocity increases until it nearly reaches the so-called “terminal velocity” due to the water drag (of course, this requires the pile to be released from an appropriate height). As the pile reaches the seabed and begins to penetrate in the soil, the velocity is reduced; when it returns to zero the penetration is completed and the final depth of the pile tip is reached.

3. Uncertainties of The Soil Parameters

Selected parameters

As stated before in (2.8), for cohesive soils in offshore applications in Campos Basin [5], the undrained soil shear strength S_u is assumed to vary linearly with depth z , in terms of S_{u0} (undrained shear strength at the mudline) and of $S_{u\kappa}$ (the rate of increase with depth). For the normally consolidated clay encountered offshore in the Campos Basin [5], typical values that

may be considered for Su_0 and Su_k are, respectively, $Su_0 = 5$ kPa and $Su_k = 2$ kPa/m. Therefore, (2.8) could be written as

$$Su(z) = 5 + 2 \cdot z \text{ (kPa)} \quad (3.1)$$

Actual values for these parameters that affect the undrained soil shear strength Su for offshore sites may be obtained from in situ tests (such as CPT—cone penetration tests [20], or vane tests based on a torsion procedure), or from laboratory tests with undisturbed samples, such as triaxial and minivane tests. It should also be recalled that the soil sensitivity Sti represents the loss of shear strength that clays suffer when remolded, and is defined as the ratio of undisturbed and remolded strengths [16]. Remolded strength values can be obtained from vane, triaxial, or minivane tests of disturbed soils.

Thus, it can be seen that values for Su (undrained shear strength) and Sti (sensitivity) are obtained from testing. Traditionally, a deterministic procedure is employed to obtain design values for these parameters, by calculating the average of values obtained from several tests. However, it is known that the results of both in-situ or laboratory tests may be influenced by several factors. The latter tests can be affected by factors such as mechanical disturbance in the soil samples, in the process of extraction and remolding; by changes in the samples during storage, and so forth. In-situ tests can also be affected by mechanical interferences, inadequate execution, and so on.

Therefore, it can be intuitively understood that there is a high degree of local soil variability, and imprecisions in the determination of the design values of these soil parameters. Large variations in the response of the torpedo pile, mainly in terms of the final penetration depth reached by the pile, may be expected due to these uncertainties. The main objective of this paper, then, is to present a methodology to take into account uncertainties and imprecision in the values of input parameters that define the physical and numerical models involved in the design and analysis of torpedo piles.

This work focuses on Su (specifically, the rate of increase with depth Su_k) and Sti . Of course, other parameters (not necessarily related only to the soil) could be considered; however, those can be dealt with in future works.

Sources of uncertainty

Before proceeding further, it is important to recall some basic concepts regarding sources of uncertainty. In soil profile modeling, they may be grouped in two types [21–23]: (1) noncognitive, random natural variability, usually referred as *aleatory* uncertainty; and (2) cognitive or *epistemic* uncertainties, that involve abstraction or subjectivity.

The first group comprises the inherent uncertainty type, due to natural heterogeneity or in-situ variability of the soil, such as varying depths of strata during soil formation, variation in mineral composition, and stress history [24]. This corresponds for instance to the natural variability of the soil strength from point to point vertically at the position where the pile is to be installed.

The second group includes epistemic uncertainties due to lack of knowledge; in this case, information about subsurface conditions is few, because soil profile characteristics must be inferred from field or laboratory investigation of a limited number of samples. It includes also uncertainties generated from sample disturbance, test imperfections, human factors, and also, when engineering properties are obtained through correlation with index properties, as

in the case of CPT tests where empirical models are used to calculate the undrained shear resistance by applying correlation factors to the cone tip resistance [24].

According to this classification, two major approaches, respectively, probabilistic or “possibilistic” can be employed to deal with uncertainties [25, 26]. Therefore, the remainder of this paper will deal with methodologies based on these approaches, to assess the sensitivity of the response to the variation of the selected parameters and mainly to incorporate, into the analysis method, techniques for the formal treatment of uncertainties. Section 4 will describe a probabilistic approach based on the Monte Carlo method and an approach based on fuzzy arithmetic (FA).

Before proceeding, some additional comments should be presented regarding these sources of uncertainty. Inherent or natural variability are random by nature and cannot be reduced by increasing the number of tests [27]. The cognitive, epistemic uncertainties are reducible; however, for offshore sites, they will usually be present since the cost of performing in-situ tests at offshore sites is very expensive. Such tests may not be performed for every installation site and sometimes the values of the parameters are even estimated or extrapolated from previous tests made at other locations. Moreover, disturbances in these few samples are very common.

As strange as it may seem to experienced geotechnical engineers, not involved in deepwater offshore activities, this is precisely what has happened in soil investigations in the Campos Basin. Those are the reasons why *epistemic uncertainties are always added to the natural variability*: the use of limited data, of data arising from disturbed soil samples, and data from locations other than the one at which the torpedo pile is to be installed. In summary, the fact that there may be no knowledge of the exact site local soil variability is the very reason why (as presented in the next section) probabilistic approaches may fail, and is the motivation of the use of the approach based on FA.

4. Approaches for Treatment of Uncertainties

4.1. Probabilistic approach: the Monte Carlo method

As mentioned before, noncognitive sources of uncertainty involve parameters that can be treated as random variables, and to which a probabilistic distribution can be associated, based on statistical data. In such cases, the probabilistic approach is traditionally recommended.

Probabilistic approaches for treatment of uncertainties can be divided in two main categories. The first one comprises statistical methods that involve simulation, such as the classical Monte Carlo simulation method and its variants. The second category comprises nonstatistical methods such as those based on perturbation techniques. For instance, the stochastic finite element method [26] falls in this latter category; it is based on expanding the random parameters around their mean values via Taylor series, in order to formulate linear relationships between some random characteristics of the response and the random input parameters.

In the implementation of the classical Monte Carlo simulation, N samples of the uncertain parameters are randomly generated using a given joint probability density function. The deterministic analysis procedure is employed for each sample of the simulation process [28], obtaining then N responses that are statistically treated to get the first two statistical moments of the response (mean and standard deviation values).

The MC method is completely general, for linear or nonlinear analyses. However, in general, the accuracy of the statistical response is only adequate when the number of sample data N is sufficiently large; therefore, it is usually considered too expensive in terms of computing time. This fact has motivated studies on variants of the classical method, involving for instance variance reduction techniques and the Neumann expansion [29].

Due to its robustness and ability to effectively treat the noncognitive, random uncertainties, the classical MC simulation method has been used to calibrate and validate all other probabilistic techniques. The studies presented in this paper will also employ this method as a benchmark to compare the performance of the approach based on fuzzy arithmetic, which will be described in Section 4.2, in the representation of the random uncertainties.

4.2. Fuzzy arithmetic (FA) approach

It is important to recall that the cognitive sources of uncertainty are related not to chance, but rather to imprecise or vague information, involving subjectivity and/or dependent on expert judgment. Moreover, the axioms of probability and statistics are not adequate to deal with such types of uncertainties, which can be more effectively treated by “possibilistic” approaches employing for instance the theory of *fuzzy sets*.

The theory of fuzzy sets was introduced by Zadeh in [30] to define classes of objects with continuous membership graduations or associations in the interval $[0, 1]$. A fuzzy set has vague limits, allowing graded changes from one class to another, instead of exact limits characteristic of ordinary or *crisp* sets. In classical Boolean algebra, the notion of false and true values is limited to 1 or zero. In fuzzy logic, values that are “more or less” true or false can be treated, defined by real numbers that vary continuously from 0 to 1.

The treatment of uncertainties that derive from imprecise information is then possible, avoiding the use of random information. Therefore, complex systems, that would be hard to model with the theory of conventional sets, can be easily modeled by fuzzy sets. The fuzzy set theory allows the representation of imprecise and uncertain measures as fuzzy numbers, defined as:

$$A = \{(x, A(x)), x \in R, A(x) \in [0, 1]\}, \quad (4.1)$$

where $x \in R$ is the numeric support of the fuzzy number A , and $A(x) \in [0, 1]$ is the membership function (MF).

Fuzzy numbers are completely characterized by their MFs, that are built based on knowledge of an expert, who can assign “low,” “probable,” or “high” values for the desired parameters. Based on this subjective information, MFs can be constructed presenting either linear or nonlinear shapes. The more usually employed shapes for engineering problems are triangular, trapezoidal, and sinusoidal; the choice will depend on the type of application, and will also follow the assessment of the expert. In this work, triangular fuzzy MFs are used, defined by estimating three values [31]:

- (i) a more reliable value, m , to which is attributed a membership degree equals to 1;
- (ii) an inferior value, a , that most certainly will be exceeded by another value, and to which is attributed a membership degree equals to 0;
- (iii) a superior value, b , that most certainly will not be exceeded by another value, and to which is also attributed a membership degree equals to 0.

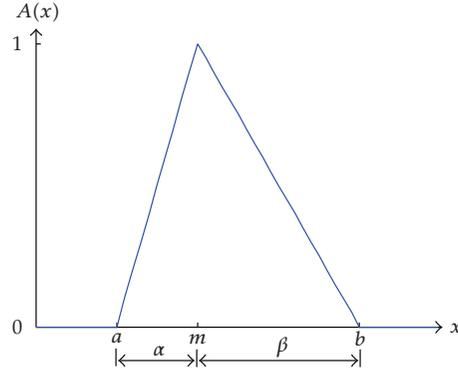


Figure 6: Triangular fuzzy number.

The membership function can then be defined as zero outside the interval $[a, b]$ of possible values; taken as linear into this range, increasing from a to m , and decreasing from m to b . This function is triangular, not necessarily symmetric, and can be defined as parameterized piecewise linear functions as:

$$A(x; a, m, b) = \begin{cases} \frac{x-a}{m-a}, & \text{if } x \in [a, m] \\ \frac{b-x}{b-m}, & \text{if } x \in [m, b] \\ 0, & \text{otherwise,} \end{cases} \quad (4.2)$$

where a and b are, respectively, the lower and upper bounds, and m is the dominant value, as illustrated in Figure 6.

Fuzzy numbers can also be defined by L (left) and R (right) MFs, resulting into the so-called L - R fuzzy numbers. In this context, a two-parameter modification of an L -type MF applies to all $x \leq m$, whereas the R -MF defines A for $x > m$, thus yielding

$$A(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & \text{if } x \leq m, \alpha > 0 \\ R\left(\frac{x-m}{\beta}\right), & \text{if } x > m, \beta > 0. \end{cases} \quad (4.3)$$

Therefore, the fuzzy number can also be identified by the notation $A = (m, \alpha, \beta)$, where α and β are the spreads of the number, which represents its uncertainty [32].

Fuzzy arithmetic (FA) operations, involving fuzzy numbers, can be used to propagate fuzziness from inputs to outputs. General operations can be deduced from the extension principle, which is used to transform fuzzy sets via functions [32], and plays a fundamental role in translating set-based concepts into their fuzzy set counterparts. However, simplified formulae can be obtained considering the L - R formulation of fuzzy numbers $A = (m, \alpha, \beta)_{LR}$ and $B = (n, \gamma, \delta)_{LR}$. The standard arithmetic operations are computed as follows.

The addition of triangular fuzzy numbers results in another triangular fuzzy number. Both addition and subtraction conserves the linearity of the numbers. These operations are expressed as, respectively,

$$\begin{aligned} A + B &= (m + n, \alpha + \gamma, \beta + \delta)_{LR'} \\ A - B &= (m - n, \alpha + \delta, \beta + \gamma)_{LR}. \end{aligned} \quad (4.4)$$

The multiplication of two fuzzy numbers produces a quadratic number. However, a linear approximation can be assumed when the spreads α and β are small in comparison to the modal or dominant values m . Therefore, this operation can be approximated to

$$\begin{aligned} A \cdot B &= (mn, m\gamma + n\alpha, m\delta + n\beta)_{LR} \quad (\text{if } A > 0, B > 0), \\ A \cdot B &= (mn, m\alpha - n\delta, m\beta - n\gamma)_{RL} \quad (\text{if } A > 0, B < 0), \\ A \cdot B &= (mn, n\alpha - m\delta, n\beta - m\gamma)_{RL} \quad (\text{if } A < 0, B > 0), \\ A \cdot B &= (mn, -n\beta - m\delta, -n\alpha - m\gamma)_{RL} \quad (\text{if } A < 0, B < 0). \end{aligned} \quad (4.5)$$

The multiplication of a fuzzy number by a scalar a is defined as:

$$\begin{aligned} aA &= (am, a\alpha, a\beta)_{LR'} \quad (\text{if } a \text{ is positive}), \\ aA &= (am, -a\beta, -a\alpha)_{RL'} \quad (\text{if } a \text{ is negative}). \end{aligned} \quad (4.6)$$

The division between two fuzzy numbers is computed as

$$\frac{A}{B} = \left(\frac{m}{n}, \frac{\delta m + \alpha n}{n^2}, \frac{\gamma m + \beta n}{n^2} \right)_{LR}. \quad (4.7)$$

To apply the FA in a given engineering problem, the uncertainty on each variable is modeled as a triangular fuzzy number; moreover, all the operations related to them have their expressions replaced by the corresponding FA expressions, as shown above on operations (4.4) to (4.7).

5. Implementation and Case Studies

Before proceeding with the study of the approaches described above, this section will begin with deterministic studies to assess the sensitivity of the response of the penetration model to the variation of the selected soil parameters. Later, in order to reach the goal of incorporating the formal treatment of uncertainties in the analysis of the penetration of torpedo piles, this section will proceed by presenting the application of the probabilistic approach based on the Monte Carlo method, followed by the implementation and application of the approach based on FA.

Recalling that uncertainties related to the penetration model involve a combination of both noncognitive (random, natural variability of soil parameters) and cognitive (epistemic, due to incomplete or imprecise information), it will be seen that, while the MC method can effectively deal only with the random uncertainties, the implementation of the fuzzy approach presented here can represent all sources of uncertainty.

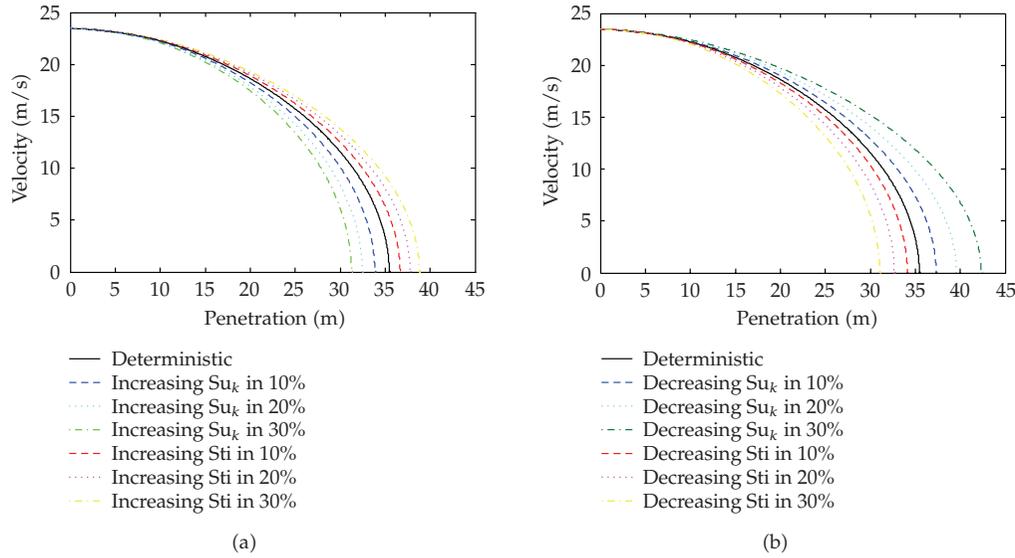


Figure 7: Deterministic sensitivity studies.

Table 3: Deterministic sensitivity studies: summary of results.

Varied parameter	Penetration (m)						
	-30%	-20%	-10%	0%	10%	20%	30%
Su_k	42.5	39.7	37.5	35.6	34.0	32.6	31.4
Sti	31.2	32.8	34.2	35.6	36.8	37.9	38.9

5.1. Deterministic sensitivity studies

In order to perform an assessment of the sensitivity of the torpedo pile penetration to the uncertainty of the selected soil parameters, a parametric study is performed by considering the same problem described in Tables 1 and 2. The penetration model is applied to deterministic and arbitrary variations on both uncertain parameters: the rate of increase with depth of the undrained shear strength (Su_k) and the soil sensitivity (Sti).

Recalling that according to Table 2, the fixed, “deterministic” values are $Su_k = 2.0$ kPa/m and $Sti = 3$. Initially, Su_k and Sti are individually increased by 10, 20, and 30%. Then, their values are reduced, also by 10, 20 and 30%. The results of the analyses for the different values of the parameters are presented in the graphs of Figure 7, corresponding to analyses where they are increased and reduced, respectively.

It should be noted that, since the drop height and the characteristics of the pile have not been changed, the behavior of the pile from the drop point until it reaches the seabed is the same as observed in Figure 4. Therefore, the graphs of Figure 7 represent only the behavior of the pile as it penetrates the soil, beginning from the depth of 200 m (that corresponds to the seabed) until it completes the penetration.

A summary of the results of Figure 7 is presented in Table 3 and Figure 8, in terms of penetration values (displacement minus the drop height) for each variation of the parameters Su_k and Sti . It can be verified that, as expected, reducing the undrained shear strength (and

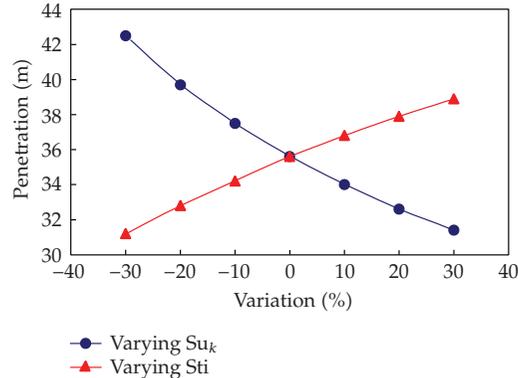


Figure 8: Deterministic sensitivity studies: summary of results.

Table 4: Statistical values from available soil data.

Parameter	Mean	Standard deviation
Su_k (kPa/m)	1.9	0.9
Sti (dimensionless)	3.2	1.0

therefore, the soil resistance) leads to the increase on the final depth values. On the other hand, decreasing the sensitivity values increases the soil resistance and, consequently, reduces the penetration of the pile.

5.2. Probabilistic analysis using the Monte Carlo method

Statistical treatment of the soil input parameters

In the probabilistic analysis using the MC method, both uncertain parameters (the undrained shear strength increase rate Su_k and soil sensitivity Sti) are varied simultaneously. Their values are randomly simulated, following a statistical distribution and its associated values of mean and standard deviation, derived from a given set of soil data from laboratory and/or in situ tests (in this case, the data were acquired from many tests performed at different sites in a certain cluster of Campos Basin). This is accomplished by performing a statistical treatment on the available data, representative of offshore fields in Campos Basin. As a result, for each uncertain parameter, the mean (which in this case is the sample average) and standard deviation values were estimated. These values are presented in Table 4.

In order to determine an appropriate probability distribution function for the data, a normality verification is performed for each parameter. Figure 9 present the results, respectively, for Su_k and Sti . It can be observed that, despite Sti data fits better than Su_k , the normal pdf is not the ideal approximation for them. Hence, other functions are fitted and verified, as presented in Figure 10. Observing this figure, it can be verified that the lognormal pdf provides a better fit for both sets of available data. Another advantage of this distribution is that it does not generate negative values for the soil parameters, which does not have physical meaning and can generate erroneous results.

The number of simulations in an MC strategy is dictated by the convergence of the mean value of the considered parameter to the deterministic design value. In the present case, 1000

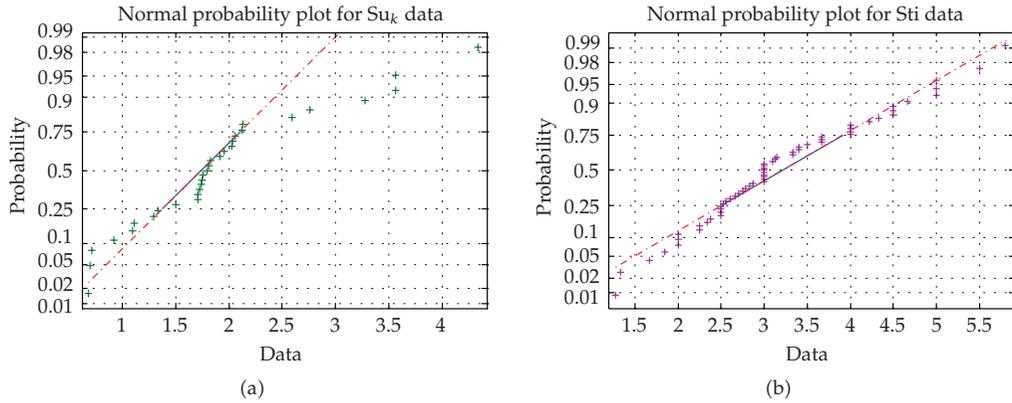


Figure 9: Normal probability verification of Su_k and Sti data.

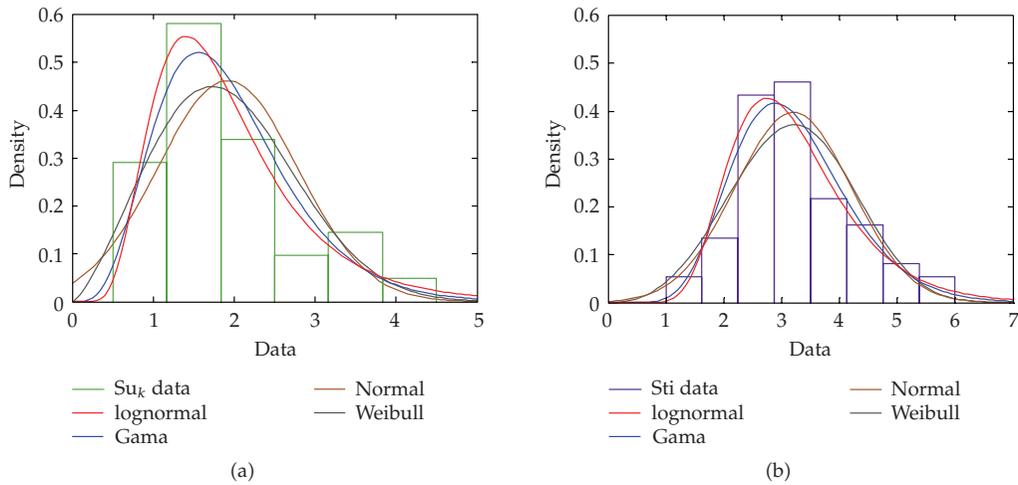


Figure 10: Probability function fitness for Su_k and Sti data.

generations were needed to obtain satisfactory convergence. Figure 11 depicts the distribution of the 1000 randomly generated values of Su_k and Sti , following the lognormal distribution.

Results

The probabilistic study then comprises a total number of 1000 analyses with the penetration model, each taking a randomly generated pair of values for the soil parameters Su_k and Sti , following the lognormal probability distribution with expected values and standard deviations given in Table 4.

The results of the 1000 analyses are then gathered to proceed to a statistical treatment, which will represent the penetration value in terms of mean and standard deviation. These results are presented in Table 5.

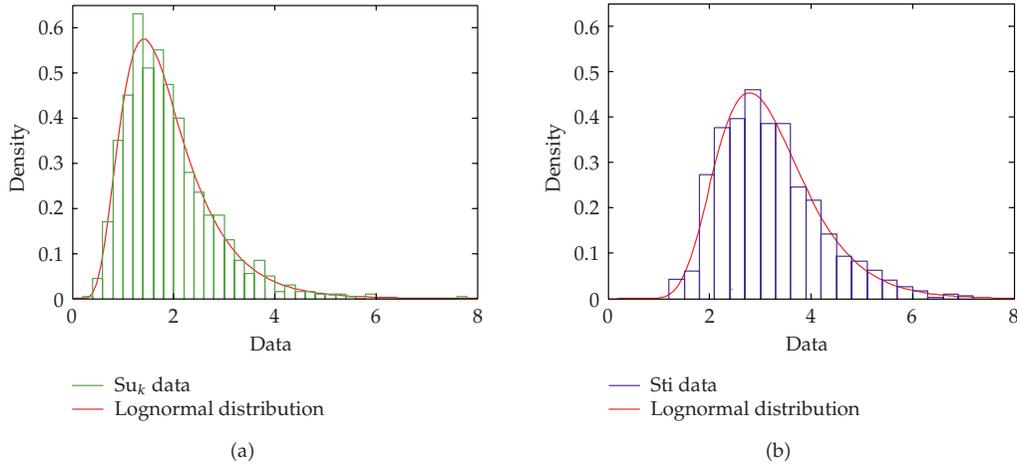


Figure 11: Lognormal distribution of Su_k and Sti .

Table 5: Probabilistic MC approach: statistical values of the results.

	Median	Mean	Standard deviation
Penetration (m)	38.5	39.8	10.1

These values will be compared with the results obtained with the approach using FA, which will be presented in Section 5.3. We recall that the mean value of 39.8 m for the penetration cannot be directly compared to the “deterministic” value of 35.6 m obtained in the previous section, since the fixed, “deterministic” values for the soil parameters were $Su_k = 2.0$ kPa/m and $Sti = 3.0$, and the mean probabilistic values gathered from the set of soil data considered are $Su_k = 1.9$ kPa/m and $Sti = 3.2$. Anyway, the results are consistent since, as could be observed in the results of the deterministic sensitivity studies summarized in Figure 8, lower values of Su_k and higher values of Sti lead to higher penetration values.

5.3. Fuzzy arithmetic: implementation and application

Implementation

In the computational implementation of the approach using FA, the uncertain variables Su_k and Sti are represented as triangular fuzzy numbers. Therefore, the computational code is altered, and all operations performed with those parameters in the solution procedure (as described in Algorithm 1 and (2.2)–(2.6)) have the traditional arithmetic operators replaced by the fuzzy operators presented in (4.4) to (4.7).

As mentioned in Section 4.2, the fuzzy operations of multiplication and division generate quadratic numbers; however, when their spreads are small, they can be approximated by linear ones. Therefore, although (4.5) to (4.7) are approximations for small spreads, they are feasible for this specific work, since the values of final penetration (dominant value, and the lower and upper bounds) are more important than the shape of the membership function.

Once these fuzzy operators are implemented in the computational code, it remains to determine the values that define the triangular membership functions, which represent Su_k

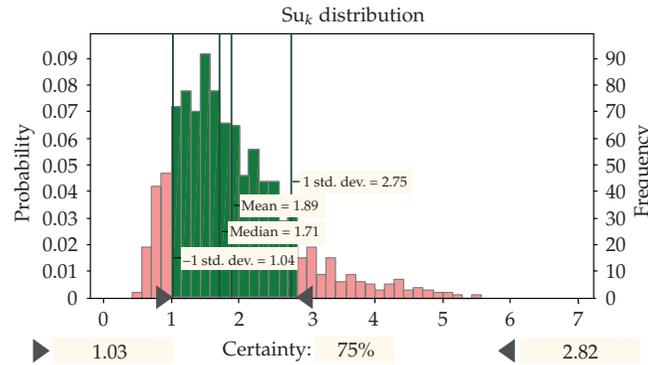


Figure 12: Definition of bounds for Su_k .

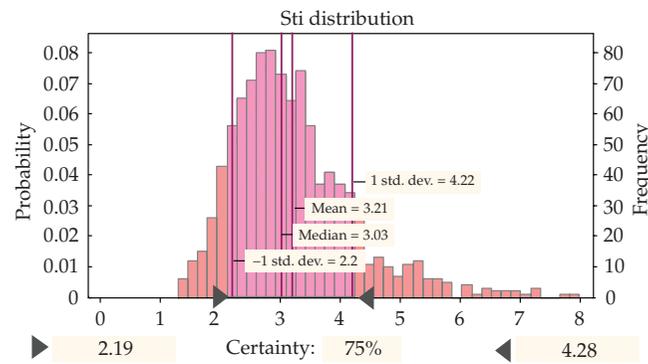


Figure 13: Definition of bounds for Sti .

and Sti as fuzzy numbers. As illustrated in Figure 6, these values are the lower and upper bounds a and b , and the dominant value m ; they can be derived by investigating the statistical distribution of the soil parameters, taking the lognormal distribution generated as described in the previous item.

The lower and upper bounds a and b can be assumed as defining an interval of confidence of 75% corresponding to one standard deviation below and above the mean. This criterion provides samples that have consistent values for the uncertain parameters (positive values, Sti greater than 1.0, etc.), and is illustrated in Figures 12 and 13, for Su_k and Sti distributions, respectively.

Regarding the “dominant” value m , the first choice could be to take the mean value; however, since the most representative value of a sample with large dispersion is the median, its value was chosen as m for each parameter. The values thus obtained for a , b , and m that define the membership functions for Su_k and Sti are presented in Table 6 and graphically represented in Figure 14.

Results

Finally, the evaluation of the uncertain response using this FA approach consists simply in performing one analysis with the penetration model. The uncertainties embedded in the fuzzy

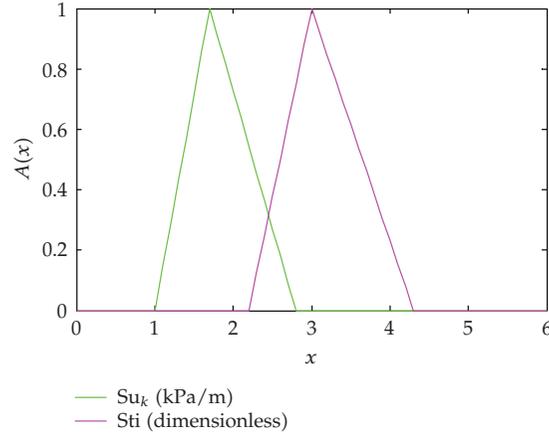


Figure 14: Triangular fuzzy representation for the soil uncertain parameters.

Table 6: Membership functions for the parameters Su_k and Sti .

	Lower bound a	Dominant value m	Upper bound b
Su_k (kPa/m)	1.0	1.7	2.8
Sti (dimensionless)	2.2	3.0	4.3

Table 7: FA approach: results of fuzzy analyses.

	Lower bound	Dominant value	Upper bound
Penetration (m)	30.1	38.6	66.3

numbers that represent the parameters Su_k and Sti are incorporated in the calculation of the terms F_T and F_S defined in (2.6), at the right-hand side of step b.1 of Algorithm 1, and therefore are updated and propagated at each time step of the solution procedure presented in that table. This fact points to the remarkable computational advantage of this approach, compared to the probabilistic MC method with conventional arithmetic that required a total number of 1000 analyses.

The results of the fuzzy analyses, in terms of lower, dominant, and upper bound for the final penetration of the torpedo pile are presented in Table 7.

5.4. Comparison of results

This section compares the results of analyses of the torpedo pile with the penetration model, considering both MC and FA approaches. Before comparing the final pile penetration, Figure 15 presents the full behavior of the pile as it penetrates the soil, in terms of penetration x velocity curves, beginning from the depth of 200 m (that corresponds to the seabed) until it completes the penetration.

Three curves are presented for each approach, corresponding to the “dominant” or “most probable” result, and a lower and upper “bounds” of the response. For the MC simulation, the “most probable” curve is represented by taking the median values of penetration and velocities at each time step of the response; the lower and upper bounds are

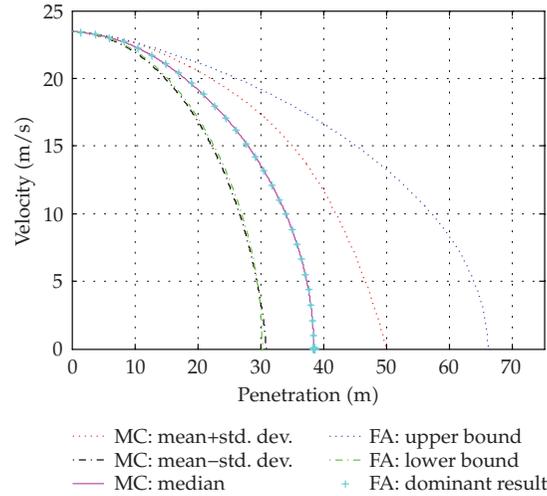


Figure 15: Comparison of results of MC and FA approaches.

Table 8: Comparison of results.

Method	Final penetration (m)		
	Mean - 1 std deviation	Median value	Mean + 1 std deviation
MC	29.7 (-22.9%)	38.5	49.9 (+29.6%)
FA	30.1 (-22.0%)	38.6	66.3 (+71.8%)

determined, respectively, by taking the mean value and subtracting or adding one standard deviation (similarly to the procedure applied to determine the bounds of the fuzzy input parameters).

For the FA approach, the “dominant” curve is represented by taking the “crisp” result, that is, the values corresponding to a degree of membership equal to one. The lower and upper bounds are determined by the support of the fuzzy set defined by the values corresponding to a membership degree greater than zero.

Table 8 summarizes and compares the results presented in Tables 5 and 7 for the MC and fuzzy approaches. Observing this table and also Figure 15, it can be observed that the “dominant” or “most probable” results for the final penetration are practically the same; the difference between the median value of the MC analyses and the dominant value of FA analyses is insignificant.

Regarding dispersion of results, it should be recalled that MC “lower” and “upper” results (defined by subtracting and adding one standard deviation to the mean value) cannot be directly compared to the spreads of the fuzzy results, where lower and upper bounds define the interval where a value can possibly represent the calculated penetration. Anyway, it can be noted that the uncertainties of the soil parameters are quite significant and can have a decisive influence in the design of the torpedo pile.

A better comparison for the final penetration can be graphically assessed in terms of the probability distribution of the MC simulation, and the membership function that characterizes

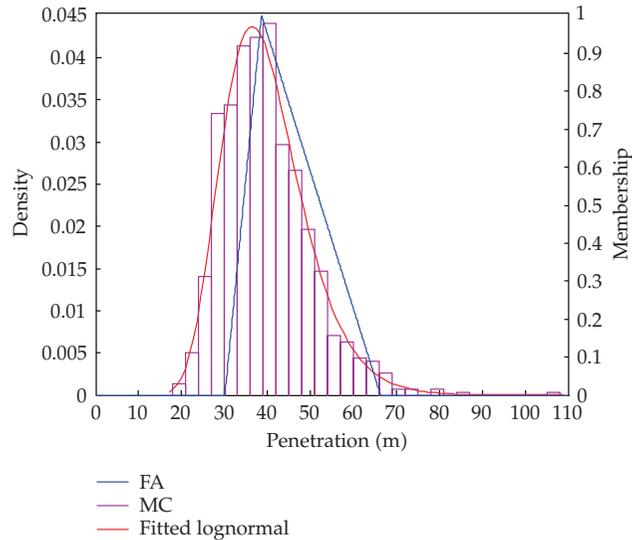


Figure 16: Final penetration obtained from MC and FA approaches.

the fuzzy number for the FA approach. Therefore, Figure 16 compares the results obtained from the MC and FA approaches, in terms of the probability distribution and membership function. For this example, while the assumed supports of the fuzzy input parameters Su_k and St_i corresponded to a certainty interval of 75% of their lognormal distribution, the support of the fuzzy number that represents the final penetration corresponds to a certainty interval of around 80% of the MC distribution.

Finally, the most remarkable comparison in the performance of both methods can be stated in terms of the total CPU time required. While the MC probabilistic approach required 1000 analyses with the penetration model using the solution procedure of Algorithm 1, only one analysis was required for the approach employing FA.

6. Final Remarks and Conclusions

The torpedo pile has been acknowledged as a very promising alternative to anchor mooring systems. It has recently been considered for use not only in mooring lines of floating production systems, but also for mobile offshore drilling units (MODUs) operating in deep and ultradeep waters. Therefore, oil exploitation companies are devoting intense research and design activities in order to deliver efficient mooring solutions using this concept.

One of the main aspects concerning the design of foundation systems are the uncertainties involved in the determination of values for the soil parameters. For conventional onshore systems, this aspect has been tackled by performing a large number of tests with soil samples. However, on deepwater offshore sites the cost of performing such tests may be very expensive, if not prohibitive; therefore, tests may not be performed for every installation site, and sometimes results of tests made at other locations are used to estimate or extrapolate the values of the soil parameters.

This fact can severely affect the effectiveness of the design and analysis of torpedo piles, leading to large discrepancies in the response of the torpedo pile, mainly in terms of

the final depth reached by the pile. Therefore, it is very important to develop and employ methodologies to properly assess the sensitivity of the response to the variation of these parameters, and to incorporate, into the analysis method, techniques for the formal treatment of the uncertainties.

The classical probabilistic Monte Carlo simulation could be considered for this purpose, since it is a sound methodology to estimate the effect of random uncertainties. Nevertheless, in the problem described in this work, there are a great amount of epistemic uncertainties in the model equations and parameters, and therefore, MC simulation results provide only a rough estimation of the uncertainty. In addition, the application of MC simulation requires excessive computational costs, as has been confirmed in the case study considered in this work. More than 1000 simulations were needed to obtain the results.

On the other hand, the computational efficiency of the fuzzy arithmetic approach is outstanding—around three orders of magnitude less. Therefore, the results of the application of the FA approach demonstrated its ability to provide low-cost approximations of the bounds of the uncertainties, and therefore, to comprise an effective design tool for the practitioner.

Future developments

In this study, only two soil parameters, the undrained shear strength and soil sensitivity, were considered as uncertain. Extensions of the fuzzy methodology presented in this work could consider the treatment of other uncertain parameters, such as for instance, the empirical maximum soil strain rate factor (S_e), the empirical soil strain rate factor (C_e), and the drag coefficient C_D considered for the calculation of the soil drag force as the pile penetrates; this latter parameter can vary for different anchor or pile shapes. Also, this work did not consider the uncertainties associated with the penetration model itself. These could also be considered, since it is a mainly empirical model and involves imprecision in its formulation.

Finally, a promising approach for the design of offshore systems would be to incorporate the pile penetration model, associated with the fuzzy methodology, in the implementation of a coupled finite element-based, time domain simulation program. In such implementation, not only the isolated torpedo pile is considered, but also a full finite-element model of all components involved in the installation of the pile (i.e., the mooring line itself and the other lines and chains, illustrated in Figures 2 and 3). The result is a complete 3D model, also submitted to environmental loadings other than dead weight (such as marine current). In such coupled model, there will be no need to “fudge” the drag coefficient C_D , to account for the presence of the mooring line and chain loop.

Such computational tool would therefore comprise an efficient tool for the design of mooring systems based on torpedo piles, and for the simulation of the procedures needed for the installation of such complex offshore system.

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References

- [1] B. R. Danziger, *Dynamic analysis of pile driving*, D.Sc. Thesis, COPPE/Federal University of Rio de Janeiro, Rio de Janeiro, RJ, Brazil, 1991.
- [2] K. Huang and M.-Y. Lee, "Experiences in classification of deepwater mooring systems for floating installations," in *Proceedings of the 30th Annual Offshore Technology Conference (OTC '98)*, vol. 2, pp. 227–236, Houston, Tex, USA, May 1998, paper 8770.
- [3] J.-L. Colliat, "Anchors for deepwater to ultradeepwater moorings," in *Proceedings of the 34th Annual Offshore Technology Conference (OTC '02)*, pp. 2695–2703, Houston, Tex, USA, May 2002, paper 14241.
- [4] P. Sparrevik, "Suction pile technology and installation in deep waters," in *Proceedings of the 20th Annual Offshore Technology Conference (OTC '02)*, pp. 2171–2179, Houston, Tex, USA, May 2002, paper 14241.
- [5] C. J. Medeiros Jr., "Torpedo anchor for deep water," in *Proceedings of the Deep Offshore Technology Conference (DOT '01)*, Rio de Janeiro, RJ, Brazil, October 2001.
- [6] C. J. Medeiros Jr., "Low cost anchor system for flexible risers in deep waters," in *Proceedings of the 20th Annual Offshore Technology Conference (OTC '02)*, pp. 1333–1337, Houston, Tex, USA, May 2002, paper 14151.
- [7] D. G. True, *Undrained vertical penetration into ocean bottom soils*, Ph.D. Thesis, University of California, Berkeley, Calif, USA, 1976.
- [8] Y. Boguslavskii, S. Drabkin, I. Juran, and A. Salman, "Theory and practice of projectile's penetration in soils," *Journal of Geotechnical Engineering*, vol. 122, no. 10, pp. 806–812, 1996.
- [9] C. W. Young, "Depth prediction for earth-penetrating projectiles," *Journal of the Soil Mechanics and Foundations*, vol. 95, no. 3, pp. 803–817, 1969.
- [10] C. W. Young, "An empirical equation for predicting penetration depth into soft sediments," in *Proceedings of the IEEE/MTS Oceans Conference (OCEANS '81)*, vol. 13, pp. 674–677, Boston, Mass, USA, September 1981.
- [11] R. L. McNeill, "Approximate method for estimating the strengths of cohesive materials from penetrator decelerations," in *Proceedings of the IEEE/MTS Oceans Conference (OCEANS '81)*, vol. 13, pp. 688–693, Boston, Mass, USA, September 1981.
- [12] R. M. Beard, "A penetrometer for deep ocean seafloor exploration," in *Proceedings of the IEEE/MTS Oceans Conference (OCEANS '81)*, vol. 13, pp. 668–673, Boston, Mass, USA, September 1981.
- [13] R. M. Beard, "Holding capacity of plate anchors," Tech. Rep. R-882, Civil Engineering Laboratory, Port Hueneme, Calif, USA, 1980.
- [14] J. R. Morison, M. P. O'Brien, J. W. Johnson, et al., "The force exerted by surface waves on piles," *Petroleum Transactions*, vol. 189, pp. 149–154, 1950.
- [15] API Recommended Practice 2A-WSD (RP 2A-WSD), "Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms—Working Stress Design," Twenty-First Edition, December 2000.
- [16] A. W. Skempton and R. D. Northey, "The sensitivity of clays," *Geotechnique*, vol. 3, no. 1, pp. 30–53, 1952.
- [17] D. G. True, "Rapid penetration into seafloor soils," in *Proceedings of the 6th Annual Offshore Technology Conference (OTC '74)*, vol. 3, pp. 607–618, Houston, Tex, USA, May 1974.
- [18] J. Chung and J. M. Lee, "A new family of explicit time integration methods for linear and non-linear structural dynamics," *International Journal for Numerical Methods in Engineering*, vol. 37, no. 23, pp. 3961–3976, 1994.
- [19] E. N. Lages, L. C. L. M. Vieira, and A. S. Ramos Jr., "A simplified model for marine soil and anchor line interaction," in *Proceedings of the 25th Iberian Latin-American Congress on Computational Methods in Engineering (XXV CILAMCE '04)*, pp. 1–15, Recife, PE, Brazil, November 2004.
- [20] A. Eltaher, Y. Rajapaksa, and K.-T. Chang, "Industry trends for design of anchoring systems for deepwater offshore structures," in *Proceedings of the Annual Offshore Technology Conference (OTC '03)*, Houston, Tex, USA, May 2003, paper 15265.
- [21] E. Hofer, M. Kloos, B. Krzykacz-Hausmann, J. Peschke, and M. Woltereck, "An approximate epistemic uncertainty analysis approach in the presence of epistemic and aleatory uncertainties," *Reliability Engineering & System Safety*, vol. 77, no. 3, pp. 229–238, 2002.
- [22] K. Durga Rao, H. S. Kushwaha, A. K. Verma, and A. Srividya, "Quantification of epistemic and aleatory uncertainties in level-1 probabilistic safety assessment studies," *Reliability Engineering & System Safety*, vol. 92, no. 7, pp. 947–956, 2007.

- [23] W. L. Oberkampf, S. M. DeLand, B. M. Rutherford, K. V. Diegertd, and K. F. Alvin, "Error and uncertainty in modeling and simulation," *Reliability Engineering & System Safety*, vol. 75, no. 3, pp. 333–357, 2002.
- [24] E. H. Vanmarcke, "Probabilistic modeling of soil profiles," *Journal of the Geotechnical Engineering Division*, vol. 103, no. 11, pp. 1227–1246, 1977.
- [25] B. S. L. P. de Lima, E. C. Teixeira, and N. F. F. Ebecken, "Probabilistic and possibilistic methods for the elastoplastic analysis of soils," *Advances in Engineering Software*, vol. 32, no. 7, pp. 569–585, 2001.
- [26] B. S. L. P. de Lima and N. F. F. Ebecken, "A comparison of models for uncertainty analysis by the finite element method," *Finite Elements in Analysis and Design*, vol. 34, no. 2, pp. 211–232, 2000.
- [27] T. J. Ross, *Fuzzy Logic with Engineering Applications*, John Wiley & Sons, New York, NY, USA, 2nd edition, 2004.
- [28] P. Bratley, L. F. Fox, and L. E. Schrage, *A Guide to Simulation*, Springer, New York, NY, USA, 1983.
- [29] F. Yamazaki and M. Shinozuka, "Neumann expansion for stochastic finite element analysis," *Journal of Engineering Mechanics*, vol. 114, no. 8, pp. 1335–1354, 1988.
- [30] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.
- [31] W. S. Dester and D. I. Blockley, "Hazard engineering," *Structural Safety*, vol. 16, no. 1-2, pp. 3–12, 1994.
- [32] W. Pedrycz and F. Gomide, *An Introduction to Fuzzy Sets Analysis and Design*, Complex Adaptive Systems, MIT Press, Cambridge, Mass, USA, 1998.