Research Article
On Classical Quotient Rings of Skew Armendariz Rings
A. R. Nasr-Isfahani and A. Moussavi
Received 7 February 2007; Accepted 2 August 2007
Recommended by Howard E. Bell

Let $R$ be a ring, $\alpha$ an automorphism, and $\delta$ an $\alpha$-derivation of $R$. If the classical quotient ring $Q$ of $R$ exists, then $R$ is weak $\alpha$-skew Armendariz if and only if $Q$ is weak $\tilde{\alpha}$-skew Armendariz.

Copyright © 2007 A. R. Nasr-Isfahani and A. Moussavi. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

1. Introduction
For a ring $R$ with a ring endomorphism, $\alpha : R \rightarrow R$ and an $\alpha$-derivation $\delta$ of $R$, that is, $\delta$ is an additive map such that $\delta(ab) = \delta(a)b + \alpha(a)\delta(b)$, for all $a, b \in R$, we denote by $R[x; \alpha, \delta]$ the skew polynomial ring whose elements are the polynomials over $R$, the addition is defined as usual and the multiplication subject to the relation $xa = \alpha(a)x + \delta(a)$ for any $a \in R$.

Rege and Chhawchharia [1] called a ring $R$ an Armendariz ring if whenever any polynomial $f(x) = a_0 + a_1 x + \cdots + a_n x^n$, $g(x) = b_0 + b_1 x + \cdots + b_m x^m \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_ib_j = 0$ for each $i, j$. This nomenclature was used by them since it was Armendariz [2, Lemma 1] who initially showed that a reduced ring (i.e., a ring without nonzero nilpotent elements) always satisfies this condition. A number of papers have been written on the Armendariz property of rings. For basic and other results on Armendariz rings, see, for example, [1–11].

The Armendariz property of rings was extended to skew polynomial rings with skewed scalar multiplication in [7].

For an endomorphism $\alpha$ of a ring $R$, $R$ is called an $\alpha$-skew Armendariz ring (or, a skew Armendariz ring with the endomorphism $\alpha$) if for $p = \sum_{i=0}^m a_i x^i$ and $q = \sum_{j=0}^n b_j x^j \in R[x; \alpha]$, $pq = 0$ implies $a_i \alpha^i(b_j) = 0$ for all $0 \leq i \leq m$ and $0 \leq j \leq n$. 
Recall that an endomorphism $\alpha$ of a ring $R$ is called rigid (see [5] and [12]) if $aa(a) = 0$ implies $a = 0$ for $a \in R$. $R$ is called an $\alpha$-rigid ring [12] if there exists a rigid endomorphism $\alpha$ of $R$. Note that any rigid endomorphism of a ring is a monomorphism, and $\alpha$-rigid rings are reduced by [12, Propositions 5 and 6].

If $R$ is an $\alpha$-rigid ring, then for $p = \sum_{i=0}^{m} a_i x^i$ and $q = \sum_{j=0}^{n} b_j x^j \in R[x; \alpha, \delta]$, $pq = 0$ if and only if $a_i b_j = 0$ for all $0 \leq i \leq m$ and $0 \leq j \leq n$ [12, Proposition 6].

Various properties of the Ore extensions have been investigated by many authors (see [1–13]). Most of these have worked either with the case $\delta = 0$ and $\alpha$-automorphism or the case where $\alpha$ is the identity. However, the recent surge of interest in quantum groups and quantized algebras has brought renewed interest in general skew polynomial rings, due to the fact that many of these quantized algebras and their representations can be expressed in terms of iterated skew polynomial rings. This development calls for a thorough study of skew polynomial rings.

Anderson and Camillo [3] assert that for a semiprime left and right Noetherian ring $R$, $R$ is Armendariz if and only if the classical right quotient ring $Q(R)$ of $R$ is reduced. Anderson and Camillo [3, Theorem 7] proved that if $R$ is a prime ring which is left and right Noetherian, then $R$ is Armendariz if and only if $R$ is reduced. Kim and Lee in [9] obtained this result under a weaker condition. They proved that if $R$ is a semiprime right and left Goldie ring, then $R$ is Armendariz if and only if $R$ is reduced. Kim and Lee also proved that if there exists the classical right quotient ring $Q(R)$ of a ring $R$, then $R$ is reduced if and only if $Q(R)$ is reduced.

In this paper, we obtain a generalized result of [3, Theorem 7] and [9, Theorem 16], for reduced rings, onto $\alpha$-skew Armendariz rings [9, Proposition 18 and Corollary 19] as corollaries.

2. The results

We first give the following definition of $\alpha$-skew Armendariz ring, and notice that our definition is compatible with Hong et al.’s [7], assuming $\delta$ to be the zero mapping.

**Definition 2.1.** Let $R$ be a ring with a ring endomorphism $\alpha$ and an $\alpha$-derivation $\delta$. $R$ is an $\alpha$-skew Armendariz ring (or, a skew Armendariz ring with the endomorphism $\alpha$) if for $p = \sum_{i=0}^{m} a_i x^i$ and $q = \sum_{j=0}^{n} b_j x^j \in R[x; \alpha, \delta]$, $pq = 0$ implies $a_i \alpha^j(b_j) = 0$ for all $0 \leq i \leq m$ and $0 \leq j \leq n$.

We notice that to extend Armendariz property to the Ore extension $R[x; \alpha, \delta]$, we do not need any more conditions. In the case $\delta = 0$, several examples of $\alpha$-skew Armendariz rings are obtained in [7], and in the same method, one can provide similar results as in [7], for the more general cases, of the Ore extension $R[x; \alpha, \delta]$. For instance, it is easy to prove that $\alpha$-rigid rings are $\alpha$-skew Armendariz.

**Definition 2.2.** Let $R$ be a ring with a ring endomorphism $\alpha$ and an $\alpha$-derivation $\delta$. $R$ is a weak $\alpha$-skew Armendariz ring, if for linear polynomials $f(x) = a_0 + a_1 x$ and $g(x) = b_0 + b_1 x \in R[x; \alpha, \delta]$, $f(x)g(x) = 0$ implies $a_i \alpha^j(b_j) = 0$ for all $0 \leq i, j \leq 1$.

Note that an $\alpha$-skew Armendariz ring is trivially a weak $\alpha$-skew Armendariz ring and a subring of an $\alpha$-skew Armendariz ring is also $\alpha$-skew Armendariz; while for the identity
endomorphism $I_R$ of a ring $R$, $R$ is Armendariz if and only if $R$ is $I_R$-skew Armendariz and $\delta = 0$.

Let $R$ be a ring with the classical right (left) quotient ring $Q$. Then each injective endomorphism $\alpha$ and $\alpha$-derivation $\delta$ of $R$ extends to $Q$, respectively, by setting $\tilde{\alpha}(c^{-1}r) = \alpha(c)^{-1}\alpha(r)$ and $\tilde{\delta}(c^{-1}r) = \alpha(c)^{-1}(\delta(r) - \delta(c)c^{-1}r)$, for each $r, c \in R$ with $c$ regular.

A ring $R$ is called right Ore given $a, b \in R$ with $b$ regular if there exist $a_1, b_1 \in R$ with $b_1$ regular such that $ab_1 = ba_1$. It is a well-known fact that $R$ is a right Ore ring if and only if there exists the classical right quotient ring of $R$.

Note that every Ore domain is an $\alpha$-skew Armendariz ring for every automorphism $\alpha$ and $\alpha$-derivation $\delta$.

Now we obtain a generalized result of [9, Theorem 16], for reduced rings, onto $\alpha$-skew Armendariz rings, by showing that the weak $\alpha$-skew Armendariz condition extends to its classical quotient ring.

**Theorem 2.3.** Let $R$ be a ring, $\alpha$ an automorphism, and $\delta$ an $\alpha$-derivation of $R$. If the classical quotient ring $Q$ of $R$ exists, then $R$ is weak $\alpha$-skew Armendariz if and only if $Q$ is weak $\tilde{\alpha}$-skew Armendariz.

**Proof.** Let $f(x) = c_0^{-1}a_0 + c_1^{-1}a_1x$ and $g(x) = s_0^{-1}b_0 + s_1^{-1}b_1x \in Q[x; \tilde{\alpha}, \tilde{\delta}]$ such that $f(x)g(x) = 0$. Then there exist $a'_i, b'_j \in R$ and regular elements $c, s \in R$ such that $c_0^{-1}a_i = c_1^{-1}a_i$ and $s_j^{-1}b_j = s_1^{-1}b'_j$. We then have $(c_0^{-1}a'_i + c_1^{-1}a'_i)(s_1^{-1}b'_1 + s_1^{-1}b'_1x) = 0$. So $(a'_0 + a'_1x)s_1^{-1}(b'_0 + b'_1x) = 0$ and hence $[a_0s_1^{-1} + a'_0\tilde{\delta}(s_1^{-1}) + a'_0\tilde{\alpha}(s_1^{-1})](b'_0 + b'_1x) = 0$. Then $[a_0s_1^{-1} - a'_0\tilde{\alpha}(s_1^{-1}) - a'_0\tilde{\delta}(s_1^{-1})](b'_0 + b'_1x) = 0$. Now there exist $d, s_2 \in R$, with $s_2$ regular, such that $\delta(s_2^{-1}) = s_2^{-1}d$. Thus $[a_0s_2^{-1} - a'_0\tilde{\alpha}(s_2^{-1})s_2^{-1}d + a'_0\tilde{\alpha}(s_2^{-1})](b'_0 + b'_1x) = 0$. There exist $s_3, s_4, s_5 \in R$, with $s_3, s_4, s_5$ regulars, such that $a_0s_2^{-1} = s_3^{-1}a''_0, a'_0\tilde{\alpha}(s_2^{-1}) = s_4^{-1}a''_0, a'_0\tilde{\alpha}(s_2^{-1}) = s_5^{-1}a''_0$. So $[s_3^{-1}a''_0 + s_4^{-1}a''_0d + s_5^{-1}a''_0x](b'_0 + b'_1x) = 0$. There exist $t, d_0, d_1, d_2 \in R$, with $t$ regular, such that $t^{-1}(d_0 - d_1d + d_2x)(b'_0 + b'_1x) = 0$. Now the Armendariz condition implies the following equations:

\begin{equation}
\begin{aligned}
(d_0 - d_1d)b'_0 &= 0, \\
(d_0 - d_1d)b'_1 &= 0, \\
d_2\alpha(b'_0) &= 0, \\
d_2\alpha(b'_1) &= 0.
\end{aligned}
\end{equation}

We have $(d_0 - d_1d + d_2x)(b'_0 + b'_1x) = 0$, so $(d_0 - d_1d)b'_0 + d_2\delta(b'_0) + ((d_0 - d_1d)b'_1 + d_2\alpha(b'_0) + d_2\delta(b'_1))x + d_2\alpha(b'_1)x^2 = 0$. Thus we get the following equations:

\begin{equation}
\begin{aligned}
(d_0 - d_1d)b'_0 + d_2\delta(b'_0) &= 0, \\
(d_0 - d_1d)b'_1 + d_2\alpha(b'_0) + d_2\delta(b'_1) &= 0, \\
d_2\alpha(b'_1) &= 0.
\end{aligned}
\end{equation}
These equations and (2.1) imply that
\[ d_2 \delta(b_0') = 0, \]
\[ d_2 \delta(b_1') = 0. \] (2.3)

Now we have
\[(d_0 - d_1) b_0' = 0 \iff t^{-1}(d_0 - d_1) b_0' = 0 \iff (s_1^{-1} a_0'' - s_2^{-1} a_1'' d) b_0' = 0 \]
\[ \iff (a_0 s^{-1} - a_1 \alpha(s)^{-1} s_2^{-1} d) b_0' = 0 \iff (a_0 s^{-1} - a_1' \alpha(s)^{-1} \delta(s) s^{-1}) b_0' = 0 \]
\[ \iff (a_0 s^{-1} + a_1' \tilde{\delta}(s^{-1})) b_0' = 0. \] (2.4)

A similar argument shows that
\[(d_0 - d_1) b_1' = 0 \iff (a_0' s^{-1} + a_1' \tilde{\delta}(s^{-1})) b_1' = 0. \] (2.5)

Also we have
\[ d_2 \delta(b_0') = 0 \iff t^{-1} d_2 \delta(b_0') = 0 \iff s_2^{-1} a_1'' \tilde{\delta}(b_0') = 0 \iff a_1' \alpha(s)^{-1} \tilde{\delta}(b_0') = 0 \]
\[ \iff a_1' \tilde{\alpha}(s^{-1}) \tilde{\delta}(b_0') = 0. \] (2.6)

A similar argument shows that
\[ d_2 \delta(b_1') = 0 \iff a_1' \tilde{\alpha}(s^{-1}) \tilde{\delta}(b_1') = 0, \] (2.7)
\[ d_2 \alpha(b_0') = 0 \iff a_1' \tilde{\alpha}(s^{-1}) \tilde{\alpha}(b_0') = 0, \] (2.8)
\[ d_2 \alpha(b_1') = 0 \iff a_1' \tilde{\alpha}(s^{-1}) \tilde{\alpha}(b_1') = 0. \] (2.9)

Now take \( h(x) = a_0' + a_1' x \) and \( k(x) = s^{-1} b_1' \in Q[x; \tilde{\alpha}, \tilde{\delta}] \), and using (2.9), (2.5), (2.7), then we get
\[ h(x)k(x) = (a_0' + a_1' x)(s^{-1} b_1') = a_0' s^{-1} b_1' + a_1' \tilde{\alpha}(s^{-1} b_1') x + a_1' \tilde{\delta}(s^{-1} b_1') \]
\[ = a_0' s^{-1} b_1' + a_1' \tilde{\delta}(s^{-1} b_1') + a_1' \tilde{\alpha}(s^{-1}) \tilde{\alpha}(b_1') x \]
\[ = a_0' s^{-1} b_1' + a_1' \tilde{\delta}(s^{-1}) b_1' + a_1' \tilde{\alpha}(s^{-1}) \tilde{\delta}(b_1') \]
\[ = (a_0' s^{-1} + a_1' \tilde{\delta}(s^{-1})) b_1' + a_1' \tilde{\alpha}(s^{-1}) \tilde{\delta}(b_1') = 0. \] (2.10)

Therefore, we have \((a_0' + a_1' x)(s^{-1} b_1') = 0\). Now there exist \( m, n \in R \) with \( n \) regular, we get \( s^{-1} b_1' = m n^{-1} \). Thus \((a_0' + a_1' x)m n^{-1} = 0\), hence \((a_0' + a_1' x)m = 0\). Since \( R \) is weak skew-Armendariz, we can deduce that \( a_0' m = a_1' \delta(m) = 0 \). But
\[ a_0' m = 0 \iff a_0' m n^{-1} = 0 \iff a_0' s^{-1} b_1' = 0. \] (2.11)
Equations (2.5) and (2.11) imply that
\[ a'_1 \widetilde{\delta} (s^{-1}) b'_1 = 0. \]  
(2.12)

Now take \( p(x) = a'_0 + a'_1 x \) and \( q(x) = s^{-1} b'_0 \), using (2.8), (2.4), (2.6) then we get
\[ p(x)q(x) = (a'_0 + a'_1 x) s^{-1} b'_0 = a'_0 s^{-1} b'_0 + a'_1 \tilde{\alpha} (s^{-1} b'_0) x + a'_1 \widetilde{\delta} (s^{-1} b'_0) \]
\[ = a'_0 s^{-1} b'_0 + a'_1 \widetilde{\delta} (s^{-1}) b'_0 + a'_1 \tilde{\alpha} (s^{-1}) \widetilde{\alpha} (b'_0) x \]
\[ = (a'_0 s^{-1} + a'_1 \widetilde{\delta} (s^{-1})) b'_0 + a'_1 \tilde{\alpha} (s^{-1}) \widetilde{\alpha} (b'_0). \]
(2.13)

So \((a'_0 + a'_1 x)s^{-1} b'_0 = 0.\) But there exist \(u,v \in R\) with \(v\) regular such that \(s^{-1} b'_0 = uv^{-1}.\) Thus \((a'_0 + a'_1 x)uv^{-1} = 0\) and hence \((a'_0 + a'_1 x)u = 0.\) The Armendariz condition implies that \(a'_0 u = 0,\) and so \(a'_0 uv^{-1} = 0.\) So we get
\[ a'_0 s^{-1} b'_0 = 0. \]  
(2.14)

By (2.14) and (2.4), we have
\[ a'_1 \widetilde{\delta} (s^{-1}) b'_1 = 0. \]  
(2.15)

By (2.14),
\[ a'_0 s^{-1} b'_0 = 0 \iff c^{-1} a'_0 s^{-1} b'_0 = 0 \iff c^{-1} a'_0 s^{-1} b'_0 = 0. \]  
(2.16)

By (2.11),
\[ a'_0 s^{-1} b'_1 = 0 \iff c^{-1} a'_0 s^{-1} b'_1 = 0 \iff c^{-1} a'_0 s^{-1} b'_1 = 0. \]  
(2.17)

By (2.6) and (2.15), we have \(a'_1 \widetilde{\delta} (s^{-1}) b'_0 = 0 = a'_1 \tilde{\alpha} (s^{-1}) \widetilde{\delta} (b'_0).\) Thus
\[ a'_1 (\text{\widetilde{\delta}} (s^{-1}) b'_0 + \tilde{\alpha} (s^{-1}) \widetilde{\delta} (b'_0)) = 0 \iff a'_1 \text{\widetilde{\delta}} (s^{-1} b'_0) = 0 \iff c^{-1} a'_1 \text{\widetilde{\delta}} (s^{-1} b'_0) = 0 \iff c^{-1} a'_1 \text{\widetilde{\delta}} (s^{-1} b'_0) = 0. \]  
(2.18)

By (2.12) and (2.7), we have \(a'_1 \text{\widetilde{\delta}} (s^{-1}) b'_1 = 0 = a'_1 \tilde{\alpha} (s^{-1}) \text{\widetilde{\delta}} (b'_1).\) Thus
\[ a'_1 \text{\widetilde{\delta}} (s^{-1}) b'_1 + a'_1 \tilde{\alpha} (s^{-1}) \text{\widetilde{\delta}} (b'_1) = 0 \iff a'_1 \text{\widetilde{\delta}} (s^{-1} b'_1) = 0 \iff c^{-1} a'_1 \text{\widetilde{\delta}} (s^{-1} b'_1) = 0. \]  
(2.19)

Using \((c^{-1} a_0 + c^{-1} a_1 x)(s_0^{-1} b'_0 + s_1^{-1} b'_1 x) = 0\) and (2.16), (2.17), (2.18), (2.19), we also have \(c^{-1} a_1 \tilde{\alpha} (s^{-1} b'_0) = 0\) and \(c^{-1} a_1 \tilde{\alpha} (s_1^{-1} b'_1) = 0.\) Therefore \(Q\) is a weak \(\tilde{\alpha}\)-skew Armendariz ring.

Now we show that skew-Armendariz rings are Abelian (i.e., every idempotent is central).

**Lemma 2.4.** Every weak \(\alpha\)-skew Armendariz ring is Abelian.
Let $R$ be a weak $\alpha$-skew Armendariz ring and let $e^2 = e$, $a \in R$. Consider the polynomials $f(x) = e - ea(1-e)x$ and $g(x) = 1 - e + ea(1-e)x \in R[x;\alpha,\delta]$. Then we have $f(x)g(x) = 0$. Since $R$ is weak skew Armendariz, $eea(1-e) = 0$. So $ea = eae$. Next let $h(x) = 1 - e - (1-e)ax$ and $k(x) = e + (1-e)ax \in R[x;\alpha,\delta]$. We have $h(x)k(x) = 0$ and since $R$ is weak skew Armendariz, it implies that $(1-e)(1-e)ae = 0$. Thus $ae = eae$ and so $ae = ea$ which implies that $R$ is Abelian.

**Corollary 2.5.** Let $R$ be a semiprime Goldie ring and $\alpha$-automorphism and $\delta$ an $\alpha$-derivation of $R$. Then the following are equivalent:

1. $R$ is weak $\alpha$-skew Armendariz;
2. $R$ is $\alpha$-skew Armendariz;
3. $Q$ is $\tilde{\alpha}$-skew Armendariz;
4. $Q$ is weak $\tilde{\alpha}$-skew Armendariz;
5. $R$ is $\alpha$-rigid;
6. $Q$ is $\tilde{\alpha}$-rigid.

**Proof.** The proof follows by Theorem 2.3. For the implication $2 \Rightarrow 5$, notice that when $R$ is a weak $\alpha$-skew Armendariz ring, then by Theorem 2.3, $Q$ is weak $\tilde{\alpha}$-skew Armendariz and hence $Q$ is Abelian by Lemma 2.4 so $Q$ is an Abelian semisimple ring and hence is reduced. Now, suppose that $\tilde{\alpha}\tilde{\alpha}(a) = 0$ for $a \in Q$. So we have $\delta(a)\tilde{\alpha}(a) = \tilde{\alpha}(a)\tilde{\delta}(\tilde{\alpha}(a)) = 0$. Now, let $h(x) = \tilde{\alpha}(a) - \tilde{\alpha}(a)x$ and $k(x) = a + \tilde{\alpha}(a)x \in Q[x;\tilde{\alpha},\tilde{\delta}]$. Then $h(x)k(x) = 0$. Since $Q$ is weak $\tilde{\alpha}$-skew Armendariz, we have $\tilde{\alpha}(a)\tilde{\alpha}(a) = 0$. But $Q$ is reduced and $\tilde{\alpha}$ is a monomorphism, therefore $a = 0$. Thus $Q$ is $\tilde{\alpha}$-rigid, so $R$ is $\alpha$-rigid.

By Corollary 2.5, it is shown that a semiprime right Goldie ring $R$ with an automorphism $\alpha$ is weak $\alpha$-skew Armendariz if and only if it is reduced.

**Acknowledgment**

The authors are thankful to the referee for a careful reading of the paper and for some helpful comments and suggestions.

**References**


A. R. Nasr-Isfahani: Department of Mathematics, Tarbiat Modares University, P.O. Box 14115-170, Tehran, Iran

Email address: a_nasr_isfahani@yahoo.com

A. Moussavi: Department of Mathematics, Tarbiat Modares University, P.O. Box 14115-170, Tehran, Iran

Email addresses: moussavi_a5@yahoo.com; moussavi_a@modares.ac.ir
Special Issue on 
Decision Support for Intermodal Transport

Call for Papers

Intermodal transport refers to the movement of goods in a single loading unit which uses successive various modes of transport (road, rail, water) without handling the goods during mode transfers. Intermodal transport has become an important policy issue, mainly because it is considered to be one of the means to lower the congestion caused by single-mode road transport and to be more environmentally friendly than the single-mode road transport. Both considerations have been followed by an increase in attention toward intermodal freight transportation research.

Various intermodal freight transport decision problems are in demand of mathematical models of supporting them. As the intermodal transport system is more complex than a single-mode system, this fact offers interesting and challenging opportunities to modelers in applied mathematics. This special issue aims to fill in some gaps in the research agenda of decision-making in intermodal transport.

The mathematical models may be of the optimization type or of the evaluation type to gain an insight in intermodal operations. The mathematical models aim to support decisions on the strategic, tactical, and operational levels. The decision-makers belong to the various players in the intermodal transport world, namely, drayage operators, terminal operators, network operators, or intermodal operators.

Topics of relevance to this type of decision-making both in time horizon as in terms of operators are:

- Intermodal terminal design
- Infrastructure network configuration
- Location of terminals
- Cooperation between drayage companies
- Allocation of shippers/receivers to a terminal
- Pricing strategies
- Capacity levels of equipment and labour
- Operational routines and lay-out structure
- Redistribution of load units, railcars, barges, and so forth
- Scheduling of trips or jobs
- Allocation of capacity to jobs
- Loading orders
- Selection of routing and service

Before submission authors should carefully read over the journal’s Author Guidelines, which are located at http://www.hindawi.com/journals/jamds/guidelines.html. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/, according to the following timetable:

<table>
<thead>
<tr>
<th>Manuscript Due</th>
<th>June 1, 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Round of Reviews</td>
<td>September 1, 2009</td>
</tr>
<tr>
<td>Publication Date</td>
<td>December 1, 2009</td>
</tr>
</tbody>
</table>

Lead Guest Editor

Gerrit K. Janssens, Transportation Research Institute (IMOB), Hasselt University, Agoralaan, Building D, 3590 Diepenbeek (Hasselt), Belgium; Gerrit.Janssens@uhasselt.be

Guest Editor

Cathy Macharis, Department of Mathematics, Operational Research, Statistics and Information for Systems (MOSI), Transport and Logistics Research Group, Management School, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussel, Belgium; Cathy.Macharis@vub.ac.be