

CONTRA-CONTINUOUS FUNCTIONS AND STRONGLY S -CLOSED SPACES

J. DONTCHEV

Department of Mathematics
University of Helsinki
00014 Helsinki 10, FINLAND

(Received June 27, 1994)

ABSTRACT. In 1989 Ganster and Reilly [6] introduced and studied the notion of LC -continuous functions via the concept of locally closed sets. In this paper we consider a stronger form of LC -continuity called contra-continuity. We call a function $f : (X, \tau) \rightarrow (Y, \sigma)$ contra-continuous if the preimage of every open set is closed. A space (X, τ) is called strongly S -closed if it has a finite dense subset or equivalently if every cover of (X, τ) by closed sets has a finite subcover. We prove that contra-continuous images of strongly S -closed spaces are compact as well as that contra-continuous, β -continuous images of S -closed spaces are also compact. We show that every strongly S -closed space satisfies FCC and hence is nearly compact.

KEY WORDS AND PHRASES. Strongly S -closed, closed cover, contra-continuous, LC -continuous, perfectly continuous, strongly continuous, FCC.

1991 AMS SUBJECT CLASSIFICATION CODE. Primary: 54C10, 54D20, Secondary: 54C08, 54G99.

1. INTRODUCTION.

The field of the mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. General Topologists have introduced and investigated many different generalizations of continuous functions. One of the most significant of those notions is LC -continuity. Ganster and Reilly [6] defined a function $f : (X, \tau) \rightarrow (Y, \sigma)$ to be LC -continuous if the preimage of every open set is locally closed. A set $A \subset (X, \tau)$ is called locally closed [4] (= FG [25]) if it can be represented as the intersection of an open and a closed set. The importance of LC -continuity is that it happens to be the dual of near continuity (= precontinuity) to continuity, i.e. a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is continuous if and only if it is LC -continuous and nearly continuous [7]. Due to this theorem we can obtain interesting and useful variations of results in functional analysis, for example theorems concerning open mappings and closed graph theorem [17, 18, 20, 27, 28].

In this paper we present a new generalization of continuity called *contra-continuity*. We define this class of functions by the requirement that the inverse image of each open set in the codomain is closed in the domain. This notion is a stronger form of LC -continuity. This definition enables us to obtain the following results (1) *Contra-continuous images of spaces having a dense finite subset are compact* and (2) *Contra-continuous, β -continuous images of S -closed spaces are compact*.

In 1976 Thompson [26] introduced the notion of S -closed spaces via Levine's semi-open sets [12]. A space (X, τ) is called S -closed if every semi-open cover has a finite subfamily the closures of whose members cover X or equivalently if every regular closed cover has a finite subcover. In what turns out

the space property of having a finite dense subset is equivalent to the following property. Every closed cover has a finite subcover. Hence this is a stronger form of S -closedness and we call spaces having this property *strongly S -closed*. Thus restating our result we have: Contra-continuous images of strongly S -closed spaces are compact. Moreover we observe that contra-continuity is properly placed between Levine's strong continuity [11] and Ganster and Reilly's LC -continuity [6]. In fact it is even a weaker form of Noiri's perfect continuity [16]. A decomposition of perfect continuity is presented by showing that a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is perfectly continuous if and only if it is contra-continuous and (nearly) continuous. An improvement of a result achieved by Singal and Mathur [22] is given by showing that continuous, contra-continuous images of almost compact spaces (= quasi- H -closed = QHC) are compact.

The notion of continuity was generalized in 1958 by Ptak [20]. He defined a function $f : (X, \tau) \rightarrow (Y, \sigma)$ to be *nearly continuous* if the inverse image of every open set in Y is nearly open in X . A subset A of a space is called *nearly open* [6] if $A \subset \text{Int } \overline{A}$, where (as well as everywhere in this paper) $\text{Int } A$ and \overline{A} denote the interior and the closure respectively of $A \subset (X, \tau)$. Nearly open sets are well-known as *preopen sets*. In [15], a topology τ^α has been introduced by defining its open sets to be the α -sets, that is the sets $A \subset X$ with $A \subset \text{Int } \overline{\text{Int } A}$. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called *α -continuous* [15] if the inverse image of every open set in Y is an α -set in X . Sometimes α -sets are called *α -open*.

In 1962 Levine introduced the notion of semi-open sets. A set $A \subset (X, \tau)$ is called *semi-open* [12] if $A \subset \overline{\text{Int } A}$. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called *semi-continuous* [12] if the inverse image of every open set in Y is semi-open in X . Note that a function is α -continuous if and only if it is semi-continuous and nearly continuous [21].

Semi-preopen sets were defined by Adrijević in 1986. A set $A \subset (X, \tau)$ is called *semi-preopen* [1] if $A \subset \overline{\text{Int } A}$. Semi-preopen sets are sometimes called *β -open*. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called *β -continuous* if the inverse image of every open set in Y is semi-preopen in X . β -continuity was recently investigated by Popa and Noiri [19]. Every semi-continuous and every nearly continuous function is β -continuous but not vice versa.

In Section 2 we study contra-continuous functions, while Section 3 is devoted to strong S -closedness. In Section 4 we show that every strongly S -closed space satisfies FCC (= semi-irreducible) and hence is nearly compact.

2. CONTRA-CONTINUOUS FUNCTIONS

DEFINITION 1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called *contra-continuous* if $f^{-1}(U)$ is closed in (X, τ) for each open set U in (Y, σ) .

The following characterization of contra-continuity can be obtained by using the same technique of the similar result involving continuity.

THEOREM 2.1. For a function $f : (X, \tau) \rightarrow (Y, \sigma)$ the following conditions are equivalent:

- (1) f is contra-continuous.
- (2) For each $x \in X$ and each closed set V in Y with $f(x) \in V$, there exists an open set U in X such that $x \in U$ and $f(U) \subset V$.
- (3) The inverse image of each closed set in Y is open in X . \square

Since closed sets are locally closed, then we have:

THEOREM 2.2. Every contra-continuous function is LC -continuous. \square

EXAMPLE 2.3. An LC -continuous function need not be contra-continuous. The identity function on the real line with the usual topology is an example of an LC -continuous function (even a continuous function) which is not contra-continuous.

REMARK 2.4. In fact contra-continuity and continuity are independent notions. Examples 2.3 above shows that continuity does not imply contra-continuity while the reverse is shown in the following example

EXAMPLE 2.5. A contra-continuous function need not be continuous. Let $X = \{a, b\}$ be the Sierpinski space by setting $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{b\}, X\}$. The identity functions $f : (X, \tau) \rightarrow (X, \sigma)$ is contra-continuous but not continuous.

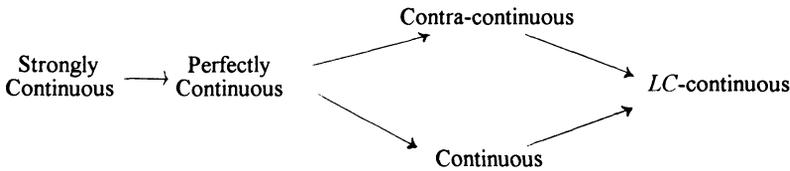
In 1960 Levine [11] defined a function $f : (X, \tau) \rightarrow (Y, \sigma)$ to be *strongly continuous* if $f(\overline{A}) \subset f(A)$ for every subset A of X or equivalently if the inverse image of every set in Y is clopen in X [11, Corollary 2]. Thus we have

THEOREM 2.6. *Every strongly continuous function is contra-continuous.* □

In 1984 Noiri [16] introduced the notion of perfect continuity between topological spaces. By definition a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called *perfectly continuous* if the inverse of every open set in Y is clopen in X . Hence.

THEOREM 2.7. *Every perfectly continuous function is contra-continuous.* □

Clearly the following diagram holds and none of the implications is reversible:



It is interesting to note that indiscrete and locally indiscrete spaces (= every open set is closed) can be characterized via perfectly continuous functions. For example it is easily seen that a space (X, τ) is indiscrete if and only if every function $f : (X, \tau) \rightarrow (Y, \sigma)$ is perfectly continuous and locally indiscrete if and only if every semi-continuous function $f : (X, \tau) \rightarrow (Y, \sigma)$ is perfectly continuous or equivalently if and only if the identity function $f : (X, \tau) \rightarrow (X, \tau)$ is perfectly continuous. In [14] locally indiscrete spaces are called *partition spaces*.

THEOREM 2.8. *For a set $A \subset (X, \tau)$ the following conditions are equivalent:*

- (1) A is clopen.
- (2) A is α -open and closed.
- (3) A is nearly open and closed.

PROOF. (1) \Rightarrow (2) and (2) \Rightarrow (3) are obvious.

(3) \Rightarrow (1) Since A is nearly open, then $A \subset \text{Int } \overline{A}$. Since A is closed, then $A \subset \text{Int } \overline{A} = \text{Int } A$ or equivalently A is open and hence clopen. □

As a consequence of the above decomposition of clopen sets we have the following decomposition of perfect continuity:

THEOREM 2.9. *For a function $f : (X, \tau) \rightarrow (Y, \sigma)$ the following conditions are equivalent:*

- (1) f is perfectly continuous.
- (2) f is continuous and contra-continuous.
- (3) f is α -continuous and contra-continuous.
- (4) f is nearly continuous and contra-continuous. □

In the case when contra-continuity is reduced to LC-continuity we have the following result proved by Ganster and Reilly in [7]: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is continuous if and only if it is

nearly-continuous and LC -continuous. In fact in this decomposition LC -continuity can be replaced by sub- LC -continuity [6].

A space (X, τ) is *almost compact* ($=$ *quasi- H -closed* $=$ QHC) if every open cover has a finite proximate subcover ($=$ subfamily the closures of whose members cover X)

THEOREM 2.10. *The image of an almost compact space under contra-continuous, nearly continuous mapping is compact.*

PROOF. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be contra-continuous and nearly continuous and let X be almost compact. Let $(V_i)_{i \in I}$ be an open cover of Y . Then $(f^{-1}(V_i))_{i \in I}$ is a closed, nearly open cover of X due to our assumptions on f . By Theorem 2.8 $(f^{-1}(V_i))_{i \in I}$ is a clopen cover of X . Since X is almost compact, then for some finite $J \subset I$ we have $X = \cup_{i \in J} f^{-1}(V_i) = \cup_{i \in J} f^{-1}(V_i)$. Since f is onto, then $Y = \cup_{i \in J} V_i$ or equivalently Y is compact. \square

COROLLARY 2.11. [22, Theorem 3.4] *The image of an almost compact ($=$ AC) space under a strongly continuous mapping is compact.*

PROOF. Every strongly continuous function is both continuous and contra-continuous (see the diagram below Theorem 2.7). \square

EXAMPLE 2.12. The composition of two contra-continuous functions need not be contra-continuous. Let $X = \{a, b\}$ be the Sierpinski space and set $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{b\}, X\}$. The identity functions $f : (X, \tau) \rightarrow (X, \sigma)$ and $f : (X, \sigma) \rightarrow (X, \tau)$ are both contra-continuous but their composition $g \circ f : (X, \tau) \rightarrow (X, \tau)$ is not contra-continuous.

However the following theorem holds. The proof is easy and hence omitted.

THEOREM 2.13. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \nu)$ be two functions. Then:*

- (i) $g \circ f$ is contra-continuous, if g is continuous and f is contra-continuous.
- (ii) $g \circ f$ is contra-continuous, if g is contra-continuous and f is continuous.
- (iii) $g \circ f$ is contra-continuous, if f and g are continuous and Y is locally indiscrete. \square

3. COVERING SPACES WITH CLOSED SETS

In this section we will give a characterization of spaces, where covers by closed subsets admit finite subcovers.

DEFINITION 2. A space is called *strongly S -closed* if every closed cover of X has a subcover.

REMARK 3.1. Note that the notion of strong S -closedness is independent from compactness. The real line with the cofinite topology is a compact space, which is not strongly S -closed, while again the real line but this time with a topology in which non-void open sets are the ones containing the origin (in such cases the origin is called a *generic point* [14]) is an example of a non-compact, strongly S -closed space. Thus a $T_{\frac{1}{2}}$ strongly S -closed space need not be finite. A space is $T_{\frac{1}{2}}$ if singletons are open or closed. However a T_1 -space is strongly S -closed if and only if it is finite.

THEOREM 3.2 *Every strongly S -closed space X is S -closed.*

PROOF. It is shown in [8, Theorem 3.2] that X is S -closed if and only if every regular closed cover of X has a finite subcover. Since every regular closed set is closed the theorem is clear. \square

Note that the reverse in the theorem above is not always true. The real line (in fact any infinite set) with the cofinite topology is an example of an S -closed space, which is not strongly S -closed, since the trivial subsets are the only regular closed sets.

THEOREM 3.3. *For a space X the following are equivalent:*

- (1) X is strongly S -closed.
- (2) X has a finite dense subset.

PROOF. (1) \Rightarrow (2) Since $\{\overline{\{x\}} : x \in X\}$ is a closed cover of X , then for some finite set $S \subset X$ we have $X = \cup_{x \in S} \overline{\{x\}} = \overline{S}$. Thus S is finite and dense in X .

(2) \Rightarrow (1) By (2) for some finite set $S = \{x_1, \dots, x_n\} \subset X$ we have $\bar{S} = X$ If $(A_i)_{i \in I}$ is a closed cover of X , then for each index $k = 1, \dots, n$ there exists an index $i(k) \in I$ such that $x_k \in A_{i(k)}$ Thus $\{x_k\} \subset A_{i(k)}$ and hence $X = \bar{S} = \cup_{k=1}^n \overline{\{x_k\}} \subset \cup_{k=1}^n A_{i(k)}$ Thus $(A_{i(1)}, \dots, A_{i(n)})$ is a finite subcover of $(A_i)_{i \in I}$ and so X is strongly S -closed \square

COROLLARY 3.4. *If X is strongly S -closed, then the set $i(X)$ of all isolated points of X is finite* \square

The following result can be easily verified Its proof is straightfoward

THEOREM 3.5 *Strong S -closedness is open hereditary.* \square

Recall that a space (X, τ) is called *d-compact* [10] if every cover of X by dense subsets has a finite subcover

THEOREM 3.6. *Every d-compact, strongly S -closed space X is finite*

PROOF. By Theorem 3 3 X has a finite dense subset S Since $\{S \cup \{x\} : x \in X \setminus S\}$ is a dense cover of X , then by assumption $X \setminus S$ is finite Thus X is finite being the union of two finite sets \square

Next we give a condition under which S -closedness implies strong S -closedness. Recall that a set $A \subset X$ is called a *semi-generalized closed set* (= sg-closed set) [2] if $sC1A \subset O$ whenever $A \subset O$ and O is semi-open Here $sC1A$ is the semi-closure of A , i.e the intersection of all semi-closed sets containing A A set is *semi-closed* if its complement is semi-open Note that the following implications hold and none of them is reversible.

$$A \text{ is closed} \Rightarrow A \text{ is semi-closed} \Rightarrow A \text{ is sg-closed}$$

A complement of an sg-closed set is called *sg-open*. It is not difficult to see that a set is regular closed if and only if it is both closed and sg-open Thus the following result holds.

THEOREM 3.7. *If every closed subset of a space X is sg-open, then X is S -closed if and only if it is strongly S -closed.* \square

THEOREM 3.8. *Contra-continuous images of strongly S -closed spaces are compact.*

PROOF. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be strongly continuous and onto. Assume that X is strongly S -closed. Let $(V_i)_{i \in I}$ be an open cover of Y Then $(f^{-1}(V_i))_{i \in I}$ is a closed cover of X , since f is contra-continuous. Thus for some finite $J \subset I$ we have $X = \cup_{i \in J} f^{-1}(V_i)$. Since f is onto, then $Y = \cup_{i \in J} V_i$ or equivalently Y is compact. \square

REMARK 3.9 In the theorem above, contra-continuity cannot be reduced to LC -continuity, since there are strongly S -closed spaces which are not compact and since the identity function is always LC -continuous.

THEOREM 3.10. *Contra-continuous, β -continuous images of S -closed spaces are compact.*

PROOF. Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra-continuous and β -continuous as well as that X is S -closed. Let $(V_i)_{i \in I}$ be an open cover of Y . Then $(f^{-1}(V_i))_{i \in I}$ is a cover of X such that for every $i \in I$, $f^{-1}(V_i)$ is closed and β -open due to assumption Thus for each $i \in I$ we have $\text{Int } \overline{f^{-1}(V_i)} \subset f^{-1}(V_i) \subset \text{Int } \overline{f^{-1}(V_i)}$ or equivalently each $f^{-1}(V_i)$ is regular closed in X . Thus since X is S -closed for some finite $J \subset I$ we have $X = \cup_{i \in J} f^{-1}(V_i)$ Then clearly, since f is onto, $Y = \cup_{i \in J} V_i$, i.e. Y is compact. \square

In the notion of the above given proof and since regular closed sets are semi-open, then the following result is obvious.

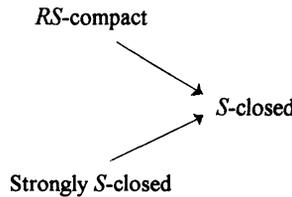
THEOREM 3.11. *Every contra-continuous, β -continuous function is semi-continuous.* \square

In 1968 Singal and Singal [23] introduced the notion of almost-continuity By definition a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called *almost-continuous* [23] if for each $x \in X$ and for each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset \text{Int } \bar{V}$ In 1974 Long and Herrington [13] proved

that $f : (X, \tau) \rightarrow (Y, \sigma)$ is almost-continuous if and only if the inverse image of every regular open set in Y is open in X . Using the same technique like in Theorem 3.8 and Theorem 3.10 one can prove easily the following result:

THEOREM 3.12. *Almost-continuous images of strongly S -closed spaces are S -closed.* \square

Finally we note that in 1980 Hong [9] introduced another stronger form of S -closedness called RS -compactness. He defined a space (X, τ) to be RS -compact [9] if every cover of X by regular semi-open sets has a finite subfamily such that the interiors of its members cover X . A set A is called *regular semi-open* [5] if for some regular open set U we have $U \subset A \subset \bar{U}$. Since regular open sets are clearly regular semi-open, then we have the following implication:



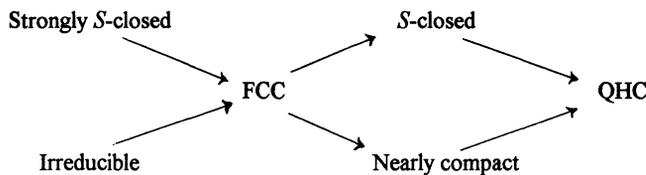
REMARK 3.13. RS -compactness and strong S -closedness are totally independent notions. An infinite space with the cofinite topology is an example of an RS -compact space which is not strongly S -closed, since the space has no non-trivial regular semi-open sets. For the reverse let X be the set of positive integers with the following topology: $\tau = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$. Note that $\{1\} \cup \{\{2, n\} : n = 3, 4, 5, \dots\}$ is a regular semi-open cover of X which does not have even a finite subcover. But X is clearly strongly S -closed, since $\{1, 2\}$ is a finite dense subset of X .

4. STRONG S -CLOSEDNESS VERSUS FCC

A space (X, τ) is an FCC -space [25] (= *semi-irreducible space* [14]) if every disjoint family of non-void open subsets of X is finite or equivalently if X has only a finite amount of regular open sets. X is *irreducible* if every two non-void open subsets of X intersect.

A space (X, τ) is *nearly compact* [22] if every open cover of X has a finite subfamily such that the interiors of its closures cover X or equivalently if every cover of X by regular open sets X has a finite subcover.

In this section we will show that the following diagram holds and none of its implications is reversible:



LEMMA 4.1. *For a subspace S of a space (X, τ) the following conditions are equivalent:*

- (1) S is FCC .
- (2) \bar{S} is FCC .

PROOF. (1) \Rightarrow (2) Let $(V_i)_{i \in I}$ be a disjoint family of non-void open subsets of \bar{S} . Then $V_i = \bar{S} \cap U_i$, where each U_i is open in X . Since $\bar{S} \cap U_i \neq \emptyset$, then also $S \cap U_i \neq \emptyset$. Then $(S \cap U_i)_{i \in I}$ is a disjoint family of non-void open subsets of S and by (1) I is finite. Hence \bar{S} is FCC .

(2) \Rightarrow (1) Let $(S \cap U_i)_{i \in I}$ be a disjoint family of non-void open subsets of S . Then $(\bar{S} \cap U_i)_{i \in I}$ is a disjoint family of non-void open subsets of \bar{S} and by (2) I is finite. Thus S is FCC. \square

COROLLARY 4.2. *If a space X has a dense FCC-subspace, then X is FCC.* \square

THEOREM 4.3. *Every strongly S -closed space (X, τ) is FCC.*

PROOF. By Theorem 3.3 X has a finite dense subspace A . Since A is finite, then A is FCC. By Corollary 4.2 X is FCC. \square

THEOREM 4.4. *Every irreducible space (X, τ) is FCC.*

PROOF. By Theorem 14 in [14] X is irreducible if and only if the only regular open subsets of X are the trivial ones. \square

Straight from the definitions we have:

THEOREM 4.5. *If a space (X, τ) is FCC, then X is both S -closed and nearly compact.*

Finally we point out that an example of an FCC-space which is not strongly S -closed is an infinite cofinite space. Thus even an irreducible space need not be strongly S -closed. On the other hand a strongly S -closed space need not be irreducible: a finite discrete space with at least two points is an easy example

REFERENCES

- [1] ANDRIJEVIĆ, D., *Semi-preopen sets*, *Mat. Vesnik*, **38** (1986), 1, 24-32.
- [2] BHATTACHARYA, P. and LAHIRI, B.K., *Semi-generalized closed sets in topology*, *Indian J. Math.*, **29** (1987), 3, 375-382.
- [3] BLUMBERG, H., *New properties of real functions*, *Trans. Amer. Math. Soc.*, **24** (1922), 113-128.
- [4] BOURBAKI, N., *General Topology*, Part 1, Addison-Wesley, Reading, Mass., 1966.
- [5] CAMERON, D., *Properties of S -closed spaces*, *Proc. Amer. Math. Soc.*, **72** (1978), 581-586.
- [6] GANSTER, M. and REILLY, I.L., *Locally closed sets and LC -continuous functions*, *Internat. J. Math. Math. Sci.*, **3** (1989), 417-424.
- [7] GANSTER, M. and REILLY, I., *A decomposition of continuity*, *Acta Math. Hungar.*, **56** (1990), 299-30.
- [8] HERRMAN, R.A., *RC -convergence*, *Proc. Amer. Math. Soc.*, **75** (1979), 311-317.
- [9] HONG, W.C., *RS -compact spaces*, *J. Korean Math. Soc.*, **17** (1980), 39-43.
- [10] JANKOVIĆ, D., REILLY, I. and VAMANAMURTHY, M., *On strongly compact topological spaces*, *Questions Answers Gen. Topology*, **6** (1) (1988), 29-40.
- [11] LEVINE, N., *Strong continuity in topological spaces*, *Amer. Math. Monthly*, **67** (1960), 269.
- [12] LEVINE, N., *Semi-open sets and semi-continuity in topological spaces*, *Amer. Math. Monthly*, **70** (1963), 36-41.
- [13] LONG, P.E. and HERRINGTON, L.L., *Properties of almost-continuous functions*, *Boll. Un. Mat. Ital.*, **10** (1974), 336-342.
- [14] NIEMINEN, T., *On ultrapseudocompact and related spaces*, *Ann. Acad. Sci. Fenn. Ser. A I. Math.*, **3** (1977), 185-205.
- [15] NJÅSTAD, O., *On some classes of nearly open sets*, *Pacific J. Math.*, **15** (1965), 961-970.
- [16] NOIRI, T., *Supercontinuity and some strong forms of continuity*, *Indian J. Pure Appl. Math.*, **15** (1984), 3, 241-250.
- [17] NOLL, D., *Baire spaces and graph theorems*, *Proc. Amer. Math. Soc.*, **96** (1986), 141-151.
- [18] PETTIS, B.J., *Closed graph and open mapping theorems in certain topologically complete spaces*, *Bull. London Math. Soc.*, **6** (1974), 37-41.
- [19] POPA, V. and NOIRI, T., *On β -continuous functions*, *Real Anal. Exchange*, **18** (1992/1993), 544-548.
- [20] PTAK, V., *Completeness and open mapping theorem*, *Bull. Soc. Math. France*, **86** (1958), 41-74.

- [21] REILLY, I.L. and VAMANAMURTHY, M.K., On α -continuity in topological spaces, *Acta Math. Hungar*, **45** (1-2) (1985), 27-32.
- [22] SINGAL, M.K. and MATHUR, A., On nearly-compact spaces, *Boll. Un. Mat. Ital., Serie IV* (4-6) (1969), 702-710.
- [23] SINGAL, M.K. and SINGAL, A.R., Almost-continuous mappings, *Yokohama Math. J.*, **16** (1968), 63-73.
- [24] STONE, A.H., Absolute FG spaces, *Proc. Amer. Math. Soc.*, **80** (1980), 515-520.
- [25] STONE, A.H., Hereditarily compact spaces, *Amer. J. Math.*, **82** (1960), 900-916.
- [26] THOMPSON, T., S -closed spaces, *Proc. Amer. Math. Soc.*, **60** (1976), 335-338.
- [27] WILHELM, M., Relations among some closed graph and open mapping theorems, *Colloq. Math.*, **42** (1979), 387-394.
- [28] WILHELM, M., Some negative examples concerning nearly continuity, *Comment. Math.*, **26** (1986), 187-194.