ABSTRACT. A study of prolongations of F-structure to the tangent bundle of order 2 has been presented.

KEY WORDS AND PHRASES. Prolongations, tangent bundle, integrable, lift, F-structure.

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1. INTRODUCTION.

Let $F$ be a nonzero tensor field of type $(1,1)$ and of class $c^\infty$ on an $n$-dimensional manifold $V_n$ such that \[ F^K + (-)^K + 1 F = 0 \quad \text{and} \quad F^W + (-)^W + 1 F \neq 0 \quad \text{for} \quad 1 < W < K \] (1.1) where $K$ is a fixed positive integer greater than 2. Such a structure on $V_n$ is called an $F$-structure of rank $r$ and degree $K$. If the rank of $F$ is constant and $r = r(F)$, then $V_n$ is called an $F$-structure manifold of degree $K(\geq 3)$. The case when $K$ is odd has been considered in this paper.

Let the operators on $V_n$ be defined as follows [1]:

\[ I = (-)^K F^{K-1} \quad \text{and} \quad m = I + (-)^K + 1 F^{K-1} \] (1.2)

where $I$ denotes the identity operator on $V_n$.

From the operators defined by (1.2) we have

\[ l + m = I \quad \text{and} \quad l^2 = l; \quad \text{and} \quad m^2 = m \] (1.3)

For $F$ satisfying (1.1), there exist complementary distributions $L$ and $M$ corresponding to the projection operators $l$ and $m$ respectively.

If rank $(F) = \text{constant on } V_n$ then $\dim L = r$ and $\dim M = (n - r)$. We have the following results [1]

\[ Fl = lF = F \quad \text{and} \quad Fm = mF = 0 \] (1.4a)

\[ F^{K-1}l = -l \quad \text{and} \quad F^{K-1}m = 0 \] (1.4b)

2. PROLONGATIONS OF F-STRUCTURE IN THE TANGENT BUNDLE OF ORDER 2.

Let $V_n$ be an $n$-dimensional differentiable manifold of class $c^\infty$ and $T_p(V_n) = \bigcup_{p \in V_n} T_p$ is the tangent bundle over the manifold $V_n$.

Let us denote $T'_2(V_n)$, the set of all tensor fields of class $c^\infty$ and of the type $(r,s)$ in $V_n$ and $T(V_n)$ be the tangent bundle over $V_n$. 
Let us introduce an equivalence relation \( \sim \) in the set of all differentiable mappings \( F: \mathbb{R} \rightarrow V \), where \( \mathbb{R} \) is the real line. Let \( r \geq 1 \) be a fixed integer. If two mappings \( F: \mathbb{R} \rightarrow V \) and \( G: \mathbb{R} \rightarrow V \) satisfy the conditions
\[
\frac{dF^h(t)}{dt} = \frac{dG^h(t)}{dt}, \quad \frac{dF^r(t)}{dt} = \frac{dG^r(t)}{dt},
\]
the mapping \( F \) and \( G \) being represented respectively by \( X^h = F^h(t) \) and \( X^h = G^h(t), \) \((t \in \mathbb{R})\) with respect to local coordinates \( X^h \) in a coordinate neighborhood \((U, X^h)\) containing the point \( P = F(0) = G(0) \), then we say that the mapping \( F \) is equivalent to \( G \). Each equivalence class determined by the equivalence relation \( \sim \) is called an \( r \)-jet of \( V \) and denoted by \( J^r_p(F) \). The set of all \( r \)-jets of \( V \) is called the tangent bundle of order \( r \) and denoted by \( T_r(V) \). The tangent bundle \( T_2(V) \) of order 2 has the natural bundle structure over \( V \), its bundle projection \( \pi_2: T_2(V) \rightarrow V \) being defined by \( \pi_2(J^2_p(F)) = P \). If we introduce a mapping such that \( P = P(0) \), then \( T_2(V) \) has a bundle structure over \( T(V) \) with projection \( \pi_{12} \).

Let us denote \( T_2(V) \), the second order tangent bundle over \( V \) and let \( F^{II} \) be the second lift of \( F \) in \( T_2(V) \). The second lift \( F^{II} \) which belong to \( T_2(T_2(V)) \) has component of the form [3]
\[
F^{II} = \begin{bmatrix}
F^h & 0 & 0 \\
y^h & F^h & 0 \\
x^h F^h + (1/2)y^h y^h F^h & y^h F^h & F^h
\end{bmatrix}
\]
with respect to the induced coordinates in \( T_2(V) \), \( F^h \) being local components of \( F \) in \( V \).

Now we obtain the following results on the second lift of \( F \) satisfying (1.1).

For any \( F, G \in T^1_2(V) \), the following holds [3]:
\[
(G^{II}F^{II})X^{II} = G^{II}(FX^{II}),
\]
\[
= G^{II}(FX)^{II}
\]
\[
= (G(FX))^{II}
\]
\[
= (GF)^{II}X^{II}
\]
for every \( X \in T^1_2(V) \),
\[
(2.2)
\]
therefore we have
\[
G^{II}F^{II} = (GF)^{II}
\]
If \( P(s) \) denote a polynomial of variable \( s \), then we have
\[
(P(F))^{II} = P(F^{II}), \text{ where } F \in T^1_2(V)
\]
\[
(2.3)
\]
We have the following theorem:

**THEOREM 2.1.** The second lift \( F^{II} \) defines a \( F \)-structure in \( T_2(V) \) iff \( F \) defines a \( F \)-structure in \( V \).

**PROOF.** Let \( F \) satisfy (1.1) then \( F \) defines \( F \)-structure in \( V \) satisfying
\[
F^K + (-)^{K+1}F = 0,
\]
which in view of equation (2.3) yields
Therefore $F^{II}$ defines a $F$-structure in $T_2(V_n)$. The converse can be proved in a similar manner.

**THEOREM 2.2.** The second lift $F^{II}$ is integrable in $T_2(V_n)$, iff $F$ is integrable in $V_n$.

**PROOF.** Let us denote $N_{II}$ and $N$, the Nijenhuis tensors of $F^{II}$ and $F$ respectively. Then we have [2]

$$N_{II}(X,Y) = (N(X,Y))^{II} \tag{2.5}$$

We know that $F$-structure is integrable in $V_n$, iff

$$N(X,Y) = 0,$$

which in view of (2.5) is equivalent to

$$N_{II}(X,Y) = 0. \tag{2.6}$$

Thus $F^{II}$ is integrable, iff $F$ is integrable in $V_n$.

**THEOREM 2.3.** The second lift $F^{II}$ of $F$ is partially integrable in $T_2(V_n)$, iff $F$ is integrable in $V_n$.

**PROOF.** We know that for $F$ to be partially integrable in $V_n$, the following holds [2]:

$$N(l_{II}X_{II}, l_{II}Y_{II}) = 0$$

and

$$N(m_{II}X_{II}, m_{II}Y_{II}) = 0,$$

which, in view of equation (2.5), takes the form

$$N_{II}(l_{II}X_{II}, l_{II}Y_{II}) = 0$$

and

$$N_{II}(m_{II}X_{II}, m_{II}Y_{II}) = 0. \tag{2.7}$$

where $l_{II}, m_{II}$ are operators in $T_2(V_n)$ which define the distribution $L^{II}$ and $M^{II}$ respectively. Thus equation (2.7) gives the condition for $F^{II}$ to be partially integrable.

The converse follows in a similar manner.

**REFERENCES**

Intermodal transport refers to the movement of goods in a single loading unit which uses successive various modes of transport (road, rail, water) without handling the goods during mode transfers. Intermodal transport has become an important policy issue, mainly because it is considered to be one of the means to lower the congestion caused by single-mode road transport and to be more environmentally friendly than the single-mode road transport. Both considerations have been followed by an increase in attention toward intermodal freight transportation research.

Various intermodal freight transport decision problems are in demand of mathematical models of supporting them. As the intermodal transport system is more complex than a single-mode system, this fact offers interesting and challenging opportunities to modelers in applied mathematics. This special issue aims to fill in some gaps in the research agenda of decision-making in intermodal transport.

The mathematical models may be of the optimization type or of the evaluation type to gain an insight in intermodal operations. The mathematical models aim to support decisions on the strategic, tactical, and operational levels. The decision-makers belong to the various players in the intermodal transport world, namely, drayage operators, terminal operators, network operators, or intermodal operators.

Topics of relevance to this type of decision-making both in time horizon as in terms of operators are:
- Intermodal terminal design
- Infrastructure network configuration
- Location of terminals
- Cooperation between drayage companies
- Allocation of shippers/receivers to a terminal
- Pricing strategies
- Capacity levels of equipment and labour
- Operational routines and lay-out structure
- Redistribution of load units, railcars, barges, and so forth
- Scheduling of trips or jobs
- Allocation of capacity to jobs
- Loading orders
- Selection of routing and service

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