ON $\pi$-s-IMAGES OF METRIC SPACES

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We establish the characterizations of metric spaces under compact-covering (resp., pseudo-sequence-covering, sequence-covering) $\pi$-s-maps by means of cfp-covers (resp., sfp-covers, cs-covers) and $\sigma$-strong networks.

1. Introduction and definitions

In 1966, Michael [11] introduced the concept of compact-covering maps. Since many important kinds of maps are compact-covering, such as closed maps on paracompact spaces, much work has been done to seek the characterizations of metric spaces under various compact-covering maps, for example, compact-covering (open) $s$-maps, pseudo-sequence-covering (quotient) $s$-maps, sequence-covering (quotient) $s$-maps, and compact-covering (quotient) $s$-maps, see [3, 9, 12, 15, 16]. $\pi$-map is another important map which was introduced by Ponomarev [13] in 1960 and correspondingly, many spaces, including developable spaces, weak Cauchy spaces, $g$-developable spaces, and semimetrizable spaces, were characterized as the images of metric spaces under certain quotient $\pi$-maps, see [1, 4, 6, 7].

The purpose of this paper is to establish the characterizations of metric spaces under compact-covering (resp., pseudo-sequence-covering, sequence-covering) $\pi$-s-maps by means of cfp-covers (resp., sfp-covers, cs-covers) and $\sigma$-strong networks.

In this paper, all spaces are Hausdorff, and all maps are continuous and surjective. $\mathbb{N}$ denotes the set of all natural numbers. $\omega$ denotes $\mathbb{N} \cup \{0\}$. $\tau(X)$ denotes a topology on $X$. For a collection $\mathcal{P}$ of subsets of a space $X$ and a map $f : X \rightarrow Y$, denote $\{ f(P) : P \in \mathcal{P} \}$ by $f(\mathcal{P})$. For the usual product space $\prod_{i \in \mathbb{N}} X_i$, $\pi_i$ denotes the projective $\prod_{i \in \mathbb{N}} X_i$ onto $X_i$.

For a sequence $\{ x_n \}$ in $X$, denote $\langle x_n \rangle = \{ x_n : n \in \mathbb{N} \}$.

Definition 1.1. Let $f : X \rightarrow Y$ be a map.

(1) $f$ is called a compact-covering map [11] if each compact subset of $Y$ is the image of some compact subset of $X$.

(2) $f$ is called a sequence-covering map [14] if whenever $\{ y_n \}$ is a convergent sequence in $Y$, then there exists a convergent sequence $\{ x_n \}$ in $X$ such that each $x_n \in f^{-1}(y_n)$.
(3) $f$ is called a pseudo-sequence-covering map [3] if each convergent sequence (including its limit point) of $Y$ is the image of some compact subset of $X$.

(4) $f$ is called an $s$-map, if $f^{-1}(y)$ is separable in $X$ for any $y \in Y$.

(5) $f$ is called a $\pi$-map [13], if $(X,d)$ is a metric space, and for each $y \in Y$ and its open neighborhood $V$ in $Y$, $d(f^{-1}(y),M \setminus f^{-1}(V)) > 0$.

(6) $f$ is called a $\pi$-$s$-map, if $f$ is both $\pi$-map and $s$-map.

It is easy to check that compact maps on metric spaces are $\pi$-$s$-maps.

**Definition 1.2.** Let $\{\mathcal{P}_n\}$ be a sequence of covers of a space $X$ such that $\mathcal{P}_{n+1}$ refines $\mathcal{P}_n$ for each $n \in \mathbb{N}$.

(1) $\bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is called a $\sigma$-strong network [5] for $X$ if for each $x \in X$, $\langle \text{st}(x, \mathcal{P}_n) \rangle$ is a local network of $x$ in $X$. If every $\mathcal{P}_n$ satisfies property $P$, then $\bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is called a $\sigma$-strong network consisting of $P$-covers.

(2) $\{\mathcal{P}_n\}$ is called a weak development for $X$ if for each $x \in X$, $\langle \text{st}(x, \mathcal{P}_n) \rangle$ is a weak neighborhood base of $x$ in $X$.

**Definition 1.3 [2].** Let $X$ be a space.

(1) Let $\{x_n\}$ be a convergent sequence in $X$, and $P \subset X$. $\{x_n\}$ is eventually in $P$ if whenever $\{x_n\}$ converges to $x$, then $\{x\} \cup \{x_n : n \geq m\} \subset P$ for some $m \in \mathbb{N}$.

(2) Let $x \in P \subset X$. $P$ is called a sequential neighborhood of $x$ in $X$ if whenever a sequence $\{x_n\}$ in $X$ converges to $x$, then $\{x_n\}$ is eventually in $P$.

(3) Let $P \subset X$. $P$ is called a sequentially open subset in $X$ if $P$ is a sequential neighborhood of $x$ in $X$ for any $x \in P$.

(4) $X$ is called a sequential space if each sequentially open subset in $X$ is open.

**Definition 1.4 [10].** Let $\mathcal{P}$ be a collection of subsets of a space $X$.

(1) $\mathcal{P}$ is called a cfp-cover (i.e., compact-finite-partition cover) of compact subset $K$ in $X$ if there are a finite collection $\{K_\alpha : \alpha \in J\}$ of closed subsets of $K$ and $\{P_\alpha : \alpha \in J\} \subset \mathcal{P}$ such that $K = \bigcup\{K_\alpha : \alpha \in J\}$ and each $K_\alpha \subset P_\alpha$.

(2) $\mathcal{P}$ is called a cfp-cover for $X$ if for any compact subset $K$ of $X$, there exists a finite subcollection $\mathcal{P}^* \subset \mathcal{P}$ such that $\mathcal{P}^*$ is a cfp-cover of $K$ in $X$.

(3) $\mathcal{P}$ is called an sfp-cover (i.e., sequence-finite-partition cover) for $X$ if for any convergent sequence (including its limit point) $K$ in $X$, there exists a finite subcollection $\mathcal{P}^* \subset \mathcal{P}$ such that $\mathcal{P}^*$ is a cfp-cover of $K$ in $X$.

(4) $\mathcal{P}$ is called a cs-cover for $X$, if every convergent sequence in $X$ is eventually in some element of $\mathcal{P}$.

2. Results

**Theorem 2.1.** A space $X$ is the compact-covering $\pi$-$s$-image of a metric spaces if and only if $X$ has a $\sigma$-strong network consisting of point-countable cfp-covers.

**Proof.** To prove the only if part, suppose $f : (M,d) \to X$ is a compact-covering $\pi$-$s$-map, where $(M,d)$ is a metric space. For each $n \in \mathbb{N}$, put $\overline{\mathcal{F}}_n = \{f(B(z,1/n)) : z \in M\}$, where $B(z,1/n) = \{y \in M : d(z,y) < 1/n\}$. Obviously, $\bigcup\{\overline{\mathcal{F}}_n : n \in \mathbb{N}\}$ is a $\sigma$-strong network for $X$. In fact, for each $x \in X$, and its open neighborhood $U$, since $f$ is a $\pi$-map, then there exists $n \in \mathbb{N}$ such that $d(f^{-1}(x),M \setminus f^{-1}(U)) > 1/n$. 

We can pick $m \in \mathbb{N}$ such that $m \geq 2n$. If $z \in M$ with $x \in f(B(z,1/m))$, then
\[
 f^{-1}(x) \cap B(z,1/m) \neq \emptyset.
\] (2.1)
If $B(z,1/m) \notin f^{-1}(U)$, then
\[
d(f^{-1}(x), M \setminus f^{-1}(U)) \leq \frac{2}{m} \leq \frac{1}{n},
\] (2.2)
which is a contradiction. Thus $B(z,1/m) \subset f^{-1}(U)$, so $f(B(z,1/m)) \subset U$. Hence $st(x, \mathcal{F}_m) \subset U$. Therefore $\bigcup \{ \mathcal{F}_n : n \in \mathbb{N} \}$ is a $\sigma$-strong network for $X$.

For each $n \in \mathbb{N}$, let $\mathcal{B}_n$ be a locally finite open refinement of $\{B(z,1/n) : z \in M\}$. Since locally finite collections are closed under finite intersections, we can assume that $\mathcal{B}_{n+1}$ refines $\mathcal{B}_n$ for each $n \in \mathbb{N}$. Put $\mathcal{P}_n = f(\mathcal{B}_n)$. Obviously, $\mathcal{P}_{n+1}$ refines $\mathcal{P}_n$. Since $f$ is an $s$-map, each $\mathcal{P}_n$ is point-countable in $X$. Because $\mathcal{P}_n$ refines $\mathcal{F}_n$ for each $n \in \mathbb{N}$, then $\bigcup \{ \mathcal{P}_n : n \in \mathbb{N} \}$ is also a $\sigma$-strong network for $X$.

We now show that each $\mathcal{P}_n$ is a cfp-cover for $X$. Suppose $K$ is compact in $X$, since $f$ is compact-covering, then $f(L) = K$ for some compact subset $L$ of $M$. Since $\mathcal{B}_n$ is an open cover of $L$ in $M$, $\mathcal{B}_n$ have a finite subcover $\mathcal{B}_L$. Thus $\mathcal{B}_L$ can be precisely refined by some finite cover of $L$ consisting of closed subsets of $L$, denoted by $\{L_\alpha : \alpha \in J_n\}$. Put $\mathcal{P}_n^K = f(\mathcal{B}_L)$, since $\mathcal{P}_n^K$ is precisely refined by closed cover $\{f(L_\alpha) : \alpha \in J_n\}$ of $K$, then $\mathcal{P}_n^K$ is a cfp-cover of $K$ in $X$. Hence each $\mathcal{P}_n$ is a cfp-cover for $X$.

To prove the if part, suppose $\bigcup \{ \mathcal{P}_i : i \in \mathbb{N} \}$ is a $\sigma$-strong network for $X$ consisting of point-countable cfp-covers. For each $i \in \mathbb{N}$, $\mathcal{P}_i$ is a point-countable cfp-cover for $X$. Let $\mathcal{P}_i = \{P_\alpha : \alpha \in \Lambda_i\}$, endow $\Lambda_i$ with the discrete topology, then $\Lambda_i$ is a metric space. Put
\[
 M = \left\{ \alpha = (\alpha_i) \in \prod_{i \in \mathbb{N}} \Lambda_i : \langle P_\alpha \rangle \text{ forms a local network at some point } x_\alpha \text{ in } X \right\},
\] (2.3)
and endow $M$ with the subspace topology induced from the usual product topology of the collection $\{\Lambda_i : i \in \mathbb{N}\}$ of metric spaces, then $M$ is a metric space. Since $X$ is Hausdorff, $x_\alpha$ is unique in $X$. For each $\alpha \in M$, we define $f : M \rightarrow X$ by $f(\alpha) = x_\alpha$. For each $x \in X$ and $i \in \mathbb{N}$, there exists $\alpha_i \in \Lambda_i$ such that $x \in P_{\alpha_i}$. Since $\bigcup \{ \mathcal{P}_i : i \in \mathbb{N} \}$ is a $\sigma$-strong network for $X$, then $\{P_{\alpha_i} : i \in \mathbb{N}\}$ is a local network of $x$ in $X$. Put $\alpha = (\alpha_i)$, then $\alpha \in M$ and $f(\alpha) = x$. Thus $f$ is surjective. Suppose $\alpha = (\alpha_i) \in M$ and $f(\alpha) = x \in U \in \tau(X)$, then there exists $n \in \mathbb{N}$ such that $P_{\alpha_n} \subset U$. Put
\[
 V = \{ \beta \in M : \text{the } n\text{th coordinate of } \beta \text{ is } \alpha_n \},
\] (2.4)
then $V$ is an open neighborhood of $\alpha$ in $M$, and $f(V) \subset P_{\alpha_n} \subset U$. Hence $f$ is continuous. For each $\alpha, \beta \in M$, we define
\[
d(\alpha, \beta) = \begin{cases} 0, & \alpha = \beta, \\ \max \{1/k : \pi_k(\alpha) \neq \pi_k(\beta)\}, & \alpha \neq \beta, \end{cases}
\] (2.5)
then $d$ is a distance on $M$. Because the topology of $M$ is the subspace topology induced from the usual product topology of the collection $\{\Lambda_i : i \in \mathbb{N}\}$ of discrete spaces, thus $d$
is a metric on \( M \). For each \( x \in U \in \tau(X) \), there exists \( n \in \mathbb{N} \) such that \( st(x, \mathcal{P}_n) \subset U \). For \( \alpha \in f^{-1}(x) \), \( \beta \in M \), if \( d(\alpha, \beta) < 1/n \), then \( \pi_i(\alpha) = \pi_i(\beta) \) whenever \( i \leq n \). So \( x \in P_{\pi_n(\alpha)} = P_{\pi_n(\beta)} \). Thus,

\[
f(\beta) \in \bigcap_{i \in \mathbb{N}} P_{\pi_i(\beta)} \subset P_{\pi_n(\beta)} \subset U.
\]

Hence

\[
d(f^{-1}(x), M \setminus f^{-1}(U)) \geq \frac{1}{n}.
\]

Therefore \( f \) is a \( \pi \)-map.

For each \( x \in X \), it follows from the point-countable property of \( \mathcal{P}_i \) that \( \{ \alpha \in \Lambda_i : x \in P_{\alpha} \} \) is countable. Put

\[
L = \left( \bigcap_{i \in \mathbb{N}} \{ \alpha \in \Lambda_i : x \in P_{\alpha} \} \right) \cap M,
\]

then \( L \) is a hereditarily separable subspace of \( M \), and \( f^{-1}(x) \subset L \). Thus \( f^{-1}(x) \) is separable in \( M \), that is, \( f \) is an \( s \)-map.

We will prove that \( f \) is compact-covering. Suppose \( K \) is compact in \( X \). Since each \( \mathcal{P}_n \) is a cfp-cover for \( X \), there exists finite subcollection \( \mathcal{P}^K_n \) such that it is a cfp-cover of \( K \) in \( X \). Thus there are a finite collection \( \{ K_{\alpha} : \alpha \in J_n \} \) of closed subsets of \( K \) and \( \{ P_{\alpha} : \alpha \in J_n \} \subset \mathcal{P}^K_n \) such that \( K = \bigcup \{ K_{\alpha} : \alpha \in J_n \} \) and each \( K_{\alpha} \subset P_{\alpha} \). Obviously, each \( K_{\alpha} \) is compact in \( X \). Put

\[
L = \left\{ (\alpha_i) : \alpha_i \in J_i, \bigcap_{i \in \mathbb{N}} K_{\alpha_i} \neq \emptyset \right\},
\]

then

(i) \( L \) is compact in \( M \).

In fact, for all \( (\alpha_i) \notin L \), \( \bigcap_{i \in \mathbb{N}} K_{\alpha_i} = \emptyset \). From \( \bigcap_{i \in \mathbb{N}} K_{\alpha_i} = \emptyset \), there exists \( n_0 \in \mathbb{N} \) such that \( \bigcap_{i=1}^{n_0} K_{\alpha_i} = \emptyset \). Put

\[
W = \left\{ (\beta_i) : \beta_i \in J_i, \beta_i = \alpha_i, 1 \leq i \leq n_0 \right\},
\]

then \( W \) is an open neighborhood of \( (\alpha_i) \) in \( \prod_{i \in \mathbb{N}} J_i \), and \( W \cap L = \emptyset \). Thus \( L \) is closed in \( \prod_{i \in \mathbb{N}} J_i \). Since \( \prod_{i \in \mathbb{N}} J_i \) is compact in \( \prod_{i \in \mathbb{N}} \Lambda_i \), \( L \) is compact in \( M \).

(ii) \( L \subset M \), \( f(L) = K \).

In fact, for all \( (\alpha_i) \in L \), \( \bigcap_{i \in \mathbb{N}} K_{\alpha_i} \neq \emptyset \). Pick \( x \in \bigcap_{i \in \mathbb{N}} K_{\alpha_i} \), then \( (P_{\alpha_i}) \) is a local network of \( x \) in \( X \), so \( (\alpha_i) \in M \). This implies \( L \subset M \).

For all \( x \in K \), for each \( i \in \mathbb{N} \), pick \( \alpha_i \in J_i \) such that \( x \in K_{\alpha_i} \). Thus \( f((\alpha_i)) = x \), so \( K \subset f(L) \). Obviously, \( f(L) \subset K \). Hence \( f(L) = K \).

In a word, \( f \) is compact-covering.

**Corollary 2.2.** A space \( X \) is the compact-covering, quotient, and \( \pi \)-image of a metric space if and only if \( X \) has a weak-development consisting of point-countable cfp-covers.
Thus there exists \( n \) neighborhood base of \( X \) countable cs-covers for each \( x \in X \) and \( H \in \mathcal{P} \). To show that each \( x \in X \), put \( \{ P_n : n \in \mathbb{N} \} \) is a sequential neighborhood base of \( x \) in \( X \). Obviously, \( X \) is a sequential space. Thus \( \text{st}(x, \mathcal{P}_n) \) is a weak neighborhood base of \( x \) in \( X \). Hence \( \{ \mathcal{P}_n \} \) is a weak-development for \( X \).

To prove the if part, suppose \( X \) has a weak development consisting of point-countable cs-covers. From Theorem 2.1, \( X \) is the image of a metric space under a compact-covering \( \pi-s \)-map \( f \). Obviously, \( X \) is sequential. By [8, Proposition 2.1.16], \( f \) is quotient.

Similar to the proofs of Theorem 2.1 and Corollary 2.2, we have the following theorem.

**Theorem 2.3.** A space \( X \) is the pseudo-sequence-covering \( \pi-s \)-image of a metric space if and only if \( X \) has a \( \sigma \)-strong network consisting of point-countable sfp-covers.

**Corollary 2.4.** A space \( X \) is the pseudo-sequence-covering, quotient, and \( \pi-s \)-image of a metric space if and only if \( X \) has a weak-development consisting of point-countable sfp-covers.

**Theorem 2.5.** A space \( X \) is the sequence-covering \( \pi-s \)-image of a metric space if and only if \( X \) has a \( \sigma \)-strong network consisting of point-countable cs-covers.

**Proof.** To prove the only if part, suppose \( f : (M, d) \to X \) is a sequence-covering \( \pi-s \)-map, where \( (M, d) \) is a metric space. Similar to the proof of Theorem 2.1, we can show that \( \bigcup \{ \mathcal{P}_n : n \in \mathbb{N} \} \) is a \( \sigma \)-strong network consisting of point-countable covers. It suffices to show that each \( \mathcal{P}_n \) is a cs-cover for \( X \). Suppose \( \{ x_n \} \) converges to \( x \in X \). Since \( f \) is sequence-covering, then there exists a convergent sequence \( \{ z_i \} \) such that each \( z_i \in f^{-1}(x_i) \). Suppose \( \{ z_i \} \to z \), then \( z \in f^{-1}(x) \) and \( z \in B \) for some \( B \in \mathcal{P}_n \). Thus \( \{ z_i \} \) is eventually in \( B \), so \( \{ x_i \} \) is eventually in \( f(B) \in \mathcal{P}_n \). Hence each \( \mathcal{P}_n \) is a cs-cover for \( X \).

To prove the if part, suppose \( \bigcup \{ \mathcal{P}_i : i \in \mathbb{N} \} \) is a \( \sigma \)-strong network consisting of point-countable cs-covers for \( X \). For each \( i \in \mathbb{N} \), \( \mathcal{P}_i \) is a point-countable cs-cover for \( X \). Let \( \mathcal{P}_i = \{ P_a : \alpha \in \Lambda_i \} \). Similar to the proof of Theorem 2.1, we can show that \( f \) is a \( \pi-s \)-map. It suffices to show that \( f \) is sequence-covering. Suppose \( \{ x_n \} \) converges to \( x \) in \( X \). For each \( i \in \mathbb{N} \), since \( \mathcal{P}_i \) is a cs-cover for \( X \), then there exists \( P_a \in \mathcal{P}_i \) such that \( \{ x_n \} \) is eventually in \( P_a \). For each \( n \in \mathbb{N} \), if \( x_n \in P_a \), let \( \alpha_{in} = \alpha_i \); if \( x_n \notin P_a \), pick \( \alpha_{in} \in \Lambda_i \) such that \( x_n \in P_{\alpha_{in}} \). Thus there exists \( n_i \in \mathbb{N} \) such that \( \alpha_{in} = \alpha_i \) for all \( n > n_i \). So \( \{ \alpha_{in} \} \) converges to \( \alpha_i \). For each \( n \in \mathbb{N} \), put

\[
\beta_n = (\alpha_{in}) \in \prod_{i \in \mathbb{N}} \Lambda_i \tag{2.11}
\]

then \( (\beta_n) \in f^{-1}(x_n) \) and \( \{ \beta_n \} \) converges to \( x \). Thus \( f \) is sequence-covering.

Similar to the proof of Corollary 2.2, we have the following corollary.

**Corollary 2.6.** A space \( X \) is the sequence-covering, quotient, and \( \pi-s \)-image of a metric space if and only if \( X \) has a weak-development consisting of point-countable cs-covers.

We give examples to illustrate the theorems of this paper.
Example 2.7. Let $Z$ be the topological sum of the unit interval $[0,1]$, and the collection 
$\{S(x) : x \in [0,1]\}$ of $2^\omega$ convergent sequence $S(x)$. Let $X$ be the space obtained from $Z$
by identifying the limit point of $S(x)$ with $x \in [0,1]$, for each $x \in [0,1]$. Then, from [8, Example 2.9.27], or see [3, Example 9.8], we have the following facts.

(1) $X$ is the compact-covering, quotient compact image of a locally compact metric
space.

(2) $X$ has no point-countable cs-network.

The above facts together with [9, Theorem 1] yield the following conclusion: compact-
covering (quotient) $\pi$-s-images of metric spaces are not sequence-covering (quotient)
$\pi$-s-images of metric spaces.

Example 2.8. Let $X$ be a sequential fan $S_\omega$ (see [8, Example 1.8.7]), then $X$ is a Fréchet
and $\aleph_0$-space. So $X$ is the sequence-covering $s$-image of a metric space. Because $X$ is
not $g$-first countable, thus $X$ is not the pseudo-sequence-covering $\pi$-image of a metric
space. Hence the following holds: sequence-covering (resp., pseudo-sequence-covering)
$s$-images of metric spaces are not sequence-covering (resp., pseudo-sequence-covering)
$\pi$-s-images of metric spaces.

Example 2.9. Let $X$ be a Gillman-Jerison space $\psi(\mathbb{N})$ (see [8, Example 1.8.4]). Since $X$
is developable, then $X$ is the sequence-covering, quotient $\pi$-image of a metric space by [10,
Corollary 3.1.12]. But $X$ has no point-countable cs*-networks. Then, it follows from [8,
Theorem 2.7.5] that $X$ is not the pseudo-sequence-covering $s$-image of a metric space. Thus,

(1) sequence-covering (quotient) $\pi$-images of metric spaces are not sequence-
covering (quotient) $\pi$-s-images of metric spaces,

(2) pseudo-sequence-covering (quotient) $\pi$-images of metric spaces are not pseudo-
sequence-covering (quotient) $\pi$-s-images of metric spaces.

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References

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