REVIEW of the book

Lessons in Geometry I Plane Geometry

by

Jacques Hadamard

Translated from the French by Mark Saul

Published by American Mathematical Society and Education Development Center in 2008. This is a translation of the 13th. edition. This book resulted (in 1898) from the need of textbooks, resources for the teaching of Mathematics. Gaston Darboux was named as editor of the set of these textbooks and he commissioned Jacques Hadamard to write the material for Geometry since he had taught on the high school level. Hadamard saw this work as important as he revised it twelve times. So, we come to this edition that was first published in 1947 and furnished the source of the translation of the present book.

The Hadamard’s vision of Geometry is remarkably fresh, even after 100 years; the classical (axiomatic) approach is permanently balanced with the changes that appeared in the teaching of Geometry. Nowadays, especially in working with the beginners, we tend to relay, more and more, on the practice and intuition rather than on abstract (formal) arguments, even if the last are more precise and correct. In other words, the so called ”pure” mathema-
tics becomes - at least regarding the teaching methods - applied mathematics. On the other hand, Hadamard is too experienced (he lived till the age of 97) as a mathematician and as a mathematical ideas teacher to forget the importance of the "Euclidean methods" as he called them. Once made the first steps in studying the Geometry (Euclidean Geometry), we must return to the axiomatic model and revisit those early starts and complete them. So, in the first chapter, the one devoted to angles, we can see a "scholastic" exposition in the Euclid's rigorous logic tradition. In the same tradition are written: most of the Book I, especially Chapter III, Note A (On the Methods of Geometry), Note B (On Euclid's Postulate), Note D, (On the Notion of Area), and many other smaller parts one can see everywhere in the book. The book contains: Translators Preface, four Authors Prefaces (to the 1st, 2nd, 8th, and 12th editions), Introduction (containing some basic definitions and notations), four Books, three Notes, Miscellaneous Problems, an Appendix devoted to Malfattis Problem and no Bibliography, references being made as footnotes.

- Book I "On The Straight Line" with seven chapters. After each chapter Hadamard proposes a set of exercises; at the end of each Book there are also presented some (more complicated) exercises (we will return below to this important issue).

- Book II "On the Circle" contains six chapters the last one being "On the Motion of Figures".

- Book III "On Similarity" having also seven chapters and other seven chapters grouped in "Complements to Book III". The reason for this is that the material from the Complements: Cross Ratio, Poles and Polars with re-
pect to a circle, Inversions, Cyclic Quadrilaterals, Peaucelliers Inverter, etc is far away from the Euclidean spirit. This Part has the same construction as a Book and we do not know the reasons why is this not a Book for itself.

- Book IV ”On Areas” has ”only” four chapters but Note D completes it.

- Note A ”On the Methods of Geometry” is the place where J. Hadamard can be seen as a Master of mathematical exposition.

- Note B ”On Euclids Postulate” needs no more explanations.

- Note C ”On the Problem of Tangent Circles” shows us that Gergonnes method for finding the circles tangent to three given circles does not apply in all possible cases and is presented a complete solution.

- Note D ”On the Notion of Area” is completing the Book IV solving the the logical fault of ”defining” the notion of area in Euclidean setting.

- The part called ”Miscellaneous Problems” contains eighty (more difficult) problems, some of them being taken from various contests.

- Appendix ”On Malfatti’s Problem”. Here is presented a general solution of this famous problem first solved by Steiner.

The book is very easy to read since every single paragraph, exercise, theorem etc. has assigned a number. Number 1 belongs to the Introduction and is a definition of volume and the number 422 ends the ”Miscellaneous Problems”.

In Hadamard’s opinion exercises are very important. Increasing the number of exercises as much as possible was the only rule that guided the Author in this work as he said in the Preface of the first edition. The problems (more than 450 of them) are well-suited to exploration using the
modern tools of dynamic geometry software. For this reason, the present edition includes a CD of dynamic solutions to select problems, created using Texas Instruments’ TI-Nspire[trademark] Learning Software. The TI-Nspire[trademark] documents demonstrate connections among problems and will allow the reader to explore and interact with Hadamard’s Geometry in new ways. In the same modern style the publishers posted, at http://www.ams.org/bookpages/mbk-57/List-of-errors.pdf, a list of errors.

In conclusion, the Hadamards book is surprisingly modern even after 100 years have passed and is a rich source of mathematical and pedagogical ideas with a special emphasis on the solving of problems as the best way to study Geometry (and not only). Even if the book was originally intended for the use of the pre-college teachers it is useful as well to gifted high school students and college students, in particular, to people interested in geometry from different standpoints.

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