A COUPLED MAGNETO-THERMO-ELASTIC PROBLEM IN A PERFECTLY CONDUCTING ELASTIC HALF-SPACE WITH THERMAL RELAXATION

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ABSTRACT. In the present paper we consider the magneto-thermo-elastic wave produced by a thermal shock in a perfectly conducting elastic half-space. Here the Lord-Shulman theory of thermoelasticity [1] is used to account for the interaction between the elastic and thermal fields. The solution obtained in analytical form reduces to those of Kaliski and Nowacki [2] when the coupling between the temperature and strain fields and the relaxation time are neglected. The results also agree with those of Massalas and Dalamangas [3] in absence of the thermal relaxation time.

KEY WORDS AND PHRASES. Magneto-thermoelastic wave; Thermal relaxation time.

1980 AMS SUBJECT CLASSIFICATION CODE.

1. INTRODUCTION.

Kaliski and Nowacki [2] investigated the problem of magneto-thermo-elastic disturbances generated by a thermal shock in a perfectly conducting elastic half-space in contact with a vacuum. It was assumed that both in the medium and in the vacuum there acted an initial magnetic field parallel to the plane boundary of the half-space and there was no influence of coupling between temperature and strain fields.

Later, Massalas and Dalamangas [3] considered the same problem where the coupling between the temperature and strain fields was considered. Very recently Chatterjee and Roy Choudhuri [4] extended the problem [3] in generalized thermo-elasticity of Green and Lindsay taking into account the two relaxation times.

In the present paper we extend the problem [3] in generalized thermoelasticity by using the thermal relaxation time of Lord-Shulman theory [1]. The solutions for temperature distribution, deformation and perturbed magnetic field in the vacuum are obtained in analytical form in the first power of the magnetothermo-elastic coupling parameter $\varepsilon$ and relaxation parameter $\tau_o'$. In absence of $\varepsilon$, $\tau_o'$ the solutions agree with those in [2] and in absence of $\tau_o'$, the results agree with those in [3].
Surface stress for different times is calculated and graphically presented. It is believed that this particular problem has not been considered earlier.

2. PROBLEM FORMULATION.

We assume that a magneto-thermo-elastic wave is produced in an elastic half-space \( x_1 > 0 \) due to the thermal shock \( \theta(0, t) = \theta_0 \) applied on \( x_1 = 0 \) where \( \theta_0 \) is a constant and \( H(t) \) is the Heaviside function. We also assume that in both the media there is an initial magnetic field acting in the direction of \( x_3 \)-axis. The simplified equations of slowly moving bodies in electrodynamics after linearization are the following:

\[
\begin{align*}
\mathbf{E} &= \frac{\mathbf{j}}{\epsilon_0} \\
\mathbf{H} &= \frac{\mathbf{H}_0}{c} \\
\partial_t \mathbf{E} &= -\frac{\mathbf{j}}{\epsilon_0} \\
\partial_t \mathbf{H} &= -\frac{\mu_0}{\epsilon_0} \left( \mathbf{E} \times \mathbf{H}_0 \right)
\end{align*}
\]  

(2.1)

where \( \mathbf{E} \) denotes the electric field, \( \mathbf{H} \) is the perturbation of the magnetic field, \( \mathbf{H}_0 \) is the initial constant magnetic field, \( \mathbf{j} \) is the current density vector, \( u \) denotes the displacement vector, \( \mu_0 \) is the magnetic permeability, \( \sigma \) is the electric conductivity and \( c \) is the velocity of light. The displacement equation of motion in thermo-elasticity including the electromagnetic effect after linearization is,

\[
\mu \nabla^2 \mathbf{u} + (\lambda + 2\mu) \nabla \cdot \mathbf{u} + \frac{\mu_0}{4\pi} \left( \mathbf{E} \times \mathbf{H}_0 \right) - \gamma \nabla \theta = \rho \mathbf{u}''.
\]

(2.2)

Also the modified form of Fourier's law of heat conduction taking into account the thermal relaxation time \([1]\) is

\[
\rho c_v \nabla \cdot (\mathbf{u}_0 \nabla \theta) + \gamma T_0 (\lambda + \nu \nabla) = K \theta_{0,11}, \quad (i=1, 2, 3)
\]

(2.3)

where \( \lambda, \mu \) are the Lame' constants, \( \gamma \) is equal to \((3\lambda+4\mu) \alpha_T \), \( \alpha_T \) is the co-efficient of linear thermal expansion, \( \theta \) is equal to \( T - T_0 \); \( T_0, T \) are the reference and absolute temperature of the body respectively; \( K \) is the co-efficient of heat conduction; \( \rho \) is the mass density; \( c_v \) is the specific heat at constant volume; \( \tau_0 \) is the relaxation time. The magneto-thermo-elastic wave propagated in the medium \( x_1 > 0 \) is assumed to depend on \( x_1 \) and time \( t \).

For \( \mathbf{H}_0 = (0, 0, H_3) \) equations (2.1) reduce to

\[
\begin{align*}
\mathbf{E}' &= -\frac{\mu_0 H_3}{c} (0, u_1, 0), \\
\mathbf{H}' &= \frac{\mathbf{E}_2}{\epsilon_0}, \\
\mathbf{j} &= \frac{\mu_0}{4\pi} (0,0, \mathbf{E}_2 \times \mathbf{H}_0), \\
\mathbf{u}' &= \frac{c^2}{\epsilon_0} (0, -\frac{\partial \theta}{\partial x_1}, 0).
\end{align*}
\]

(2.4)

Equations (2.2) and (2.3) then lead to

\[
(\lambda + 2\mu) \frac{\partial^2 u_1}{\partial x_1^2} - \gamma \frac{\partial \theta}{\partial x_1} = \rho \mathbf{u}''
\]

(2.5)
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\[ \nabla \cdot \left( \frac{\partial \varepsilon}{\partial \varepsilon} + \frac{\partial \theta}{\partial \theta} \right) + \gamma \left( \frac{\partial u_1}{\partial x} + \frac{\partial u_1}{\partial x} \right) = K \left( \frac{\partial^2 \varepsilon}{\partial x^2} + \frac{\partial^2 \theta}{\partial x^2} \right) \]  

where \( a_o = \sqrt{\frac{\mu_0 H_3^2}{4\pi \rho}} \) is the Alfvén wave velocity. For convenience, we shall use the notations \( u_1 = u, x_1 = x \).

In vacuum the system of equations of electrodynamics are

\[ \left( -\frac{\partial^2}{\partial x^2} \hat{c} \right) h_3^0 = 0 \]

\[ \left( -\frac{\partial^2}{\partial x^2} \hat{c} \right) E_2^0 = 0 \]

where \( x' = -x \).

The components \( T_{11} \) and \( T_{11}^0 \) of Maxwell's stress tensor in elastic medium and in vacuum are

\[ T_{11} = -\frac{\mu_0}{4\pi} h_3 H_3 \text{ and } T_{11}^0 = -\frac{1}{4\pi} h_3^0 H_3. \]

The normal mechanical and thermal stress is

\[ \sigma_{11} = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma \theta. \]

The boundary conditions to be satisfied are

\[ \sigma_{11} + T_{11} = T_{11}^0 = 0, \quad x = x' = 0 \]  

\[ E_2 = E_2^0, \quad x = x' = 0 \]  

\[ \theta(0, t) = \theta_0(t). \]

3. SOLUTION OF THE PROBLEM.

To find the solution of the problem we now introduce the following notations and non-dimensional variables

\[ c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_0^2 = \frac{a_o^2}{c_1^2}, \quad \xi = \frac{c_0 x}{\kappa}, \quad \tau = \frac{c_0 t}{\kappa}. \]
The equations (2.5) - (2.7) and boundary conditions (2.8) - (2.10) become

\[
\frac{\partial^2 U}{\partial \xi^2} - \frac{\partial^2 U}{\partial \tau^2} = 0, \quad \xi > 0
\]

(3.1)

\[
\frac{\partial^2 \tau}{\partial \xi^2} - \tau_o \frac{\partial^2 \tau}{\partial \tau^2} - \frac{\partial^2 U}{\partial \xi^2} = 0, \quad \xi > 0
\]

(3.2)

\[
\frac{\partial^2 h_3}{\partial \xi^2} - \beta^2 \frac{\partial^2 h_3}{\partial \tau^2} = 0, \quad \xi' > 0
\]

(3.3)

\[
\frac{\partial u}{\partial \xi} + \beta_1 h_3 = 0, \quad \xi = \xi' = 0
\]

(3.4)

\[
\beta_2 \frac{\partial^2 U}{\partial \tau^2} - \frac{\partial^2 h_3}{\partial \tau^2} = 0, \quad \xi = \xi' = 0
\]

(3.5)

\[
Z(\xi, \tau) = \frac{\theta}{T_o} H(\tau),
\]

(3.6)

where \( \beta_1 = \frac{H_3}{4\pi T_o}, \quad \beta_2 = \frac{\mu H_3 T_o}{\rho c^2}, \quad \beta = \frac{C_o}{c}, \quad \xi' = -\xi. \)

Initial conditions in the new variables are

\[
U(\xi, 0) = 0, \quad Z(\xi, 0) = 0, \quad \frac{\partial U(\xi, 0)}{\partial \xi} = 0.
\]

We now introduce a potential function \( \phi \) defined by

\[
U = \frac{\partial \phi}{\partial \xi}.
\]

(3.7)

Using (3.7) in (3.1) and then integrating we get

\[
z(\xi, \tau) = \left( \frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \tau^2} \right) \phi \text{ in } \xi > 0.
\]

(3.8)
Using (3.7), the equation (3.2) leads to

\[ \frac{\partial^2 z}{\partial \xi^2} - \frac{3}{2} \frac{\partial z}{\partial \tau} - \tau'_0 \frac{\partial^2 z}{\partial \tau^2} - \varepsilon \frac{\partial^3 \phi}{\partial \xi^3} - \varepsilon \tau'_0 \frac{\partial^4 \phi}{\partial \xi^4} = 0 \]  

(3.9)

In the Laplace transform domain the equations (3.8), (3.9) and (3.3) become

\[ \tilde{z}(\xi,s) = (-\frac{\partial^2}{\partial \xi^2} - s^2) \tilde{\phi}, \quad \xi > 0 \]  

(3.10)

\[ (-\frac{\partial^2}{\partial \xi^2} - s - \tau'_0 s^2) \tilde{z} = \varepsilon s(1+\tau'_0 s) \frac{\partial^2 \phi}{\partial \xi^2}, \quad \xi > 0 \]  

(3.11)

\[ \tilde{n}_3 = c_3 e^{\beta \xi}, \quad \xi > 0. \]  

(3.12)

In Laplace transform domain, the boundary conditions (3.4) - (3.6) reduce to

\[ \frac{\partial}{\partial \xi} \tilde{Z}(o,s) - \frac{\partial}{\partial \xi} \tilde{Z}(o,s) = 0, \quad \xi = 0 \]  

(3.13)

\[ \frac{\partial}{\partial \xi} \tilde{Z}(o,s) - \frac{\partial}{\partial \xi} \tilde{Z}(o,s) = 0, \quad \xi = \xi' = 0 \]  

(3.14)

\[ \tilde{Z}(o,s) = \frac{\theta}{\tau_0} \frac{1}{s}. \]  

(3.15)

Eliminating \( \tilde{z} \) from (3.10) and (3.11) we get

\[ \frac{\partial^4 \phi}{\partial \xi^4} - (1+\varepsilon+s+(1+\varepsilon) \tau'_0 s) \frac{\partial^2 \phi}{\partial \xi^2} + s^3(1+\tau'_0) \frac{\partial \phi}{\partial \xi} = 0. \]  

(3.16)

The equation (3.16) reduces to (31) in [4] on setting \( \alpha = \alpha' = \tau'_0 \).

The general solution of (3.16) vanishing at \( \xi = \infty \)

\[ \tilde{\phi}(\xi,s) = C_1 e^{-\lambda_1 \xi} + C_2 e^{-\lambda_2 \xi}, \quad \xi > 0 \]  

(3.17)

where \( \lambda_1, \lambda_2 \) are given by the roots of the equation

\[ \lambda^4 - s \{1+\varepsilon+s+(1+\varepsilon) \tau'_0 s\} \lambda^2 + s^3(1+\tau'_0) = 0. \]  

(3.18)

Hence

\[ \lambda_{1,2} = \left( \frac{\varepsilon}{2} \{s+1+\varepsilon+\tau'_0 s \} \pm [(1+\varepsilon) \tau'_0 + \tau'_0^2 + 2 \varepsilon \tau'_0 + 2 \varepsilon \tau'_0^2 - 2 \tau'_0 s]^2 \right)^{1/2} + 2(\varepsilon - 1+2 \varepsilon \tau'_0 + \tau'_0 + \varepsilon \tau'_0) s + (1+\varepsilon)^{1/2}. \]  

(3.19)
The equation (3.19) agrees with that of (34) in [4] for $\alpha' = \alpha'' = \tau'_0$. For $\alpha' = \alpha'' = 0$, the equations (3.16), (3.19) are in agreement with that of (24) in [3]. Thus the equations (3.1), (3.2), (3.16), (3.19) are more general in the sense that they incorporate the effect of thermal relaxation time of Lord-Shulman theory.

From (3.10) using (3.17) we have

$$
\bar{z}(\xi, s) = C_1(\lambda_1^2 - s^2) e^{-\lambda_1 \xi} + C_2(\lambda_2^2 - s^2) e^{-\lambda_2 \xi}, \xi > 0.
$$

(3.20)

From the boundary conditions (3.13) - (3.15) taking into account (3.17) and (3.20) we obtain a linear algebraic system with respect to $C_1, C_2$ and $C_3$ as

$$
C_1 s^2 + C_2 s^2 + \beta_1 C_3 = 0, \text{ at } \xi = \xi' = 0
$$

(3.21)

$$
\beta_2 s\lambda_1 C_1 + \beta_2 s\lambda_2 C_2 + \beta C_3 = 0, \text{ at } \xi = \xi' = 0
$$

(3.22)

$$
C_1(\lambda_1^2 - s^2) + C_2(\lambda_2^2 - s^2) = \frac{\theta_0}{T_0 s}.
$$

(3.23)

The constants $C_i (i=1,2,3)$ being determined by (3.21) - (3.23), the solutions for $\bar{\psi}, \bar{z}, \bar{h}_3^0$ are given by

$$
\bar{\psi}(\xi, s, \xi', \tau'_0) = \frac{\theta_0}{T_0} \frac{(s \beta + \beta_1 \beta_2 \lambda_2) e^{-\lambda_1 \xi} - (s \beta + \beta_1 \beta_2 \lambda_1) e^{-\lambda_2 \xi}}{s(\lambda_1 - \lambda_2)(\beta_1 \beta_2 s^2 + \beta(\lambda_1 + \lambda_2)s + \beta_1 \beta_2 \lambda_1 \lambda_2)}
$$

(3.24)

$$
\bar{z}(\xi, s, \xi', \tau'_0) = \frac{\theta_0}{T_0} \frac{\lambda_2 e^{-\lambda_2 \xi} - (\lambda_2 - s^2)(s \beta + \beta_1 \beta_2 \lambda_2) e^{-\lambda_2 \xi}}{s(\lambda_1 - \lambda_2)(\beta_1 \beta_2 s^2 + \beta(\lambda_1 + \lambda_2)s + \beta_1 \beta_2 \lambda_1 \lambda_2)}
$$

(3.25)

$$
\bar{U}(\xi, s, \xi', \tau'_0) = \frac{\theta_0}{T_0} \frac{\lambda_2 e^{-\lambda_2 \xi} - (s \beta + \beta_1 \beta_2 \lambda_2) e^{-\lambda_1 \xi}}{s(\lambda_1 - \lambda_2)(\beta_1 \beta_2 s^2 + \beta(\lambda_1 + \lambda_2)s + \beta_1 \beta_2 \lambda_1 \lambda_2)}, \xi > 0
$$

(3.26)

$$
\bar{h}_3^0(\xi, s, \xi', \tau'_0) = \frac{\theta_0}{T_0} \frac{s \beta \lambda_2 e^{-\lambda_2 \xi}}{\beta_1 \beta_2 s^2 + \beta(\lambda_1 + \lambda_2)s + \beta_1 \beta_2 \lambda_1 \lambda_2}, \xi' > 0.
$$

(3.27)

Since $\epsilon, \tau'_0 < 1$ for small thermo-elastic couplings, we expand the functions $\bar{z}, \bar{U}, \bar{h}_3^0$ into Maclaurian's series and retain the first two terms in the series expansion to obtain
\[ \begin{align*}
\tilde{e}(\xi, s, \epsilon, \tau_0') &= \frac{\beta e^{-\xi\sqrt{s}}}{\sqrt{s}(s-1)^2} + \frac{\beta_1 \beta_2 e^{-\xi\sqrt{s}}}{(\beta + \beta_1 \beta_2)(s-1)^2} + \frac{\beta_1 \beta_2 e^{-\xi\sqrt{s}}}{(\beta + \beta_1 \beta_2)\sqrt{s}(s-1)^2} + \frac{e^{-\xi\sqrt{s}}}{s(s-1)^2} \\
+ \frac{\beta_1 \beta_2}{(\beta + \beta_1 \beta_2)s(s-1)^2} + \frac{\xi e^{-\xi\sqrt{s}}}{2\sqrt{s}(s-1)^2} - \frac{e^{-\xi\sqrt{s}}}{2s(s-1)^2} - \beta e^{-\xi\sqrt{s}} \\
+ \tau_0' \left[ -\frac{\xi e^{-\xi\sqrt{s}}}{2\sqrt{s}(s-1)^2} \right] 
\end{align*} \]

\[ \begin{align*}
\tilde{U}(\xi, s, \epsilon, \tau_0') &= \frac{\epsilon e^{-\xi\sqrt{s}}}{\sqrt{s}(s-1)} - \frac{\beta e^{-\xi\sqrt{s}}}{(\beta + \beta_1 \beta_2)s(s-1)} - \frac{\beta \beta_1 \beta_2 e^{-\xi\sqrt{s}}}{(\beta + \beta_1 \beta_2)s\sqrt{s}(s-1)^2} + \frac{e^{-\xi\sqrt{s}}}{2(\beta + \beta_1 \beta_2)s \sqrt{s}(s-1)^2} \\
+ \frac{\xi \beta_1 \beta_2 e^{-\xi\sqrt{s}}}{2(\beta + \beta_1 \beta_2)s(s-1)^2} + \frac{4 e^{-\xi\sqrt{s}}}{2(\beta + \beta_1 \beta_2)s(s-1)^2} + \frac{\xi \beta_1 \beta_2 e^{-\xi\sqrt{s}}}{2(\beta + \beta_1 \beta_2)s(s-1)^2} + \frac{\xi \beta_1 \beta_2 e^{-\xi\sqrt{s}}}{2(\beta + \beta_1 \beta_2)s(s-1)^2} \\
+ \frac{\beta_1 \beta_2 e^{-\xi\sqrt{s}}}{2(\beta + \beta_1 \beta_2)s\sqrt{s}(s-1)(\sqrt{s}-1)^2} + \frac{\beta e^{-\xi\sqrt{s}}}{2(\beta + \beta_1 \beta_2)s\sqrt{s}(s-1)(\sqrt{s}-1)^2} - \frac{\beta e^{-\xi\sqrt{s}}}{2(\beta + \beta_1 \beta_2)s\sqrt{s}(s-1)(\sqrt{s}-1)^2} \\
+ \frac{\beta \beta_1 \beta_2 e^{-\xi\sqrt{s}}}{2(\beta + \beta_1 \beta_2)s\sqrt{s}(s-1)(\sqrt{s}-1)^2} + \tau_0' \left[ \frac{\epsilon e^{-\xi\sqrt{s}}}{2\sqrt{s}(s-1)} - \frac{\xi e^{-\xi\sqrt{s}}}{2(s-1)} - \frac{\beta_1 \beta_2 e^{-\xi\sqrt{s}}}{(\beta + \beta_1 \beta_2)\sqrt{s}(s-1)} \right] \\
- \frac{\beta e^{-\xi\sqrt{s}}}{(\beta + \beta_1 \beta_2)(s-1)^2} + \frac{\beta e^{-\xi\sqrt{s}}}{2(\beta + \beta_1 \beta_2)\sqrt{s}(s-1)^2} \right]
\end{align*} \]

\[ \begin{align*}
\tilde{h}_3^0(\xi', s, \epsilon, \tau_0') &= \frac{\beta e^{-\beta \xi'}}{\sqrt{s}(\sqrt{s}+1)^3} - \frac{\epsilon \beta_1 \beta_2 e^{-\beta \xi'}}{2(\beta + \beta_1 \beta_2)^2 \sqrt{s}(\sqrt{s}+1)^3} \\
- \tau_0' \left[ \frac{\beta e^{-\beta \xi'}}{2(\beta + \beta_1 \beta_2)(\sqrt{s}+1)^2} \right]
\end{align*} \]

Taking inverse Laplace transform we obtain (Chatterjee (Roy) and Roy Choudhuri [4], Hetnarski [5], Oberhettiner and Badil [6]),
\[ Z(\xi, \tau, \varepsilon, \tau') = \frac{\theta_0}{\pi} \left[ \text{erfc} \left( \frac{\xi}{\sqrt{2\tau}} \right) + \varepsilon \left( \frac{\beta_1}{\beta_1 + \beta_2} (\tau - \xi) - \frac{\beta_2}{\beta_1 + \beta_2} \right) \right] H(\tau - \xi) + \frac{\beta_1 \beta_2}{(\beta_1 + \beta_2) \sqrt{\pi}} \left( \tau - \frac{\xi}{\sqrt{2\tau}} \right) \]

\[ + \left( \tau - \xi - \frac{1}{2} \right) e^{(\tau - \xi)} \text{erf} \left( \frac{\xi}{\sqrt{2\tau}} \right) H(\tau - \xi) + \tau f_1(\xi, \tau) - \frac{\xi}{2} f_2(\xi, \tau) - \frac{\xi}{2} f'_3(\xi, \tau) + \text{erfc} \left( \frac{\xi}{\sqrt{2\tau}} \right) \]

\[ + \frac{\beta_1 \beta_2}{(\beta_1 + \beta_2)^2} \left[ f_1(\xi, \tau) - \text{erfc} \left( \frac{\xi}{\sqrt{2\tau}} \right) + \frac{\xi}{2} f_2(\xi, \tau) - \frac{1}{2} f'_3(\xi, \tau) \right] \]

\[ - \frac{\beta}{(\beta_1 + \beta_2)^2} \left[ \frac{\xi}{2} \left( \xi^2 - 2\tau \right) - \frac{5}{4} e^{\frac{\xi^2}{4\tau}} \right] \]

\[ (3.31) \]

\[ U(\xi, \tau, \varepsilon, \tau') = \frac{\theta_0}{\pi} f_2(\xi, \tau) - 2\sqrt{\pi} e^{\left( \frac{\xi}{2\tau} \right)} + \xi \text{erfc} \left( \frac{\xi}{\sqrt{2\tau}} \right) - \frac{\beta}{(\beta_1 + \beta_2)^2} \left( \tau - \frac{\xi}{\sqrt{2\tau}} \right) H(\tau - \xi) \]

\[ - \frac{\beta_1 \beta_2}{(\beta_1 + \beta_2)^2} \left[ e^{(\tau - \xi)} \text{erf} \left( \frac{\xi}{\sqrt{2\tau}} \right) - 2\sqrt{\pi} \right] H(\tau - \xi) + \varepsilon \left( \frac{\beta_1}{\beta_1 + \beta_2} \right) f_1(\xi, \tau) - f_2(\xi, \tau) \]

\[ + 2\sqrt{\pi} e^{\left( \frac{\xi}{4\tau} \right)} - \xi \text{erfc} \left( \frac{\xi}{2\tau} \right) + \frac{\xi \beta_1}{(\beta_1 + \beta_2)^2} \left[ \frac{\tau f_1(\xi, \tau) - \frac{\xi}{2} f_2(\xi, \tau) - \frac{\xi}{2} f'_3(\xi, \tau) + \text{erfc} \left( \frac{\xi}{\sqrt{2\tau}} \right) \right] \]

\[ - \frac{\beta}{(\beta_1 + \beta_2)^2} \left[ \tau f_2(\xi, \tau) - f_1(\xi, \tau) + \text{erfc} \left( \frac{\xi}{\sqrt{2\tau}} \right) \right] \]

\[ + \left( \frac{\xi}{2} \right) f_2(\xi, \tau) - \frac{f_1(\xi, \tau) + \text{erfc} \left( \frac{\xi}{\sqrt{2\tau}} \right)}{2\tau} \]

\[ + \frac{\xi \beta_1}{(\beta_1 + \beta_2)^2} \left[ \frac{\tau f_1(\xi, \tau) - \frac{\xi}{2} f_2(\xi, \tau) - \frac{\xi}{2} f'_3(\xi, \tau) + \text{erfc} \left( \frac{\xi}{\sqrt{2\tau}} \right) \right] \]

\[ + \frac{\xi}{2} f_4(\xi, \tau) + 4\sqrt{\pi} e^{\left( \frac{\xi}{4\tau} \right)} - \text{erfc} \left( \frac{\xi}{\sqrt{2\tau}} \right) - (2\tau - \xi - 2)e^{(\tau - \xi)} \text{erfc} \left( \frac{\xi}{\sqrt{2\tau}} \right) \]

\[ + \frac{\beta}{(\beta_1 + \beta_2)^2} \left[ \left( \tau - \frac{3}{2} \right) \text{erf} \left( \frac{\xi}{\sqrt{2\tau}} \right) - \frac{5}{2} (\tau - \xi - 1) e^{(\tau - \xi)} - \frac{\xi}{2} \right) \]

\[ + \frac{3}{2} \left( \tau - \xi \right) + \left( \tau - \xi \right)^2 \text{erf} \left( \frac{\xi}{\sqrt{2\tau}} \right) + \frac{\beta_1 \beta_2}{(\beta_1 + \beta_2)^2} \left[ \left( \tau - \xi \right) \right] \text{erf} \left( \frac{\xi}{\sqrt{2\tau}} \right) \]

\[ - \frac{\beta}{(\beta_1 + \beta_2)^2} \left[ \text{erf} \left( \frac{\xi}{\sqrt{2\tau}} \right) + \left( \tau - \xi \right)^2 \text{erf} \left( \frac{\xi}{\sqrt{2\tau}} \right) \right] \]
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\[ + \frac{\beta^2}{2(\beta+\beta_1 \beta_2)^2} [ \frac{5}{2} \tau \xi + 2(\tau+\xi)^2 - 2] H(\tau-\xi) + \frac{\beta^2}{2(\beta+\beta_1 \beta_2)^2} \left[ - \frac{3}{2} \xi^2 \left( \xi^2 - \xi \right) + \frac{1}{2} \left( 5 \tau - 5 \xi - 3 \right) e^{(\tau-\xi)} \right] \]

\[ + \frac{5}{2} \left( \tau - \xi \right) e^{(\tau-\xi)} \text{erf} \sqrt{\tau-\xi} - \frac{3}{2} \left( \tau - \xi \right) + \frac{3}{2} \left( \tau - \xi \right) + \left( \tau - \xi \right)^2 - 1 \right] H(\tau-\xi) \]

\[ - \frac{\beta}{2(\beta+\beta_1 \beta_2)} \left[ f_5(\xi, \tau) - 4 \sqrt{\pi} e^{-(\frac{x}{\sqrt{\tau}})^2} \text{erfc} \left( \frac{x}{\sqrt{\tau}} \right) - 2 \sqrt{\pi} \left( 2+\xi - 2\tau + \xi^2 - 2 \right) e^{(\tau-\xi)} \text{erf} \left( \frac{x}{\sqrt{\tau}} \right) \right] \]

\[ + \frac{\beta \beta_2}{2(\beta+\beta_1 \beta_2)} \left[ \left( \tau - \xi \right) + \frac{9}{4} \sqrt{\tau-\xi} + \frac{1}{2} \left( 7 \tau - 7 \xi - 5 \right) e^{(\tau-\xi)} - \frac{1}{2} \left( 7 \tau - 7 \xi - \frac{11}{2} \right) e^{(\tau-\xi)} \text{erf} \sqrt{\tau-\xi} \right] \]

\[ + \frac{\beta}{2(\beta+\beta_1 \beta_2)} \left[ \left( \tau - \xi \right) + \left( \tau - \xi \right)^2 \right] H(\tau-\xi) + \frac{\beta}{2(\beta+\beta_1 \beta_2)} e^{(\tau-\xi)} \text{erf} \sqrt{\tau-\xi} + \frac{2}{2} \left( \tau - \xi \right) H(\tau-\xi) \]

\[ - \frac{\beta}{2(\beta+\beta_1 \beta_2)} \left[ \left( \tau - \xi \right) + \frac{9}{4} \sqrt{\tau-\xi} + \frac{1}{2} \left( 7 \tau - 7 \xi - 5 \right) e^{(\tau-\xi)} - \frac{1}{2} \left( 7 \tau - 7 \xi - \frac{11}{2} \right) e^{(\tau-\xi)} \text{erf} \sqrt{\tau-\xi} \right] \]

\[ + \frac{\beta}{2(\beta+\beta_1 \beta_2)} \left[ \left( \tau - \xi \right) + \left( \tau - \xi \right)^2 \right] H(\tau-\xi) + \frac{\beta}{2(\beta+\beta_1 \beta_2)} e^{(\tau-\xi)} \text{erf} \sqrt{\tau-\xi} + \frac{2}{2} \left( \tau - \xi \right) H(\tau-\xi) \] \]

(3.32)

\[ h_3(\xi', \tau, \varepsilon, \tau_0) = \frac{\theta}{\tau_0} \left[ \beta_2 \frac{\beta_2}{\beta+\beta_1 \beta_2} e^{(\tau-\xi')} \text{erf} \sqrt{\tau-\xi'} H(\tau-\xi') - \frac{\beta_2}{2(\beta+\beta_1 \beta_2)} \left( 2(\tau-\xi') \right) \right] \]

\[ + \left[ 1 - 2(\tau-\xi')^2 \right] e^{(\tau-\xi')} \text{erf} \sqrt{\tau-\xi'} H(\tau-\xi') + \tau_0 \left[ \frac{\beta_2}{2(\beta+\beta_1 \beta_2)} \left( \frac{1}{\sqrt{\pi}} \right) \right] \]

\[ - 2(\tau-\xi') e^{(\tau-\xi')} \text{erf} \sqrt{\tau-\xi'} + 2 \sqrt{\beta_2/\pi} H(\tau-\xi') \] \]

(3.33)

where the functions \( f_1(\xi, \tau), i=1,2,3,4,5 \) are given by

\[ f_1(\xi, \tau) = \frac{\tau}{2} \left[ e^{-\xi \text{erfc}(\frac{\xi}{\sqrt{\tau}})} + e^{\xi \text{erfc}(\frac{\xi}{\sqrt{\tau}})} \right] \]
\[ f_2(\xi, \tau) = \frac{e^\xi}{2} \left[ e^{-\xi} \text{erfc}(\frac{\tau}{2\sqrt{\tau}} - \sqrt{\tau}) - e^{\xi} \text{erfc}(\frac{\tau}{2\sqrt{\tau}} + \sqrt{\tau}) \right] \]

\[ f_3^I(\xi, \tau) = \text{erfc}(\frac{\xi}{2\sqrt{\tau}}) + 2\sqrt{\tau} e^{-\frac{\tau^2}{4\tau}} e^{(2\tau^2-\xi) \text{erfc}(\frac{\tau}{2\sqrt{\tau}} - \sqrt{\tau})} \]

\[ f_3^{II}(\xi, \tau) = \text{erfc}(\frac{\xi}{2\sqrt{\tau}}) - 2\sqrt{\tau} e^{-\frac{\tau^2}{4\tau}} e^{(2\tau^2+\xi) \text{erfc}(\frac{\tau}{2\sqrt{\tau}} + \sqrt{\tau})} \]

\[ f_4(\xi, \tau) = \int_0^\tau e^{\tau} \left[ 2\sqrt{\tau} e^{-\frac{\tau^2}{4\tau}} + [2(\tau^2-\xi) \text{erfc}(\frac{\tau}{2\sqrt{\tau}} - \sqrt{\tau}) \right] \text{erfc}(\frac{\tau}{2\sqrt{\tau}} - \sqrt{\tau}) \right] \text{d}m \]

\[ f_5(\xi, \tau) = \int_0^\tau e^{\tau} \left[ 2\sqrt{\tau} e^{-\frac{\tau^2}{4\tau}} - [2(\tau^2+\xi) \text{erfc}(\frac{\tau}{2\sqrt{\tau}} + \sqrt{\tau}) \right] \text{d}m \]

where \( \text{erf} \) and \( \text{erfc} \) denote the error function and complementary error function respectively.

4. NUMERICAL RESULT.

The surface stress is given by

\[
- \frac{T_{11}^o}{\frac{\epsilon H_3}{0 \pi \beta_3} \beta_2} = e^\tau (1-\text{erf} \sqrt{\tau} - \frac{\tau}{2(1+\beta_3)} (-2 \tau \sqrt{\frac{\tau}{\pi}} + (1-2\tau^2) e^\tau (1-\text{erf} \sqrt{\tau}) \}
\]

\[
- \frac{T_{11}^o}{\frac{\epsilon H_3}{0 \pi \beta_3} \beta_2} = \frac{1}{2\sqrt{\pi} \tau} \tau e^\tau (1-\text{erf} \sqrt{\tau} + \sqrt{\frac{\tau}{\pi}}) \]

where \( \beta_3 = \frac{\beta_1 \beta_2}{\beta} \).

If there is no coupling between the electromagnetic field and strain field, \( H_3 = 0, \beta_2 = 0, \beta_3 > 0 \) and \( \beta \) is finite so that \( T_{11}^o = 0 \) on \( \xi = a_0 \).

In presence of the electromagnetic field and strain field, the surface stress is given by

\[
- \frac{T_{11}^o}{\frac{\epsilon H_3}{0 \pi \beta_3} \beta_2} = X(\tau, \xi, \xi') \]

where \( X(\tau, \xi, \xi') \) is a function of \( \tau, \xi, \xi' \).
We can assume \( \beta_3 \ll 1 \) since \( c \gg 1 \) and \( a_0 \) and \( C_0 \) are finite. We take \( \beta_3 = 0.05 \).

For numerical calculation we take the material of the half-space to be copper for which \( c = 0.0168 \). If we assume that a representative value of the relaxation time \( \tau_0 \) is \( 10^{-11} \) (see [7]), then the non-dimensional thermal wave speed in copper should be approximately equal to 0.66.

Then \( \tau_0' = 2.3 \) (for thermal properties and sound wave speed in copper, see ref. [8]).

Surface stress \( X \) for various values of times \( \tau \) are exhibited in the following table and also graphically represented.

<table>
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<th>( \tau )</th>
<th>( 10X )</th>
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<tr>
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</tr>
<tr>
<td>1.5</td>
<td>7.7</td>
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<tr>
<td>2.0</td>
<td>11.8</td>
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<tr>
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<td>16.5</td>
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<td>23.2</td>
</tr>
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<td>4.0</td>
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<tr>
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<td>28.3</td>
</tr>
<tr>
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<td>30.0</td>
</tr>
<tr>
<td>6.0</td>
<td>31.5</td>
</tr>
</tbody>
</table>

Surface stress \( 10X \) decreases as time \( \tau \) increases. One division corresponds to 0.5.
REFERENCES


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<td>May 1, 2009</td>
</tr>
<tr>
<td>First Round of Reviews</td>
<td>August 1, 2009</td>
</tr>
<tr>
<td>Publication Date</td>
<td>November 1, 2009</td>
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</tbody>
</table>

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