A NOTE ON ALMOST CONTRA-PRECONTINUOUS FUNCTIONS

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New characterizations of almost contra-precontinuity are presented. These characterizations are used to develop a new weak form of almost contra-precontinuity. This new weak form is then used to extend several results in the literature concerning almost contra-precontinuity.

1. Introduction

Almost contra-precontinuous functions were introduced by Ekici [7] and recently have been investigated further by Noiri and Popa [13]. The purpose of this note is to develop some new characterizations of almost contra-precontinuous functions and to introduce a new weak form of almost contra-precontinuity, which we call subalmost contra-precontinuity. It is shown that subalmost contra-precontinuity implies subalmost weak continuity and is independent of subweak continuity. Subalmost contra-precontinuity is used to extend several results in the literature concerning almost contra-precontinuity. For example, we show that the graph of a subalmost contra-precontinuous function with a Hausdorff codomain is P-regular and that the domain of a subalmost contra-precontinuous injection with a weakly Hausdorff codomain is pre-T_1. These results extend the analogous results for an almost contra-precontinuous function.

2. Preliminaries

The symbols X and Y denote topological spaces with no separation axioms assumed unless explicitly stated. All sets are considered to be subsets of topological spaces. The closure and interior of a set A are signified by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A set A is regular open if $A = \mathrm{Int}(\mathrm{Cl}(A))$. A set A is preopen [12] (resp., semiopen [11], β-open [1]) provided that $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$ (resp., $A \subseteq \mathrm{Cl}(\mathrm{Int}(A))$, $A \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$). A set is θ-open provided that it contains a closed neighborhood of each of its points. A set A is preclosed (resp., semiclosed, β-closed, regular closed, θ-closed) if its complement is preopen

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(resp., semiopen, β-open, regular open, θ-open). The preclosure [8] of A, denoted by $p\mathrm{Cl}(A)$, is the intersection of all preclosed sets containing A. The semiclosure [5] of a set A denoted by $s\mathrm{Cl}(A)$, and β-closure [2] of a set A denoted by $\beta\mathrm{Cl}(A)$ are defined analogously. The θ-semi-closure [9] of a subset A of a space X, denoted by $s\mathrm{Cl}_\theta(A)$, is the set of all $x \in X$ such that $\mathrm{Cl}(V) \cap A \neq \emptyset$ for every semiopen subset V of X containing x. The set of all preopen subsets of a space X is denoted by $\mathrm{PO}(X)$ and the collection of all preopen subsets of X containing a fixed point x is denoted by $\mathrm{PO}(X,x)$. The sets $\mathrm{SO}(X)$, $\mathrm{SO}(X,x)$, $\beta\mathrm{O}(X)$, $\beta\mathrm{O}(X,x)$, $\mathrm{PC}(X)$, and $\mathrm{RO}(X)$ are defined analogously. Finally, if an operator is used with respect to a proper subspace, then a subscript will be added to the operator. Otherwise, it is assumed that the operator refers to the space X or Y.

Definition 2.1. A function $f : X \to Y$ is said to be almost contra-precontinuous [7] if $f^{-1}(V) \in \mathrm{PC}(X)$ for every $V \in \mathrm{RO}(Y)$.

Definition 2.2. A function $f : X \to Y$ is said to be subweakly continuous [14] (resp., subalmost weakly continuous [3], subweakly β-continuous [4]) provided that there is an open base $\mathcal{B}$ for the topology on Y such that for every $V \in \mathcal{B}$, $\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ (resp., $p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$, $\beta\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$).

Definition 2.3. A function $f : X \to Y$ is said to be semicontinuous [11] if $f^{-1}(V) \in \mathrm{SO}(X)$ for every open subset V of Y.

3. Almost contra-precontinuous functions

Noiri and Popa proved the following characterizations of almost contra-precontinuity.

Theorem 3.1 (Noiri and Popa [13]). *For a function $f : X \to Y$, the following properties are equivalent:*
 (a) *f is almost contra-precontinuous;*
 (b) *$f(p\mathrm{Cl}(A)) \subseteq s\mathrm{Cl}_\theta(f(A))$ for every subset A of X;*
 (c) *$p\mathrm{Cl}(f^{-1}(B)) \subseteq f^{-1}(s\mathrm{Cl}_\theta(B))$ for every subset B of Y.*

We extend these characterizations by showing that Theorem 3.1(c) can be stated for open sets only. The following lemmas will be useful.

Lemma 3.2. *If V is an open set, then $s\mathrm{Cl}_\theta(V) = s\mathrm{Cl}(V)$.*

Proof. Obviously $s\mathrm{Cl}(V) \subseteq s\mathrm{Cl}_\theta(V)$. Suppose that $x \notin s\mathrm{Cl}(V)$. Then there exists $U \in \mathrm{SO}(X,x)$ such that $U \cap V = \emptyset$. Then, since V is open, $\mathrm{Cl}(U) \cap V = \emptyset$. Therefore $x \notin s\mathrm{Cl}_\theta(V)$. Hence $s\mathrm{Cl}_\theta(V) \subseteq s\mathrm{Cl}(V)$. □

Lemma 3.3 (Di Maio and Noiri [6]). *If V is an open set, then $s\mathrm{Cl}(V) = \mathrm{Int}(\mathrm{Cl}(V))$.*

Theorem 3.4. *For a function $f : X \to Y$, the following conditions are equivalent:*
 (a) *f is almost contra-precontinuous;*
 (b) *$p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\mathrm{Cl}_\theta(V))$ for every open subset V of Y;*
 (c) *$p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\mathrm{Cl}(V))$ for every open subset V of Y;*
 (d) *$p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))$ for every open subset V of Y;*
 (e) *$\mathrm{Cl}(\mathrm{Int}(f^{-1}(V))) \subseteq f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))$ for every open subset V of Y.*

Proof. (a)⇒(b) follows from Theorem 3.1(c).

(b)⇒(c) follows from Lemma 3.2.

(c)⇒(d) follows form Lemma 3.3.

(d)⇒(e). Since $p\text{Cl}(f^{-1}(V)) = f^{-1}(V) \cup \text{Cl}(\text{Int}(f^{-1}(V)))$, it follows from (d) that $\text{Cl}(\text{Int}(f^{-1}(V))) \subseteq f^{-1}(\text{Int}(\text{Cl}(V)))$.

(e)⇒(a). Let $V \in \text{RO}(Y)$. Then by (e), $\text{Cl}(\text{Int}(f^{-1}(V))) \subseteq f^{-1}(\text{Int}(\text{Cl}(V))) = f^{-1}(V)$. Therefore $f^{-1}(V)$ is preclosed, which proves that f is almost contra-precontinuous. □

The next result is an immediate consequence of Theorems 3.1 and 3.4.

Theorem 3.5. *Let $f : X \to Y$ be a function and let $\mathcal{S}$ be any collection of subsets of Y containing the open sets. Then f is almost contra-precontinuous if and only if $p\text{Cl}(f^{-1}(S)) \subseteq f^{-1}(s\text{Cl}_\theta(S))$ for every $S \in \mathcal{S}$.*

Corollary 3.6. *For a function $f : X \to Y$, the following properties are equivalent:*

(a) *f is almost contra-precontinuous;*

(b) *$p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}_\theta(V))$ for every $V \in \text{SO}(Y)$;*

(c) *$p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}_\theta(V))$ for every $V \in PO(Y)$;*

(d) *$p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}_\theta(V))$ for every $V \in \beta O(Y)$.*

4. Subalmost contra-precontinuous functions

We define a function $f : X \to Y$ to be subalmost contra-precontinuous provided that there exists an open base $\mathcal{B}$ for the topology on Y such that $p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}(V))$ for every $V \in \mathcal{B}$. Obviously almost contra-precontinuity implies subalmost contra-precontinuity. The following example shows that the converse does not hold.

Recall that a space X is extremally disconnected (ED) if the closure of every open set is open in X.

Example 4.1. Let X be a non-ED, T_1-space and let $Y = X$ have the discrete topology. The identity mapping $f : X \to Y$ is subalmost contra-precontinuous with respect to the base for Y consisting of the singleton sets. However, f is not almost contra-precontinuous. Note that for $y \in Y$, $p\text{Cl}_X(f^{-1}(\{y\})) = \{y\}$. Also, since X is non-ED, there exists an open set V of X such that $\text{Cl}_X(V)$ is not open. Then $f^{-1}(s\text{Cl}_Y(V)) = V$, but $p\text{Cl}_X(f^{-1}(V)) = \text{Cl}_X(V)$.

Since $s\text{Cl}(A) \subseteq \text{Cl}(A)$ for every set A, it follows that subalmost contra-precontinuity implies subalmost weak continuity, and hence it also implies subweak β-continuity. The following example shows that subalmost contra-precontinuity and subalmost weak continuity are not equivalent.

Example 4.2. Let $X = \{a,b,c\}$ have the topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$. The identity mapping $f : X \to X$ is obviously subalmost weakly continuous (in fact, continuous). However, f is not subalmost contra-precontinuous because any base for τ must contain $\{a\}$ and $p\text{Cl}(f^{-1}(\{a\})) \nsubseteq f^{-1}(s\text{Cl}(\{a\}))$.

Since the function in Example 4.2 is obviously subweakly continuous, we see that subweak continuity does not imply subalmost contra-precontinuity. The following example completes the proof that subalmost contra-precontinuity is independent of subweak continuity.

Example 4.3. Let X be an indiscrete space with at least two points and let $Y = X$ have the discrete topology. Since $p\,\mathrm{Cl}(\{x\}) = \{x\}$ for very $x \in X$, the identity mapping $f : X \to Y$ is subalmost contra-precontinuous with respect to the base for Y consisting of the singleton sets. However, since every singleton set of X is dense, f is not subweakly continuous.

The following characterizations of subalmost contra-precontinuity are analogous to those in Theorem 3.4 for almost contra-precontinuity.

Theorem 4.4. *For a function $f : X \to Y$, the following conditions are equivalent:*

(a) *f is subalmost contra-precontinuous;*

(b) *there exists an open base $\mathcal{B}$ for Y such that* $p\,\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\,\mathrm{Cl}_\theta(V))$ *for every $V \in \mathcal{B}$;*

(c) *there exists an open base $\mathcal{B}$ for Y such that* $p\,\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))$ *for every $V \in \mathcal{B}$;*

(d) *there exists an open base $\mathcal{B}$ for Y such that* $\mathrm{Cl}(\mathrm{Int}(f^{-1}(V))) \subseteq f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))$ *for every $V \in \mathcal{B}$.*

Theorem 4.5. *If $f : X \to Y$ is subalmost weakly continuous and satisfies the additional property that images of preclosed sets are open, then f is subalmost contra-precontinuous.*

Proof. Since f is subalmost weakly continuous, there exists an open base $\mathcal{B}$ for Y such that $p\,\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Since images of preclosed sets are open, we have $f(p\,\mathrm{Cl}(f^{-1}(V))) \subseteq \mathrm{Int}(\mathrm{Cl}(V))$ or $p\,\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))$. Therefore by Theorem 4.4, f is subalmost contra-precontinuous. □

Since subweak continuity implies subalmost weak continuity, we have the following result.

Corollary 4.6. *If $f : X \to Y$ is subweakly continuous and satisfies the additional property that images of preclosed sets are open, then f is subalmost contra-precontinuous.*

Theorem 4.7. *If $f : X \to Y$ is subalmost contra-precontinuous and semicontinuous, then f is subweakly continuous.*

Proof. Since f is subalmost contra-precontinuous, there exists an open base $\mathcal{B}$ for the topology on Y such that $p\,\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\,\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Because f is semicontinuous, $f^{-1}(V)$ is semiopen for every $V \in \mathcal{B}$, and hence $p\,\mathrm{Cl}(f^{-1}(V)) = \mathrm{Cl}(f^{-1}(V))$ for every $V \in \mathcal{B}$. Finally, since $s\,\mathrm{Cl}(A) \subseteq \mathrm{Cl}(A)$ for every set A, we have $\mathrm{Cl}(f^{-1}(V)) = p\,\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\,\mathrm{Cl}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$. Therefore, f is subweakly continuous. □

5. Graph-related properties of subalmost contra-precontinuous functions

By the graph of a function $f : X \to Y$, we mean the subset $\mathrm{G}(f) = \{(x, f(x)) : x \in X\}$ of the product space $X \times Y$.

Definition 5.1. The graph of a function $f : X \to Y$, $\mathrm{G}(f)$ is said to be P-regular [7] provided that for every $(x, y) \in X \times Y - \mathrm{G}(f)$, there exist a preclosed subset U of X and regular open subset V of Y such that $(x, y) \in U \times V \subseteq X \times Y - \mathrm{G}(f)$.

Theorem 5.2. *If $f : X \to Y$ is subalmost contra-precontinuous and Y is Hausdorff, then the graph of f, $G(f)$ is P-regular.*

Proof. Let $(x,y) \in X \times Y - G(f)$. Then $y \neq f(x)$. Let $\mathcal{B}$ be an open base for Y such that $p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))$ for every $V \in \mathcal{B}$. Since Y is Hausdorff, there exist disjoint open sets V and W such that $f(x) \in V$, $y \in W$, and $V \in \mathcal{B}$. Then, since $\mathrm{Int}(\mathrm{Cl}(V)) \cap \mathrm{Int}(\mathrm{Cl}(W)) = \emptyset$, it follows that $(x,y) \in p\mathrm{Cl}(f^{-1}(V)) \times \mathrm{Int}(\mathrm{Cl}(W)) \subseteq X \times Y - G(f)$, which proves that $G(f)$ is P-regular. □

Corollary 5.3 (Ekici [7, Theorem 17]). *If $f : X \to Y$ is almost contra-precontinuous and Y is Hausdorff, then $G(f)$ is P-regular.*

Recall that the graph function of a function $f : X \to Y$ is the function $g : X \to X \times Y$ given by $g(x) = (x, f(x))$ for every $x \in X$.

Theorem 5.4. *Let $f : (X,\tau) \to (Y,\sigma)$ be a function and let $\mathcal{B}$ be an open base for σ. Let $\mathcal{C} = \{U \times V : U \in \tau, V \in \mathcal{B}\}$. If the graph function of f, $g : X \to X \times Y$ is subalmost contra-precontinuous with respect to $\mathcal{C}$, then f is subalmost contra-precontinuous with respect to $\mathcal{B}$.*

Proof. If $V \in \mathcal{B}$, then $p\mathrm{Cl}(f^{-1}(V)) = p\mathrm{Cl}(g^{-1}(X \times V)) \subseteq g^{-1}(s\mathrm{Cl}(X \times V)) = g^{-1}(X \times s\mathrm{Cl}(V)) = f^{-1}(s\mathrm{Cl}(V))$. Hence f is subalmost contra-precontinuous with respect to $\mathcal{B}$. □

If we let $\mathcal{B} = \sigma$ in Theorem 5.4, we obtain the following result.

Corollary 5.5. *If the graph function of $f : X \to Y$ is subalmost contra-precontinuous with respect to the usual base for the product topology for the product space $X \times Y$, then f is almost contra-precontinuous.*

Corollary 5.6 (Ekici [7, Theorem 4]). *If the graph function of $f : X \to Y$ is almost contra-precontinuous, then f is almost contra-precontinuous.*

Recall that a space X is said to be zero-dimensional provided that X has a clopen base.

Theorem 5.7. *If the function $f : X \to Y$ is subalmost contra-precontinuous and X is zero-dimensional, then the graph function of f, $g : X \to X \times Y$ is subalmost contra-precontinuous.*

Proof. Let $\mathcal{B}$ be an open base for Y such that $p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))$ for every $V \in \mathcal{B}$. Then $\mathcal{C} = \{U \times V : U \subseteq X$ is clopen and $V \in \mathcal{B}\}$ is a base for $X \times Y$. For $U \times V \in \mathcal{C}$, we have $p\mathrm{Cl}(g^{-1}(U \times V)) = p\mathrm{Cl}(U \cap f^{-1}(V)) \subseteq U \cap p\mathrm{Cl}(f^{-1}(V)) \subseteq \mathrm{Int}(\mathrm{Cl}(U)) \cap f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))) = g^{-1}(\mathrm{Int}(\mathrm{Cl}(U)) \times \mathrm{Int}(\mathrm{Cl}(V))) = g^{-1}(\mathrm{Int}(\mathrm{Cl}(U \times V)))$. Therefore the graph function g is subalmost contra-precontinuous. □

Remark 5.8. In Theorem 5.7 the requirement that X be zero-dimensional can be replaced by the assumption that X is an ED space.

6. Additional properties of subalmost contra-precontinuous functions

The following generalizations of the T_1 and Hausdorff properties will be useful.

Definition 6.1. A space X is said to be pre-T_1 [10] provided that for every pair of distinct points x and y of X, there exist preopen sets U and V containing x and y, respectively, with $y \notin U$ and $x \notin V$.

Definition 6.2. A space X is said to be weakly Hausdorff [15] if each element of X is an intersection of regular closed sets.

THEOREM 6.3. *If $f : X \to Y$ is a subalmost contra-precontinuous injection and Y is weakly Hausdorff, then X is pre-T_1.*

Proof. Let x_1 and x_2 be distinct points in X. Then $f(x_1) \neq f(x_2)$, and since Y is weakly Hausdorff, there exists a regular closed subset F of Y such that $f(x_1) \in F$ and $f(x_2) \notin F$. Then $f(x_2) \in X - F$, which is regular open. Let $\mathcal{B}$ be an open base for Y such that $p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Then let $V \in \mathcal{B}$ such that $f(x_2) \in V \subseteq Y - F$. Then $x_2 \notin X - p\mathrm{Cl}(f^{-1}(V))$, which is preopen. Also $f(x_1) \in F$, which is regular closed and therefore also semiopen. Since $F \cap V = \emptyset$, it follows that $f(x_1) \notin s\mathrm{Cl}(V)$, and hence $x_1 \notin f^{-1}(s\mathrm{Cl}(V))$. Then $x_1 \in X - f^{-1}(s\mathrm{Cl}(V)) \subseteq X - p\mathrm{Cl}(f^{-1}(V))$. Therefore, $X - p\mathrm{Cl}(f^{-1}(V))$ is a preopen set containing x_1 but not x_2, which proves that X is pre-T_1. □

COROLLARY 6.4 (Ekici [7, Theorem 11]). *If $f : X \to Y$ is an almost contra-precontinuous injection and Y is weakly Hausdorff, then X is pre-T_1.*

The following example shows that the restriction of a subalmost contra-precontinuous function is not necessarily subalmost contra-precontinuous.

Example 6.5. Let $X = \{a, b, c, d\}$ have the topology $\tau = \{X, \emptyset, \{a, b\}\}$ and let $Y = X$ have the discrete topology. Since the singleton subsets of X are preclosed [10], the identity mapping $f : X \to Y$ is subalmost contra-precontinuous with respect to the base for Y consisting of the singleton sets. However, if $A = \{a, c\}$, then $f|_A : A \to Y$ fails to be subalmost contra-precontinuous.

Next we show that the restriction of a subalmost contra-precontinuous function to a semiopen set is subalmost contra-precontinuous. The following lemma will be useful.

LEMMA 6.6 (Baker [3]). *If $B \subseteq A \subseteq X$ and A is semiopen in X, then $p\mathrm{Cl}_A(B) \subseteq p\mathrm{Cl}(B)$.*

THEOREM 6.7. *If $f : X \to Y$ is subalmost contra-precontinuous with respect to the open base $\mathcal{B}$ for Y and A is a semiopen subset of X, then $f|_A : A \to Y$ is subalmost contra-precontinuous with respect to $\mathcal{B}$.*

Proof. Let $V \in \mathcal{B}$. Then using Lemma 6.6, we see that $p\mathrm{Cl}_A(f|_A^{-1}(V)) \subseteq A \cap p\mathrm{Cl}(f|_A^{-1}(V)) = A \cap p\mathrm{Cl}(f^{-1}(V) \cap A) \subseteq A \cap p\mathrm{Cl}(f^{-1}(V)) \cap p\mathrm{Cl}(A) = A \cap p\mathrm{Cl}(f^{-1}(V)) \subseteq A \cap f^{-1}(s\mathrm{Cl}(V)) = f|_A^{-1}(s\mathrm{Cl}(V))$. Hence, $f|_A : A \to Y$ is subalmost contra-precontinuous with respect to $\mathcal{B}$. □

If we take $\mathcal{B}$ to be the topology on Y in Theorem 6.7, we obtain the following result.

COROLLARY 6.8 (Ekici [7, Theorem 2]). *If $f : X \to Y$ is almost contra-precontinuous and A is a semiopen subset of X, then $f|_A : A \to Y$ is almost contra-precontinuous.*

Theorem 6.9. *If $f : X \to Y$ is subalmost contra-precontinuous and A is an open subset of Y with $f(X) \subseteq A$, then $f : X \to A$ is subalmost contra-precontinuous.*

Proof. Let $\mathcal{B}$ be an open base for Y such that $p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Then $\mathcal{C} = \{V \cap A : V \in \mathcal{B}\}$ is an open base for the relative topology on A. For $V \in \mathcal{B}$, we have $p\mathrm{Cl}(f^{-1}(V \cap A)) = p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))) = f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)) \cap A)$. Now we show that $\mathrm{Int}(\mathrm{Cl}(V)) \cap A \subseteq \mathrm{Int}_A(\mathrm{Cl}_A(V \cap A))$.

Let $y \in \mathrm{Cl}(V) \cap A$ and let $W \subseteq A$ be open in the relative topology on A with $y \in W$. Since A is open in Y, we see that W is open in Y. Because $y \in \mathrm{Cl}(V)$, $V \cap W \neq \varnothing$. Then $W \cap (V \cap A) = W \cap V \neq \varnothing$, and hence $y \in \mathrm{Cl}_A(V \cap A)$. Then $\mathrm{Cl}(V) \cap A \subseteq \mathrm{Cl}_A(V \cap A)$, and therefore $\mathrm{Int}(\mathrm{Cl}(V) \cap A) \subseteq \mathrm{Int}(\mathrm{Cl}_A(V \cap A))$. Since $\mathrm{Int}(\mathrm{Cl}(V) \cap A) = \mathrm{Int}(\mathrm{Cl}(V)) \cap A$ and $\mathrm{Int}(\mathrm{Cl}_A(V \cap A)) \subseteq \mathrm{Int}_A(\mathrm{Cl}_A(V \cap A))$, it follows that $\mathrm{Int}(\mathrm{Cl}(V)) \cap A \subseteq \mathrm{Int}_A(\mathrm{Cl}_A(V \cap A))$.

Recall that we established in the first part of the proof that $p\mathrm{Cl}(f^{-1}(V \cap A)) \subseteq f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)) \cap A)$. Therefore $p\mathrm{Cl}(f^{-1}(V \cap A)) \subseteq f^{-1}(\mathrm{Int}_A(\mathrm{Cl}_A(V \cap A)))$, which by Theorem 4.4 proves that $f : X \to A$ is subalmost contra-precontinuous with respect to the base $\mathcal{C}$. □

Theorem 6.10. *If $f : X \to Y$ is subalmost contra-precontinuous, then for every θ-open (resp., θ-closed) subset W of Y, $f^{-1}(W)$ is a union of preclosed sets (resp., an intersection of preopen sets).*

Proof. Let $\mathcal{B}$ be an open base for Y such that $p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Let W be a θ-open set of Y and let $x \in f^{-1}(W)$. Let $V \in \mathcal{B}$ such that $f(x) \in V \subseteq \mathrm{Cl}(V) \subseteq W$. Then $x \in p\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\mathrm{Cl}(V)) \subseteq f^{-1}(\mathrm{Cl}(V)) \subseteq f^{-1}(W)$. Since $p\mathrm{Cl}(f^{-1}(V))$ is preclosed, it follows that $f^{-1}(W)$ is a union of preclosed sets. An argument using complements will prove the remaining part of the theorem. □

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb, and R. A. Mahmoud, *β-open sets and β-continuous mapping*, Bulletin of the Faculty of Science. Assiut University. A **12** (1983), no. 1, 77–90.

[2] M. E. Abd El-Monsef, R. A. Mahmoud, and E. R. Lashin, *β-closure and β-interior*, Journal of Faculty of Education, Ain Shams University **10** (1986), 235–245.

[3] C. W. Baker, *On a weak form of almost weakly continuous functions*, Demonstratio Mathematica **33** (2000), no. 4, 865–872.

[4] ———, *Subweakly β-continuous functions*, Far East Journal of Mathematical Sciences **8** (2003), no. 2, 151–160.

[5] S. G. Crossley and S. K. Hildebrand, *Semi-closure*, The Texas Journal of Science **22** (1971), 99–112.

[6] G. Di Maio and T. Noiri, *On s-closed spaces*, Indian Journal of Pure and Applied Mathematics **18** (1987), no. 3, 226–233.

[7] E. Ekici, *Almost contra-precontinuous functions*, Bulletin of the Malaysian Mathematical Sciences Society. Second Series **27** (2004), no. 1, 53–65.

[8] S. N. El-Deeb, I. A. Hasanein, A. S. Mashhour, and T. Noiri, *On p-regular spaces*, Bulletin Mathématique de la Société des Sciences Mathématiques de la République Socialiste de Roumanie **27(75)** (1983), no. 4, 311–315.

[9] J. E. Joseph and M. H. Kwack, *On S-closed spaces*, Proceedings of the American Mathematical Society **80** (1980), no. 2, 341–348.

[10] A. Kar and P. Bhattacharyya, *Some weak separation axioms*, Bulletin of the Calcutta Mathematical Society **82** (1990), no. 5, 415–422.

[11] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, The American Mathematical Monthly **70** (1963), no. 1, 36–41.

[12] A. S. Mashhour, M. E. Abd El-Monsef, and S. N. El-Deeb, *On precontinuous and weak precontinuous mappings*, Proceedings of the Mathematical and Physical Society of Egypt (1982), no. 53, 47–53.

[13] T. Noiri and V. Popa, *Some properties of almost contra-precontinuous functions*, Bulletin of the Malaysian Mathematical Sciences Society. Second Series **28** (2005), no. 2, 107–116.

[14] D. A. Rose, *Weak continuity and almost continuity*, International Journal of Mathematics and Mathematical Sciences **7** (1984), no. 2, 311–318.

[15] T. Soundararajan, *Weakly Hausdorff spaces and the cardinality of topological spaces*, General Topology and Its Relations to Modern Analysis and Algebra, III (Proc. Conf., Kanpur, 1968), Academia, Prague, 1971, pp. 301–306.

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